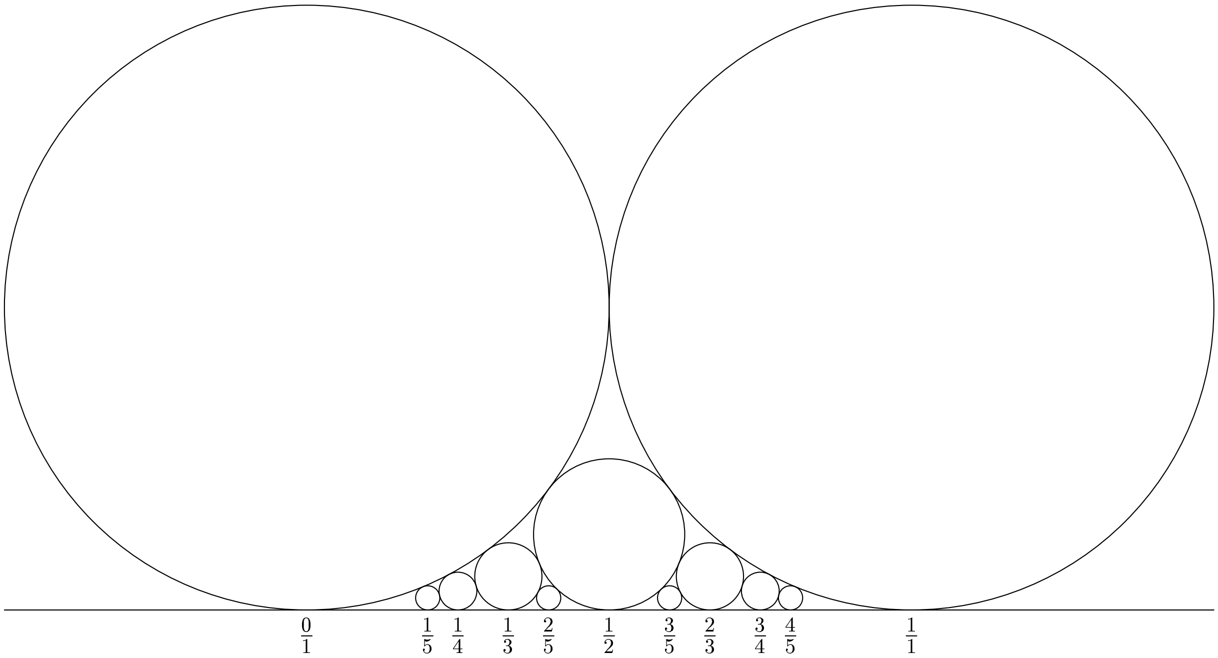
# Farey Circles




The Farey circle associated to a fraction $p/q$, which *must* be written in lowest terms, is the circle of diameter $1/q^2$ which touches the horizontal axis at the point $p/q$. These circles are non-overlapping and the two circles associated to $p/q > r/s$ touch precisely when

$$\det \begin{pmatrix} p & r \\ q & s \end{pmatrix} = ps - rq = 1.$$

Furthermore, given two such touching circles, there is a unique circle which is below and touching both of them: the circle associated to

$$\frac{p}{q} \boxplus \frac{r}{s} = \frac{p+r}{q+s}.$$

Thus we can generate the circles recursively in a way that enables one to see that the vertical line through any irrational point must meet infinitely many circles, i.e. every irrational $\lambda$ satisfies

$$\left| \lambda - \frac{p}{q} \right| < \frac{1}{2q^2}$$

for infinitely many fractions $p/q$. The fractions which occur here are convergents in the continued fraction expansion for $\lambda$, but not all convergents occur.

ADK, 11 Dec 97