SimpleIntegrands

February 12, 2018

Setting up iPython

```
In [6]: import numpy as np
        import matplotlib
        import matplotlib.pyplot as plt
        from pylab import *
```

We choose an integrand as (say) a function of W_t, which we will approximate by Simple functions, and compute the integral.

```
In [7]: #Normal Increments
        noiseT = lambda N,T: np.random.normal(0,sqrt(T/float(N)),N)

        # Number of time steps:
        N = 100000
        t = np.linspace(0,1,N+1) # time discretisation
        # append a 0 to the start of the noise vector, and compute the cumulative sum:
        W = cumsum(np.hstack((arange(1),noiseT(N,1))))

        # Function defining integrand:
        f = lambda x: x

        ff = np.vectorize(f)
```

Compute the integral $\int_0^1 f(W_s)\,\mathrm{d}W_s$, by approximating $f(W_s)$ at intervals of size Δt.

```
In [8]: def Ito(W,f,Dt):
                "This computes the Ito integral of W against itself"
                M = W.size
                NN = int(np.ceil(Dt*M))
                f_vec = np.vectorize(f)
                I1 = f(W[0:(M+1):NN])
                I1 = I1.repeat(NN)
                I1 = I1[0:M]
                I2 = I1[0:(M-1)]*np.diff(W[0:(M+1)])
                I2 = concatenate((range(0,1),cumsum(I2)))
                return I1, I2
```

Plot the resulting processes and the integral:

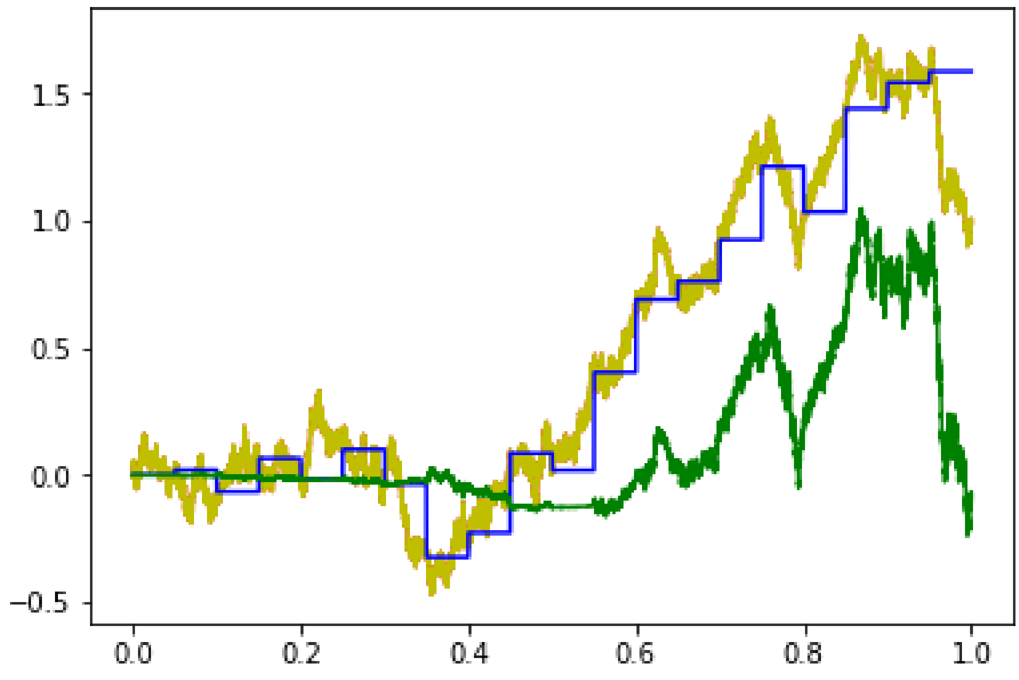
In [11]:
```
%matplotlib inline

Simp, Integral = Ito(W,f,0.05)

fig = plt.figure()
axes = fig.add_axes([0.1, 0.1, 0.8, 0.8])

axes.plot(t, W , 'r')
axes.plot(t, ff(W), 'y')
axes.plot(t, Simp, 'b')
axes.plot(t, Integral, 'g')
```

Out[11]:
```
[<matplotlib.lines.Line2D at 0x7f98b72cae80>]
```



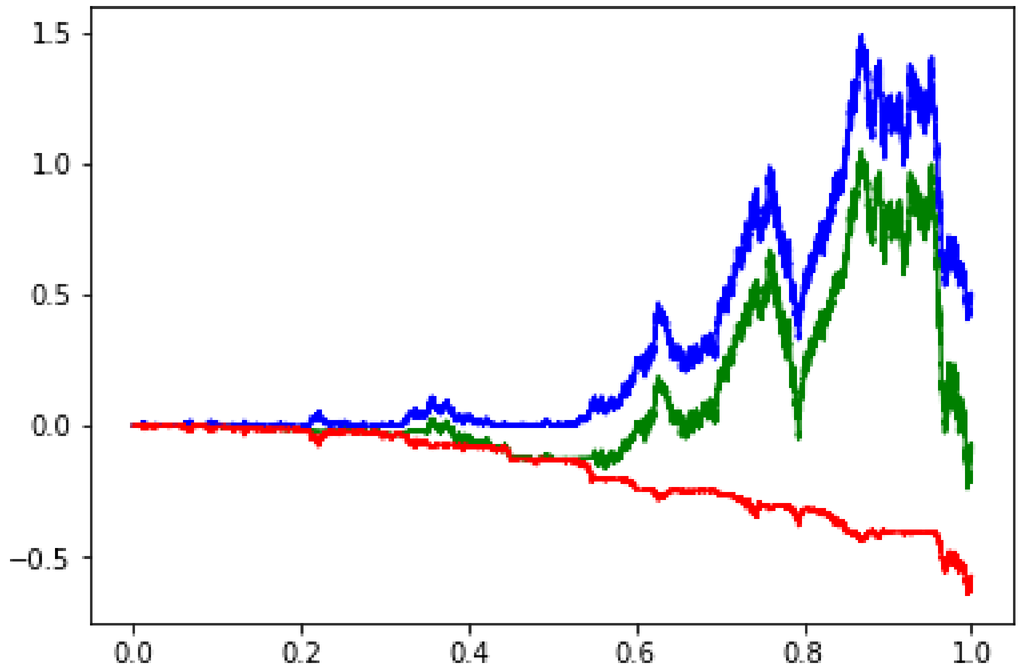
Let's compare the resulting integral with a candidate solution:

In [12]:
```
g = lambda x: x*x/2.0
gg = np.vectorize(g)

fig = plt.figure()
axes = fig.add_axes([0.1, 0.1, 0.8, 0.8])

axes.plot(t, Integral, 'g')
axes.plot(t, gg(W), 'b')
axes.plot(t,Integral-gg(W),'r')
```

Out[12]: [<matplotlib.lines.Line2D at 0x7f98b7220e10>]



What we expect to see (assuming we've chosen g correctly) is that for small time-steps, the approximation is good, and the red line is zero (or at least, a nice function of t).