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Recent progress in the shell buckling theory

In has been known that the rigidity of a shell (for instance under compression) is closely related to the optimal Korn's constant in the nonlinear first Korn's inequality (geometric rigidity estimate) for $W^{1,2}$ fields under the appropriate Dirichlet type boundary conditions (arising from the nature of the compression). In their recent work, Frisecke, James and Mueller (2002, 2006) derived a geometric rigidity estimate for plates, which gave rise to a derivation of a hierarchy of plate theories for different scaling regimes of the elastic energy depending on the thickness of the plate. FJM-type theories have been derived by Gamma-convergence and rely on L^p compactness arguments and of course the underlying nonlinear Korn's inequality. While the rigidity of plates has been understood almost completely, the rigidity, in particular the buckling, of shells is much less well understood. This was due to the luck of rigidity estimates and L^p compactness as understood by Grabovsky and Harutyunyan (2014) for cylindrical shells. In the case of shells, when there is enough rigidity, is has been understood that actually the linear first Korn's inequality can replace the nonlinear one, Frisecke, James Mueller (2002), Grabovsky, Truskinovsky (2007). The important mathematical question is: What makes the shells more rigid than plates and how can one compare the rigidity of two different shells? In this talk we give the answer to that question by classifying shells according to the Gaussian and principal curvatures. We derive sharp first Korn's inequalities for shells of zero, positive and negative Gaussian curvature. It turns out, that for zero Gaussian curvature (one principal curvature zero, the other one never zero) the amount of rigidity is $h^{1.5}$, for negative Gaussian curvature it is $h^{4/3}$ and for positive Gaussian curvature it is h, i.e., the positive Gaussian curvature shell is the most rigid one. Here h is the shell thickness. All three exponents are completely new in ever appeared geometric rigidity estimates, however the exponents 4/3 and 1 appear in the engineering work of Tovstik and Smirnov (2000) in a different context. This is partially joint work with Yury Grabovsky (Temple University)