A NEW APPROACH TO STUDY STRONG ADVECTION PROBLEMS.

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ABSTRACT. In this talk, I shall be attempting to give an overview of a new weak convergence type tool developed by myself, Thomas Holding (Warwick) and Jeffrey Rauch (Michigan) to handle multiple scales in advection-diffusion type models used in the turbulent diffusion theories. Loosely speaking, our strategy is to recast the advection-diffusion equation in moving coordinates dictated by the flow associated with a mean advective field. Crucial to our analysis is the introduction of a fast time variable. We introduce a notion of "convergence along mean flows" which is a weak multiple scales type convergence – in the spirit of two-scale convergence theory. We have used ideas from the theory of "homogenization structures" developed by G. Nguetseng. We give a sufficient structural condition on the "Jacobain matrix" associated with the flow of the mean advective field which guarantees the homogenization of the original advection-diffusion problem as the microscopic lengthscale vanishes. We also show the robustness of this structural condition by giving an example where the failure of such a structural assumption leads to a degenerate limit behaviour. More details on this new tool in homogenization theory can be found in the following paper:

T. Holding, H. Hutridurga, J. Rauch. Convergence along mean flows, in press SIAM J Math. Anal., arXiv e-print: arXiv:1603.00424, (2016).

In a sequel to the above mentioned work, we are preparing a work where we address the growth in the Jacobain matrix – termed as Lagrangian stretching in Fluid dynamics literature – and its consequences on the vanishing microscopic lengthscale limit. To this effect, we introduce a new kind of multiple scales convergence in weighted Lebesgue spaces. This helps us recover some results in Freidlin-Wentzell theory.

This talk aims to present both these aspects of our work in an unified manner.

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