

Optimising Group Sequential and Adaptive Designs:

Where Frequentist meets Bayes

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Group Sequential and Adaptive Designs

Group sequential and adaptive clinical trial designs have been proposed for a number of important applications:

Early stopping for efficacy or futility,

Sample size modification,

Treatment selection and testing (seamless Phase 2/3 trials),

Population selection and testing (enrichment designs).

There are usually options to choose from within such a design.

How should one make such choices and assess the end result?

Choosing a group sequential or adaptive design

A Phase 3 trial must protect the type I error rate.

This can be a complex problem when testing multiple null hypotheses — type I error rate must be controlled over a high-dimensional region.

We wish to be efficient, gaining high power with low sample size.

How should we make decisions:

At interim analyses?

At the final analysis?

Type I error rate is a frequentist property.

But Bayesian methods have advantages when optimising a design.

Outline of talk

1. Monitoring clinical trials

Group sequential stopping rules

Optimising the stopping boundary

2. Seamless Phase 2/3 designs

Designs that protect family-wise error rate,

Optimising decision rules and sample size allocation.

3. Enrichment designs

Adaptive enrichment in response to interim data.

Optimising the decision rule for when to enrich.

4. Conclusions

1. A group sequential clinical trial

Consider a Phase 3 clinical trial comparing a new treatment against a standard.

Let θ denote the “effect size”, a measure of the improvement in the new treatment over the standard.

We shall test the null hypothesis $H_0: \theta \leq 0$ against $\theta > 0$.

Rejecting H_0 allows us to conclude the new treatment is superior.

We allow type I error probability α for rejecting H_0 when it is true.

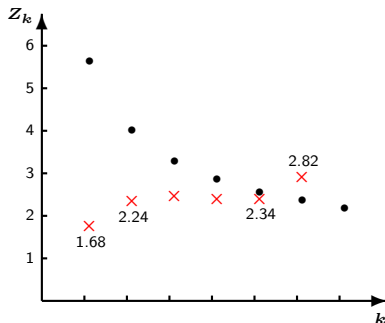
We specify power $1 - \beta$ as the probability of rejecting H_0 when $\theta = \delta$. Here δ is, typically, the minimal clinically significant treatment difference.

The trial design, including the method of analysis and stopping rule, must be set up to attain these error rates.

An early example: The BHAT trial

DeMets et al. (*Cont. Clin. Trials*, 1984) report on the Beta-Blocker Heart Attack Trial, that compared propranolol with placebo.

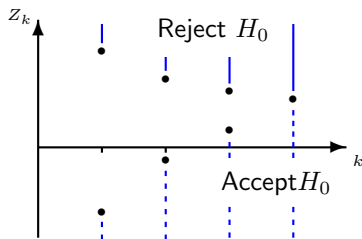
An “O’Brien and Fleming” stopping boundary was defined with overall type I error probability 0.025.



The trial stopped after the 6th of 7 planned analyses.

Group sequential tests: Stopping for futility

Adding a lower boundary allows stopping when there is little chance of a positive conclusion.



Rosner & Tsiatis (*Statistics in Medicine*, 1989) carried out retrospective analyses of 72 cancer studies of the U.S. Eastern Co-operative Oncology Group.

Had group sequential stopping rules been applied, early stopping (mostly to accept H_0) would have occurred in $\sim 80\%$ of cases.

Requirements for clinical trial designs

We seek designs which:

Achieve specified type I error rate and power,

Stop early, on average, under key parameter values,

Can be applied to a variety of response types.

We shall present distribution theory which shows that a common set of methods can be applied to many data types.

To define efficient tests, we shall formulate and solve an optimal stopping problem.

Sequential distribution theory

Let $\hat{\theta}_k$ denote the estimate of the treatment effect θ at analysis k .

Information for θ at analysis k is $\mathcal{I}_k = \{\text{Var}(\hat{\theta}_k)\}^{-1}$, $k = 1, \dots, K$.

Canonical joint distribution of $\hat{\theta}_1, \dots, \hat{\theta}_K$

In many situations, $\hat{\theta}_1, \dots, \hat{\theta}_K$ are approximately multivariate normal,

$$\hat{\theta}_k \sim N(\theta, \{\mathcal{I}_k\}^{-1}), \quad k = 1, \dots, K,$$

and

$$\text{Cov}(\hat{\theta}_{k_1}, \hat{\theta}_{k_2}) = \text{Var}(\hat{\theta}_{k_2}) = \{\mathcal{I}_{k_2}\}^{-1} \quad \text{for } k_1 < k_2.$$

References:

Jennison & Turnbull, *JASA*, 1997,

Scharfstein et al, *JASA*, 1997.

An optimal stopping problem

Consider a trial designed to test $H_0: \theta \leq 0$ vs $\theta > 0$, with:

Type I error rate α ,

Power $1 - \beta$ at $\theta = \delta$,

Up to K analyses.

A fixed sample test needs information

$$\mathcal{I}_{fix} = \{\Phi^{-1}(\alpha) + \Phi^{-1}(\beta)\}^2 / \delta^2.$$

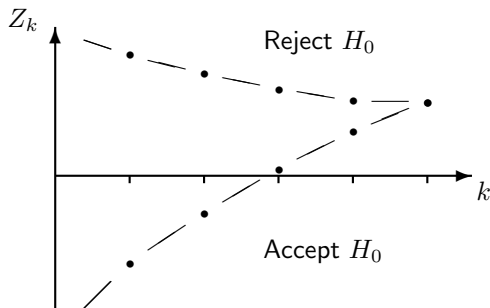
We set the maximum information to be

$$\mathcal{I}_{max} = R \mathcal{I}_{fix},$$

where $R > 1$, with equal increments between analyses.

Optimal group sequential tests

The error rates impose two constraints on the $2K - 1$ boundary points — leaving a high dimensional space of possible boundaries.



We shall look for a boundary that minimises

$$\{E_0(\mathcal{I}) + E_\delta(\mathcal{I})\}/2.$$

Finding optimal group sequential tests

We want a group sequential test of $H_0: \theta \leq 0$ vs $\theta > 0$ with

$$Pr_{\theta=0}\{\text{Reject } H_0\} = \alpha,$$

$$Pr_{\theta=\delta}\{\text{Accept } H_0\} = \beta,$$

Analyses at $\mathcal{I}_k = (k/K) \mathcal{I}_{max}$, $k = 1, \dots, K$,

Minimum possible value of $\{E_0(\mathcal{I}) + E_\delta(\mathcal{I})\}/2$.

We deal with constraints on error rates by introducing Lagrangian multipliers to create the *unconstrained problem* of minimising

$$\{E_0(\mathcal{I}) + E_\delta(\mathcal{I})\}/2 + \lambda_1 Pr_{\theta=0}\{\text{Reject } H_0\} + \lambda_2 Pr_{\theta=\delta}\{\text{Accept } H_0\}.$$

We shall find a pair of multipliers (λ_1, λ_2) such that the solution has type I and II error rates α and β , then this design will solve the *constrained problem* too.

Bayesian interpretation of the Lagrangian approach

Suppose we put a prior on θ with $Pr\{\theta = 0\} = Pr\{\theta = \delta\} = 0.5$ and specify costs of

- 1 per unit of information observed,
- $2\lambda_1$ for rejecting H_0 when $\theta = 0$,
- $2\lambda_2$ for accepting H_0 when $\theta = \delta$.

Then, the total Bayes risk is

$$\{E_0(\mathcal{I}) + E_\delta(\mathcal{I})\}/2 + \lambda_1 Pr_{\theta=0}\{\text{Reject } H_0\} + \lambda_2 Pr_{\theta=\delta}\{\text{Accept } H_0\},$$

just as in the Lagrangian problem.

An advantage of the Bayes interpretation is that it can give insight into solving the problem by using “Dynamic Programming” or “Backwards Induction”.

Solution by Dynamic Programming

Denote the posterior distribution of θ given $Z_k = z_k$ at analysis k by

$$p^{(k)}(\theta|z_k), \quad \theta = 0, \delta.$$

At the final analysis, K

There is no further sampling cost, so compare decisions

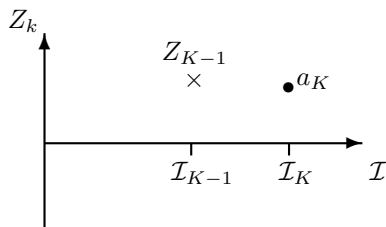
$$\text{Reject } H_0: \quad E(\text{Cost}) = 2 \lambda_1 p^{(K)}(0|z_K),$$

$$\text{Accept } H_0: \quad E(\text{Cost}) = 2 \lambda_2 p^{(K)}(\delta|z_K).$$

The boundary point a_K is the value of z_K where these expected losses are equal.

The optimum decision rule is to reject H_0 for $Z_K > a_K$.

At analysis $K - 1$



If the trial stops at this analysis, there is no further cost of sampling and the expected additional cost is

$$\text{Reject } H_0: \quad 2 \lambda_1 p^{(K-1)}(0|z_{K-1}),$$

$$\text{Accept } H_0: \quad 2 \lambda_2 p^{(K-1)}(\delta|z_{K-1}).$$

At analysis $K - 1$

If the trial continues to analysis K , the expected additional cost is

$$\begin{aligned} & 1 \times (\mathcal{I}_K - \mathcal{I}_{K-1}) \\ & + 2 \lambda_1 p^{(K-1)}(0|z_{K-1}) Pr_{\theta=0}\{Z_K > a_K | Z_{K-1} = z_{K-1}\} \\ & + 2 \lambda_2 p^{(K-1)}(\delta|z_{K-1}) Pr_{\theta=\delta}\{Z_K < a_K | Z_{K-1} = z_{K-1}\}. \end{aligned}$$

We can now define the optimal boundary points:

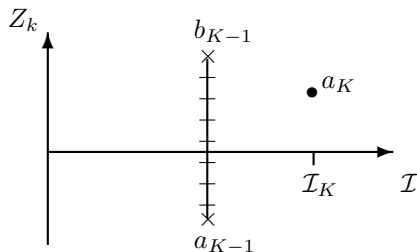
Set b_{K-1} to be the value of z_{K-1} where

$$E(\text{Cost of continuing}) = E(\text{Cost of stopping to reject } H_0).$$

Set a_{K-1} to be the value of z_{K-1} where

$$E(\text{Cost of continuing}) = E(\text{Cost of stopping to accept } H_0).$$

At analysis $K - 1$



Before leaving analysis $K - 1$, we set up a grid of points for use in numerical integration over the range a_{K-1} to b_{K-1} .

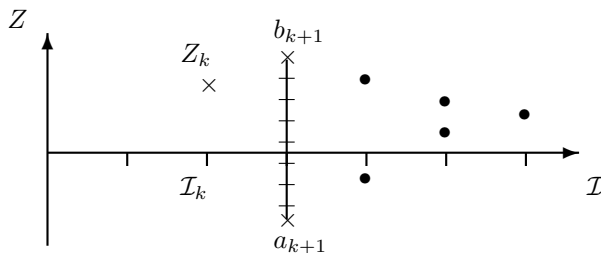
For each point, we sum over the posterior distribution of θ to calculate

$$\beta^{(K-1)}(z_{K-1}) = E(\text{Additional cost when continuing} \mid Z_{K-1} = z_{K-1}).$$

We are now ready to move back to analysis $K - 2$.

Analyses 1 to $K - 2$

We work back through analyses $k = K - 2, K - 3, \dots, 1$.



At each analysis, we find the optimal stopping boundary using knowledge of the optimal stopping rule at future analyses.

Then, for a grid of values of z_k , compute

$$\beta^{(k)}(z_k) = E(\text{Additional cost when continuing} \mid Z_k = z_k)$$

to use in evaluating the option of continuing at analysis $k - 1$.

Solving the original problem

For any given (λ_1, λ_2) we can find the Bayes optimal design and compute its type I and II error rates.

We now search for a pair (λ_1, λ_2) for which type I and type II error rates of the optimal design equal α and β , respectively.

The resulting design will be the optimal group sequential test, with the specified frequentist error rates, for our original problem.

Notes

1. The method of solving the overall problem demonstrates explicitly that good frequentist procedures should be similar to Bayes procedures.
2. The prior and costs in the final Bayes problem are a means to an end, rather than “true” costs of type I and type II errors, or costs of treating patients in the trial.

Properties of optimal designs

Tests with $\alpha = 0.025$, $1 - \beta = 0.9$, K analyses, $\mathcal{I}_{max} = R\mathcal{I}_{fix}$, equal group sizes, minimising $\{E_0(\mathcal{I}) + E_\delta(\mathcal{I})\}/2$.

Minimum values of $\{E_0(\mathcal{I}) + E_\delta(\mathcal{I})\}/2$, as a percentage of \mathcal{I}_{fix}

K	R					<i>Minimum over R</i>
	1.01	1.05	1.1	1.2	1.3	
2	80.8	74.7	73.2	73.7	75.8	73.0 at $R=1.13$
5	72.2	65.2	62.2	59.8	59.0	58.8 at $R=1.38$
10	69.2	62.2	59.0	56.3	55.1	54.2 at $R=1.6$
20	67.8	60.6	57.5	54.6	53.3	51.7 at $R=1.8$

Observe: $E(\mathcal{I}) \searrow$ as $K \nearrow$ but with diminishing returns,
 $E(\mathcal{I}) \searrow$ as $R \nearrow$ up to a point.

Solutions can be obtained for a variety of related problems:

- Other optimality criteria such as a weighted sum

$$\sum_i w_i E_{\theta_i}(\mathcal{I})$$

or an integral

$$\int f(\theta) E_{\theta}(\mathcal{I}) d\theta$$

- Optimising a set of fixed group sizes in a group sequential test
- Data dependent group sizes in a group sequential test
- Group sequential tests for a delayed response
- Testing for either superiority or non-inferiority

2. Optimising a Phase 2/3 seamless design

During Phase 2 and Phase 3 of the drug development process,

The final decision is made on the treatment specification, including the dose level,

The selected treatment is tested against control.

A seamless Phase 2/3 trial design combines these two phases:

In stage 1 (Phase 2)

Compare K “treatments” against control

Select the best treatment and, if it has performed sufficiently well, proceed to stage 2.

In stage 2 (Phase 3)

Compare the selected treatment against the control.

Optimising a Phase 2/3 seamless design

After both stages are completed, we test the null hypothesis that the selected treatment is no better than the control.

Since this treatment was selected based on data that will also be used in the final analysis, care must be taken to avoid inflating the overall type I error rate.

Design issues

We would like to optimise:

- 1 The way in which data on all treatments are combined in the final hypothesis test,
- 2 The way in which the total sample size is divided between the two stages.

Research Article

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Optimizing the data combination rule for seamless phase II/III clinical trials

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We consider seamless phase II/III clinical trials that compare K treatments with a common control in phase II then test the most promising treatment against control in phase III. The final hypothesis test for the selected treatment can use data from both phases, subject to controlling the familywise type I error rate. We show that the choice of method for conducting the final hypothesis test has a substantial impact on the power to demonstrate that an effective treatment is superior to control. To understand these differences in power, we derive decision rules maximizing power for particular configurations of treatment effects. A rule with such an optimal frequentist property is found as the solution to a multivariate Bayes decision problem. The optimal rules that we derive depend on the assumed configuration of treatment means. However, we are able to identify two decision rules with robust efficiency: a rule using a weighted average of the phase II and phase III data on the selected treatment and control, and a closed testing procedure using an inverse normal combination rule and a Dunnett test for intersection hypotheses. For the first of these rules, we find the optimal division of a given total sample size between phases II and III. We also assess the value of using phase II data in the final analysis and find that for many plausible scenarios, between 50% and 70% of the phase II numbers on the selected treatment and control would need to be added to the phase III sample size in order to achieve the same increase in power. © 2014 The Authors. *Statistics in Medicine* published by John Wiley & Sons Ltd.

Keywords: Bayes decision problem; combination test; closed testing procedure; multiple hypothesis testing; seamless phase II/III trial; treatment selection

Optimising a Phase 2/3 seamless design

Denote the K treatment effects vs control by $\theta_1, \dots, \theta_K$.

Stage 1

Randomise m_1 subjects to each of the K treatments and the control and observe their responses.

Denote the estimated treatment effects by $\hat{\theta}_{1,i}$, $i = 1, \dots, K$.

Treatment i^* with the highest $\hat{\theta}_{1,i}$ is selected for stage 2.

Stage 2

Treatment i^* is compared against control, with m_2 observations on each. The estimated treatment effect is $\hat{\theta}_{2,i^*}$.

Conclusion

A final decision is made, based on $\hat{\theta}_{1,1}, \dots, \hat{\theta}_{1,K}$ and $\hat{\theta}_{2,i^*}$.

Optimising a Phase 2/3 seamless design

There are K null hypotheses, $H_i: \theta_i \leq 0, i = 1, \dots, K$.

If dose i^* is selected for Phase 3, we focus on testing $H_{i^*}: \theta_{i^*} \leq 0$.

Family-wise error

We want strong control of the **family-wise error** rate. Then, for all vectors $\theta = (\theta_1, \dots, \theta_K)$,

$$Pr_{\theta}\{\text{Reject any true } H_i\} \leq \alpha.$$

Power

When some θ_i are greater than zero, we can define power as

$$Pr\{\text{Select treatment } j \text{ with maximum } \theta_i \text{ and reject } H_j: \theta_j \leq 0\}.$$

More generally, we can define a gain function or utility that is positive when H_{i^*} is rejected, whichever treatment is selected, but the gain increases with θ_{i^*} .

Optimising a Phase 2/3 seamless design

Family-wise error can be controlled by a Closed Testing Procedure:

Define level α tests are defined for each null hypothesis H_i ,
and for all intersections of sets of null hypotheses.

Reject H_i overall if all intersection hypotheses that include H_i
are rejected.

Theory implies the family-wise type I error rate is at most α .

Each hypothesis test can be formed as a Combination Test across
the two stages of the trial (Bauer & Köhne, *Biometrics*, 1984).

How should we test the intersection hypotheses in stage 1?

What type of combination test is best?

The best choice may depend on the K -dimensional parameter θ .

Optimising a Phase 2/3 seamless design

Hampson & Jennison (*Statistics in Medicine*, 2013) found optimal final decision rules that maximise power when $\theta = \delta v$, for various choices of vector v .

Two procedures were close to 100% efficient across a wide range of scenarios.

1. In the framework we have described, use a Dunnett test for each intersection hypothesis in stage 1 and combine Z values across stages with a weighted normal combination test.
2. Use the procedure proposed by Thall, Simon and Ellenberg (*Biometrika*, 1988).

The very best design does depend on the high-dimensional, treatment effect vector θ .

However, since we have such robustly efficient procedures, we do not need to consider Bayesian averaging over θ .

Optimising a Phase 2/3 seamless design

Hampson & Jennison also considered how best to divide a total sample size between stage 1 (m_1 observations on K treatments and control) and stage 2 (m_2 on selected treatment and control).

The choice that maximises power depends on the vector of treatment effects, θ , in particular, the largest effect $\max_i(\theta_i)$.

If $\max_i(\theta_i)$ is fairly small, a high stage 2 sample size, m_2 , is needed to give adequate power in that stage.

If $\max_i(\theta_i)$ is large, a lower m_2 may suffice and a higher m_1 increases the probability of selecting the best treatment in stage 1.

Bayesian averaging:

We do not know θ .

So, we express our expectations as a distribution for θ and choose a design with good properties averaged over this distribution.

Optimal Stage 1 group sizes in a seamless design

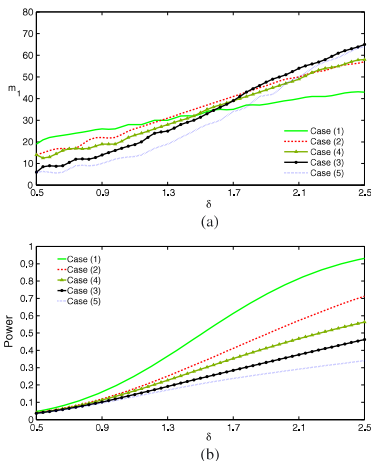


Figure 4. (a) Stage 1 group sizes maximizing the power of the TSE procedure when the total sample size is fixed at 448 and θ is a random permutation (1) of $(0, 0, 0, 0, 1)\delta$, (2) of $(0.5, 0.5, 0.5, 0.5, 1)\delta$, (3) of $(0.75, 0.75, 0.75, 0.75, 1)\delta$, (4) of $(0.3, 0.475, 0.65, 0.825, 1)\delta$ and (5) of $(0.75, 0.8125, 0.875, 0.9375, 1)\delta$. (b) Power achieved by the optimized TSE procedures. Decision rules are listed in order of decreasing power. Designs are specified with $K = 5$, $\ell = 0$, $\sigma = 5.0$ and $\alpha = 0.025$. Results are based on 1 million simulations for each scenario.

Controlling the frequentist type I error rate

Use of a closed testing procedure (CTP) and combination test guarantees control of type I error.

Optimising within this class of designs

We can (very nearly) optimise the choice of CTP and combination test for all treatment effect vectors θ simultaneously.

However, the best choice of sample sizes in stage 1 and stage 2 does depend on the vector θ .

The Bayes solution is to specify a prior distribution for the unknown θ and optimise performance integrated over this distribution.

An outer layer

If the optimised value of m_1 leads to unacceptably low average power, consider a higher total sample size for the two stages.

3. Creating an efficient enrichment design

Consider a drug designed to disrupt a disease's biological pathway. Patients with high levels of a biomarker for this pathway should gain particular benefit.

In a clinical trial with **enrichment** we

Start by comparing the new treatment against control in the full population.

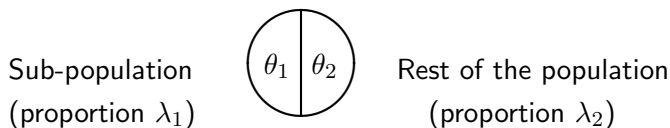
At an interim analysis, we decide whether to:

Continue recruiting from the full population,

Recruit only from the subgroup — and increase their numbers.

Results may support a licence for the full population or just for the sub-population.

Creating an efficient enrichment design



The treatment effect (difference in mean response between new treatment and control) is θ_1 in the sub-population and θ_2 in the complement of this sub-population.

The treatment effect over the full population is $\theta_3 = \lambda_1\theta_1 + \lambda_2\theta_2$.

We may wish to test either or both of:

The null hypothesis for the full population, $H_3: \theta_3 \leq 0$ vs $\theta_3 > 0$,

The null hypothesis for the sub-population, $H_1: \theta_1 \leq 0$ vs $\theta_1 > 0$.

Creating an efficient enrichment design

As in the adaptive seamless Phase 2/3 design, we want to control strongly the **family-wise error** rate.

Then, for all values of θ_1 and θ_3 ,

$$Pr_{\theta}\{\text{Reject any true } H_i\} \leq \alpha.$$

This can be achieved by a Closed Testing Procedure, involving level α tests of H_1 , H_3 and the intersection hypothesis $H_1 \cap H_3$.

Each of these tests will be constructed as a Combination Test across the two stages of the trial.

Then, general theory implies that the family-wise type I error rate is controlled at level α .

This leaves freedom to define the rule for deciding whether or not to enrich at the interim analysis.

Creating an efficient enrichment design

I have worked on this problem with Thomas Burnett.

We chose to use Simes' test for the intersection hypothesis $H_1 \cap H_3$ and an inverse normal combination test.

We specified a utility or “gain function” to optimise:

$$\text{Gain} = \lambda_1 \theta_1 \mathcal{I}(\text{Reject } H_1 \text{ only}) + \theta_3 \mathcal{I}(\text{Reject } H_3).$$

We placed a prior distribution on (θ_1, θ_2) .

We then sought the adaptive decision rule that maximises the expected gain.

Given observed treatment effects, $\hat{\theta}_1$ and $\hat{\theta}_2$, at the interim analysis, the optimal decision (to continue in the full population or to enrich in the sub-population) is that which maximises the **conditional** expected gain.

Example: An optimal enrichment design

Consider a trial with total sample size that would provide power 0.9 to detect a treatment effect in the full population if $\theta_1 = \theta_2 = 10$.

Suppose $\lambda_1 = \lambda_2 = 0.5$.

Our prior distribution for (θ_1, θ_2) is bivariate normal with

$$E(\theta_1, \theta_2) = (15, 2)$$

and

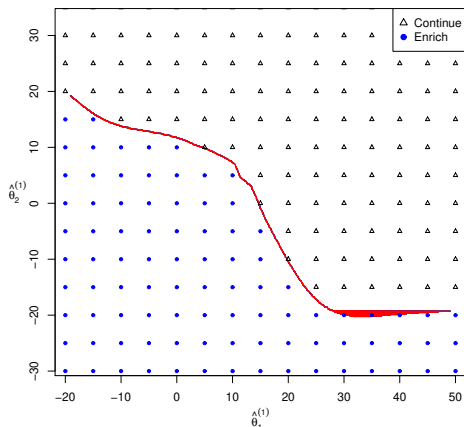
$$\text{Var}(\theta_1, \theta_2) = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}.$$

We conduct an interim analysis after half the total number of subjects have been observed.

If the decision is to “enrich”, all the remaining sample size is allocated to the sub-population.

Example: An optimal enrichment design

The optimal decision rule is:



The peculiar shape of the boundary reflects features of the Simes test applied to data at the interim analysis.

Example: An optimal enrichment design

Properties of the optimised enrichment design:

$$Pr(\text{Enrich}) = 0.41$$

$$Pr(\text{Reject } H_1 \text{ only}) = 0.38$$

$$Pr(\text{Reject } H_3) = 0.55$$

$$E(\text{Gain}) = 7.73$$

The design with no enrichment that tests both H_1 and H_3 has

$$E(\text{Gain}) = 7.55$$

The design that recruits all subjects from the sub-population from the outset, and only tests H_1 , has

$$E(\text{Gain}) = 7.41$$

Creating an efficient enrichment design

We have found examples of the gain function and prior for which the best adaptive design is superior to both simple, non-adaptive designs — but this is not always the case.

However, adaptive enrichment may have additional appeal:

If investigators differ in their prior beliefs, an optimal adaptive design for a “consensus” prior may be broadly acceptable.

An optimal design that recruits only from the sub-population may be deemed too restrictive by some investigators — and the adaptive approach allows a slower route towards this end.

When the optimal policy is to recruit from the full population (so no enrichment occurs and combination tests are not needed), the optimal adaptive design's $E(\text{Gain})$ is only slightly sub-optimal.

Controlling the frequentist type I error rate

Use of a closed testing procedure and combination tests guarantees control of family-wise type I error.

Optimising within this class of designs

Given gain and cost functions, and a prior distribution for (θ_1, θ_2) , we can compute Bayes-optimal adaptive enrichment designs.

An outer layer

Other design features that merit investigation include:

- Details of the closed testing procedure and combination tests.

- The timing of the interim analysis.

- Preferential sampling of one population when the proportions λ_1 and λ_2 are away from 0.5.

4. Overall conclusions

Controlling the frequentist type I error rate

We can apply closed testing procedures and combination tests to protect family-wise error in complex, high-dimensional settings.

We can then work on optimising other aspects of a given design.

Optimising within a class of designs

Before trying to optimise, we need to understand which properties of a design are important to the investigators.

Typically, this is done through the elicitation of their gain function, cost function, and prior distribution for unknown parameters.

Then, we can optimise by analysis, calculation or simulation.

An outer layer

Once we can optimise the central component of a design, we may re-visit higher level aspects and question initial assumptions.