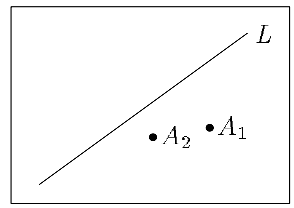
1: Experimenting with Reflections.

1. Practice finding reflections.

Draw a line L and a point A_1 not too far away. Find the reflection of A_1 in L and call it B_1. Use a ruler to check that the half-way point from A_1 to B_1 lies on the line L. Now draw another point A_2 and find its reflection B_2. Draw the line through A_1 and A_2 and the line through B_1 and B_2. You should discover that they meet on L. Why do you think this is true?

Choose the line and points with care to keep everything on the page. One option is something like this:



2. Verify Alice's observation.

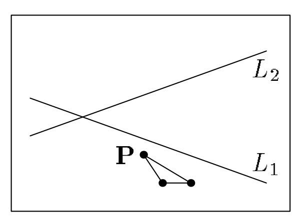
First draw a circle. Choose three points on the circle and draw the triangle with these three vertices. Draw the edges of the triangle so that they extend some way beyond the circle. Choose a fourth point A_0 (for Alice's original position) also lying on the circle and find the three reflections A_1, A_2, A_3 of A_0 in the three sides of the triangle. Try to draw a straight line through these three reflections.

You may find it helps to try one or two smaller rough drawings, using a ruler to estimate the positions of the reflections, before doing a larger accurate drawing.

3. A diversion : the effect of two reflections.

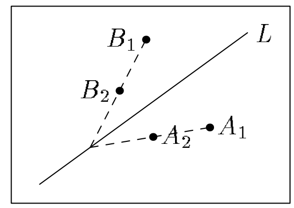
Draw two lines L_1 and L_2 and a small irregular triangle $\mathbf{P}$ with vertices P_1, P_2, P_3. Find the reflection $\mathbf{Q}$ of $\mathbf{P}$ in the first line L_1, by reflecting the three vertices P_1, P_2, P_3 to get the new vertices Q_1, Q_2, Q_3. Now reflect $\mathbf{Q}$ in the second line L_2 to get a triangle $\mathbf{R}$. How is $\mathbf{R}$ related to $\mathbf{P}$? What one operation gives the same result as these two reflections? To help you see, put the point of the compass at the intersection of L_1 and L_2 and draw the circular arcs through P_1, P_2 and P_3. What do you think would happen if the two lines L_1 and L_2 were parallel? (Try it.)

Again you need to take care not to fall off the page. Try starting from something like this:



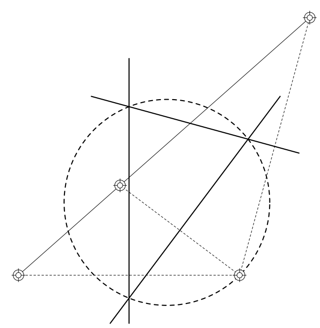
1: Experimenting with Reflections: Teacher's Notes.

1. The final picture might look like this: Reflection takes every point on A_1A_2 to a corresponding point on B_1B_2. The point where A_1A_2 meets L is fixed by the reflection, so is also on B_1B_2

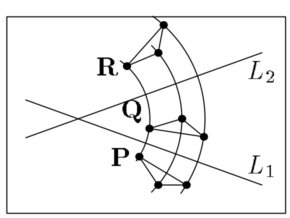


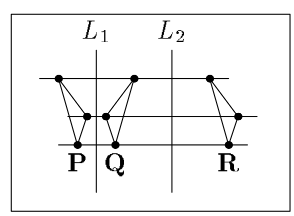
In practice, it is difficult to avoid small errors in making geometric drawings, so the two lines A_1A_2 and B_1B_2 may not meet exactly on L, but this should give the pupils an idea of what degree of accuracy to expect.

2. As always, one needs to find a size and configuration for the drawing that is nicely spaced, but stays on the page. The example shown on the overhead works OK, but I hope we will end up with a wide variety of pictures to make the result more convincing.



3. The effect of two reflections is a rotation through twice the angle between the two lines. If the two lines are parallel the effect is a translation through twice the distance between the two lines.





This exercise does not need to be completed at this stage. It is intended partly as extra drawing practice, but also to illustrate the fundamental role of reflections in plane geometry. It also shows how a translation is a limiting case of a rotation, or how a straight line is a limiting case of a circle.

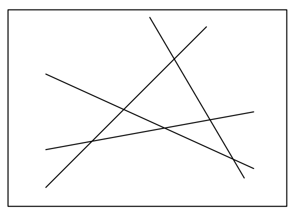
2: Circles and Triangles.

4. The circumcircle.

Draw a triangle and construct its circumcircle. To do this, draw the perpendicular bisectors of each of the three sides of the triangle. The point where these three bisectors meet is the centre of the circumcircle. Place your compass point here and adjust it to draw a circle starting at one of the vertices. (Before you draw the circle, move the compass off the page to check that it goes through the other two vertices of the triangle: you can make small adjustments to get a best fit.)

5. Four lines and four circles.

Now draw four lines, making sure that all six intersection points are on the page. Here is a typical arrangement of lines that will just keep everything on the page.

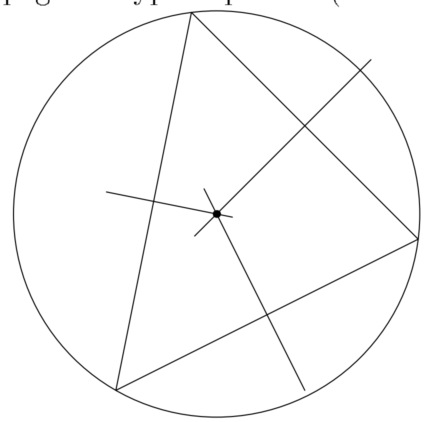


Forgetting each line in turn will give you a triangle for which you should construct the circumcircle, as above. (This will involve a lot of drawing, so you should try to draw the intermediate parts of the construction, such as the perpendicular bisectors of the sides, as lightly as possible, or even rub them out after they have been used.) When your have finished you should notice a remarkable thing: all four circumcircles pass through one point.

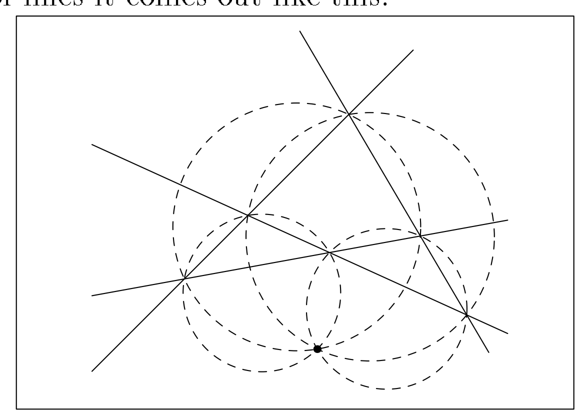
The existence of this remarkable point is the theorem of geometry that Alice discovered during her ballet practice. Conveniently there was a second movable mirror in the practice room. Can you think how she discovered the theorem?

2: Circles and Triangles: Teacher's Notes.

4. The one danger is that, if the triangle is too narrow, then the circumcircle will be too large for the page. A typical picture (where this doesn't happen) is:



5. Keeping everything on the page can be quite difficult here. For the suggested arrangement of lines it comes out like this.



As I will explain in the summing up, Alice found the remarkable point by trying to get her four reflections in the four mirrors to line up. When this happens, any three of the four points are lined up, so she is standing on any of the four circumcircles.