High-Velocity Estimates, Inverse Scattering and Topological Effects

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Abstract

In this lectures I will present results in high-velocity (or high-energy) estimates for solutions to Schrödinger equations with time-dependent methods. By time-dependent I mean that I consider wave packets, not time-periodic solutions as is the case in stationary methods. Stationary methods are a very powerful tool, but they have limitations because with these methods the localization in space of the solutions is lost and this important in many problems like scattering for N-body quantum mechanical systems where the configuration space is very large and the physical wave packet is localized is a well defined region of space at each time. Another case is topological effects in quantum mechanics, like the Aharonov-Bohm effect, where charged particles are scattered by a torodial magnet that contains a magnetic field inside and it it important to know is the wave packet (the beam) passes inside the hole of the magnet or outside it.

I will introduce this method in the simple case of potential scattering for the Schrödinger equation and I will consider the inverse problem of uniquely reconstructing the potential from the high-velocity limit of the scattering operator. Then, I will study the Aharonov-Bohm effect in three dimensions with a toroidal magnet (handle bodies) and I will present quantitave results obtained by a mixed analytical-numerical technique that proves that quantum mechanics predicts the Aharonov-Bohm effect, under the experimental conditions of the celebrated experiments of Tonomura et al., that gave a solid evidence of the existence of this effect. Furthermore, I will discuss scattering for the Schrödinger equations in two dimensions with long-range magnetic potentials, in particular the Aharonov-Bohm effect for an infinite solenoid. This will take us to the limit of the validity of the physical interpretation of scattering theory.

References

[1] M. Ballesteros, R. Weder, High-velocity estimates for the scattering operator and Aharonov-Bohm effect in three dimensions, Comm. Math. Phys. **285** (2009) 345-398.

[2] M. Ballesteros, R. Weder, The AharonovBohm effect and Tonomura et al. experiments: Rigorous results, J. Math. Phys. **50** (2009) 122108, 54 pp.

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For a review up to 2014 of results obtained with these methods in quantum mechanics, for nonlinear Schrödinger and Klein-Gordon equations, in scattering by black-hole metrics, etc. see,

[5] R. Weder, High-Velocity Estimates, Inverse Scattering and Topological Effects, *Spectral Theory and Differential Equations V. A. Marchenko's 90th Anniversary Collection*, pp.225-251. E. Khruslov, L. Pastur, D. Shepelsky, editors, American Mathematical Society, Translations Series 2, **233** Providence, 2014.

Recent results related to these lectures

[6] M. Ballesteros, R. Weder, High-velocity estimates for Schrödinger operators in two dimensions: Long-range magnetic potentials and time-dependent inverse scattering, Reviews in Mathematical Physics **27** (2015) 1550006, 54 pp.

[7] M. Ballesteros, R. Weder, Aharonov-Bohm Effect and High-Momenta Inverse Scattering for the Klein-Gordon Equation, Annals Henri Poincaré, published online (2016) DOI 10.1007/s00023-016-0466-9, 47 pp.

[8] M. Ballesteros, R. Weder, High-Momenta Estimates for the Klein-Gordon Equation: Long-Range Magnetic Potentials and Time-Dependent Inverse Scattering, J. Phys. A: Math. Theor. **49** (2016), 26 pp.

Lecture Notes

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