Graeme Milton (University of Utah)

Exact relations for Green's functions in linear PDE and boundary field equalities: a generalization of conservation laws

Many physical equations have the form $J(x) = L(x)E(x) - h(x)$ with source $h(x)$ and fields E and J satisfying differential constraints, symbolized by $E \in \mathcal{E}$ and $J \in \mathcal{J}$ where $\mathcal{E}$, and $\mathcal{J}$ are orthogonal spaces. We show that if $L(x)$ takes values in certain nonlinear manifolds $\mathcal{M}$, and coercivity and boundedness conditions hold, then the infinite body Greens function (fundamental solution) satisfies exact identities. The theory also links Greens functions of different problems. The analysis is based on the theory of exact relations for composites, as developed by Grabovsky, Milton, and Sage, but without assumptions about the length scales of variations in $L(x)$,and more general equations, such as for waves in lossy media, are allowed. For bodies, inside which $L(x) \in \mathcal{M}$, the Dirichlet-to-Neumann map giving the response also satisfies exact relations. These boundary field equalities generalize the notion of conservation laws: the field inside satisfies certain constraints that leave a wide choice in these fields, but which give identities satisfied by the boundary fields, and moreover provide constraints on the fields inside the body. This is joint work with Daniel Onofrei. The paper is available at Res. Math. Sci. (2019) 6:19, https://doi.org/10.1007/s40687-019-0179-z