Introduction to control theory

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Purpose of lecture

1. Aims: To give a brief outline of mathematical control theory, its history and context
2. Target audience: students, assuming little background in the discipline
3. Hopefully: interesting, informative, useful for rest of week

- Some structure and ideas based on those in *Control Engineering the Hidden Technology*, by K. Åström, Lund University.

What's in a name?

- Control theory
- Control engineering
- Systems and control theory
- *Mathematical* control theory
- *Mathematical* ...
- May have different meanings to experts, but here can all take as roughly the same.

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Systems and control theory

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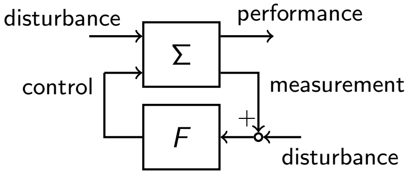

- Control theory (engineering) focusses on the analysis and design (synthesis) of controllers — feedback, optimal or otherwise — in causal dynamical systems to achieve a desired outcome.
- Systems theory is the mathematical framework for (inter)connecting dynamical objects.
- Combined systems & control theory is *the* mathematical language for describing and abstracting feedback.

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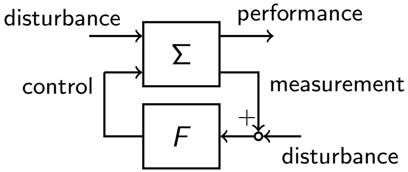

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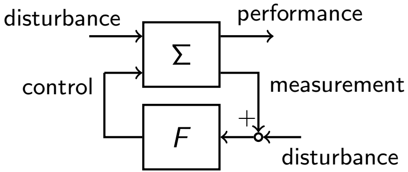

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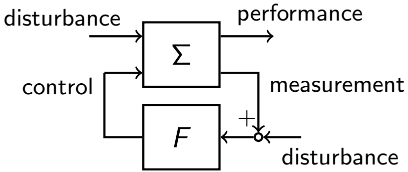

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Mathematical control theory

- For example

$$\left.\begin{aligned} \dot{x} &= f(x, u, d_1), \\ z &= g(x), \\ y &= h(x), \\ \dot{u} &= k(y, u, d_2), \end{aligned}\right\} \qquad \begin{aligned} &d_j \text{ disturbances, } x \text{ state, } u \text{ control,} \\ &y \text{ measurement, } z \text{ performance.} \end{aligned}$$

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The field of mathematical control theory concerns itself with the basic theoretical principles underlying the analysis of feedback and the design of control systems. It differs from the more classical study of dynamical systems in its emphasis on inputs (or controls) and outputs (or measurements). Linearized analysis of systems is the basic foundation of most practical control engineering and has been phenomenally successful.

Foreword by E. Sontag to *Stability and Stabilization of Nonlinear Systems* by I. Karafyllis and Z.-P. Jiang, Springer, 2011

Feedback is everywhere — including biology

Feedback is a central feature of life. The process of feedback governs how we grow, respond to stress and challenge, and regulate factors such as body temperature, blood pressure, and cholesterol level. The mechanisms operate at every level, from the interaction of proteins in cells to the interaction of organisms in complex ecologies.

The Way Life Works, M. B. Hoagland and B. Dodson, Times Books 1995

Feedback is everywhere — including evolution

Natural selection

In natural selection, those variations in the genotype that increase an organisms chances of survival and procreation are preserved and multiplied from generation to generation at the expense of less advantageous ones. Evolution often occurs as a consequence of this process.

Encyclopaedia Britannica

The action of [natural selection] is exactly like that of the centrifugal governor of the steam engine, which checks and corrects any irregularities almost before they become evident; and in like manner no unbalanced deficiency in the animal kingdom can ever reach any conspicuous magnitude, because it would make itself felt at the very first step, by rendering existence difficult and extinction almost sure soon to follow.

On the Tendency of Varieties to Depart Indefinitely From the Original Type, A. Wallace 1858

A very brief history

- Roots can be traced back to the industrial revolution
- Centrifugal governor, James Watt 1788
- Used extensively in industrial process control, 19th century
- Telecommunications, signal processing, Nyquist, Bode, 1930s
- Flight control, 20th century
  Quiz: What year did this appear in the New York Times: *Robot Piloted Plane makes Safe Crossing of the Atlantic No hands on controls from Newfoundland to Oxfordshire: Take-Off, Flight and Landing are fully Automatic.*
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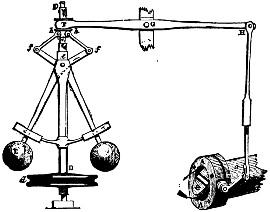

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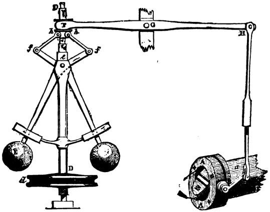

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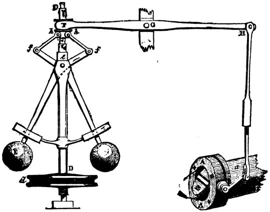

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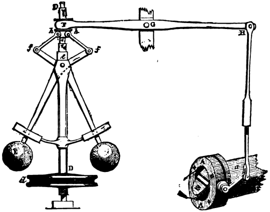

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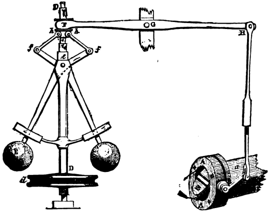
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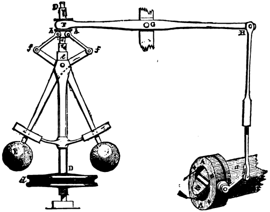

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Today

- EPSRC: *Control engineering is critical to the success of numerous engineering applications and underpins areas including power electronics, smart grids, wind turbines, aerospace, automotive, chemical processing, robotics, and manufacturing*

- According to Åström it is *the Hidden Technology*
  - ✓ Widely used
  - ✓ Very successful
  - ✗ Seldom talked about
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Image from Ibex73

Three examples: 1) PID control

- Given reference r, measurement y, we compute input u via

$$u = k_P(y(t) - r) + k_I \int_0^t (y(s) - r)\, ds + k_D \dot{y}(t)\,,$$

for parameters k_P, k_I and k_D.

- See video here
- Some "optimal" results known: minimise

$$\int_0^\infty \langle y(t), Qy(t)\rangle + \langle u(t), Ru(t)\rangle\, dt\,,$$

for positive definite R and Q, subject to

$$\dot{x} = Ax + Bu, \quad x(0) = x_0, \quad y = Cx\,,$$

over all $u \in L^2$ such that $x(t) \to 0$ as $t \to \infty$.

- Solution is given by $u = -R^{-1}B^T Px$, where P is a certain matrix solution of

$$A^T P + PA + C^T QC - PBR^{-1}B^T P = 0\,.$$

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Three examples: 2) Pontryagin minimum principle

- Assume that x is given by

$$\dot{x} = f(x, u), \quad x(0) = x_0, \quad u(t) \in \mathcal{U}, \quad t \in [0, t_f]\,, \tag{1}$$

  where t_f is the (fixed) final time, $\mathcal{U}$ is the set of admissible controls.

- Objective is to choose u to minimise

$$J = \Psi(x(t_f)) + \int_0^{t_f} L(x(t), u(t))\, dt \quad \text{subject to (1).}$$

- Key object is the Hamiltonian H given by $H(\xi, v, p) := p^T f(\xi, v) + L(\xi, v)$
- Pontryagin's minimum principle states gives a necessary condition optimality: an optimal trajectory x^*, u^* and λ^* must satisfy
  (1) Minimise H: $H(x^*, u^*, \lambda^*) \leq H(x^*, u, \lambda^*)$ for all admissible u
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Three examples: 3) Filtering, estimation, prediction

- Goal: given

$$\dot{x} = Ax + Bv, \quad x(0) = x_0, \quad z = C_1 x, \quad y = C_2 x + Dv\,,$$

determine "optimal" estimate $\hat{x}$ of x.

- Under assumptions, the estimator

$$\hat{x}' = A\hat{x} + L(y - C_2\hat{x}), \quad \hat{x}(0) = \hat{x}_0, \quad \text{with} \quad L = PC_2^T(DD^T)^{-1}\,,$$

and where $P = P^T$ is a certain solution of

$$AP + PA^T - PC_2^T(DD^T)^{-1}C_2P + BB^T = 0\,,$$

minimises

$$\sum_{k=1}^{m} \left(\int_0^\infty \|z(s) - C_1\hat{x}(s)\|^2\, ds \;:\; v = e_k\delta \right),$$

- The above estimator is the celebrated Kalman-Bucy filter (1960,61), originally introduced in a stochastic framework for recursive state estimation in stochastic systems.

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- Under assumptions, the estimator

$$\hat{x}' = A\hat{x} + L(y - C_2\hat{x}), \quad \hat{x}(0) = \hat{x}_0, \quad \text{with} \quad L = PC_2^T(DD^T)^{-1}\,,$$

and where $P = P^T$ is a certain solution of

$$AP + PA^T - PC_2^T(DD^T)^{-1}C_2P + BB^T = 0\,,$$

minimises

$$\sum_{k=1}^{m}\left(\int_0^\infty \|z(s) - C_1\hat{x}(s)\|^2\,ds \;:\; v = e_k\delta\right)\,,$$

- The above estimator is the celebrated Kalman-Bucy filter (1960,61), originally introduced in a stochastic framework for recursive state estimation in stochastic systems.

Three examples: 3) Filtering, estimation, prediction

- Goal: given

$$\dot{x} = Ax + Bv, \quad x(0) = x_0, \quad z = C_1 x, \quad y = C_2 x + Dv\,,$$

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Research at Bath

- Part of the analysis research group
- Myself, Mark Opmeer, Hartmut Logemann, and PhD students
- Current research includes
  - ▷ Systems theoretic properties of infinite-dimensional systems

$$\dot{x} = Ax + Bu, \quad y = Cx$$

    (abstract Cauchy problem). Also through frequency domain approach

$$\mathbb{C} \ni s \mapsto \mathbf{G}(s) = C(sI - A)^{-1}B \in \mathcal{L}(U, Y),$$

  - ▷ Stability properties of *nonlinear* control systems

$$\dot{x} = f(x, u)$$

  - ▷ Collaborations with engineering
  - ▷ Applications of systems and control to biology and ecology
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