The resolvent method for shear waves spectra calculation in 2D phononic crystals: dispersion equation, displacement and traction wave fields

Maria Korotyaeva

FEMTO-ST
15B Avenue des Montboucons
25030 Besanon cedex France
E-mail: korotiaeva@yahoo.de

Abstract

We propose the resolvent method for calculating the horizontally polarized shear waves spectra in the 2D phononic crystal (PC) waveguides: the free PC plate and the PC plate sandwiched between two substrates.

Since the propagator $\mathbf{M}$ over a unit cell approximated by Fourier harmonics in one coordinate can have very large components, we introduce its resolvent $\mathbf{R} = (z\mathbf{I} - \mathbf{M})^{-1}$ (z is a complex number outside of spec$\mathbf{M}$) as a numerically stable substitute. Another two key tools given in terms of the resolvent, a spectral projector $\mathbf{P}_d$ and propagator $\mathbf{M}_d$ for the decreasing modes, come into play in the case of a waveguide with a substrate.

The resolvent method providing simple dispersion and wave field equations in terms of $\mathbf{R}$, $\mathbf{P}_d$ and $\mathbf{M}_d$ has several advantages. It is of a good precision due to the exact solution in one direction, computationally cheap due to the reduction of the problem to one unit cell even in a semi-infinite substrate, and versatile since it is applicable to uniform, 1D- or 2D-periodic structures. Moreover, it is extendible to P/SV waves and 3D PC.

In numerical examples, we model low-frequency band gaps and compare them for the mirror-symmetric and perturbed profiles. The displacement and traction wave fields are calculated for the waveguides with highly contrasting matrix/inclusions stiffness values which allows us to reveal the PC geometry.

Keywords: Phononic crystals, Guided waves, Propagator, Resolvent, Spectral projector