Research statement

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I first give an outline of my current research interests. This is followed by the previous research track record.

Research interests:

1. *Wave propagation; Spectral properties of differential operators emerging in linearised elasticity and electromagnetism.*

Here I am particularly interested in the effects of interaction of the lengthscale associated with the wave motion and the characteristic size of the heterogeneities. In the language of the operator theory, the related range of questions concerns the asymptotic behaviour of operator groups/semigroups with respect to a set of parameters (heterogeneity size, contrast between components in a composite *etc.*). This approach leads to two kinds of results:

a) Understanding "effective" characteristics of media in relation to the wave motion, *e.g.* the speed of the wave motion along a boundary ("surface wave") in a layered medium;

b) Considering special "critical" scalings between the parameters involved leads to new formulations that possess properties unattainable by homogeneous media, *e.g.* localised modes in photonic crystals).

2. *Multiscale analysis; Homogenisation of families of PDEs with non-uniformly elliptic coefficients, high-contrast media and their spectral properties.*

In the "stationary" framework (*i.e.* for equilibrium equations rather than equations of dynamics), the effective description of a composite can be viewed as a convergence proof for a family of operators in a suitable representation of the Hilbert space (linear PDEs), or for a family of integral functionals (non-quadratic, possibly non-convex) on a Banach space. The conventional notions of convergence ("G-convergence", "H-convergence", "Γ-convergence") are not sufficient (at least in the form they are known by) for control of problems involving strong scale interactions, or effects of macroscopic "resonances" at the microscale.

3. A. *Applications of the theory of dissipative operators to estimating convergence rates in homogenisation; Norm-resolvent estimates.* B. *Scattering theory; Effective descriptions of periodic composite media via asymptotics of their scattering matrix.* C. *The study of systems of Maxwell equations in dissipative media and the related spatial localisation effects ("cloaking").*

These are the themes of EPSRC Fellowship "Mathematical foundations of metamaterials: homogenisation, dissipation and operator theory", which I have been awarded for July 2014 – June 2019. This is an extensive programme of research involving 2 postdoctoral researchers (2 years each) as part of the team. (Collaborators: S. Cooper, A. Kiselev)

4. *Variational methods in homogenisation;* Γ*-convergence of functionals describing composite media in nonlinear elasticity.*

I have developed a technique of "two-scale Γ-convergence", which is convenient for understanding the effective behaviour of composites whose constituents are described by non-convex energy densities that allow a high degree of contrast between deformation gradients at different material points and/or in different directions. (Collaborators: M. Cherdantsev, S. Neukamm.) I have also carried out an analysis of the behaviour of heterogeneous plates in the "bending" regime for the scaling between elastic energy and plate thickness. I have shown that when these are small in comparison with the period of the composite, an additional constraint emerges in the effective description, which corresponds to the intuitive notion that the plate mid-surface is an isometric immersion at the miscroscale. (Collaborator: M. Cherdantsev)

5. *Metamaterials: negative refraction, transformation media, thin structures.*

This is a variety of approaches aimed at obtaining new differential equations that describe continuous media with constitutive parameters taking negative values (usually in a certain frequency interval). It has been a subject of intensive analytical effort recently to show that, say, periodic elastic composites with voids whose volume fraction is close to one, exhibit on the one hand some Cosserat-type properties, and on the other hand, have spectra with infinitely many band gaps opening at high frequencies. (Collaborator: S. Pastukhova). Another example involves a combination of a special parameter-dependent change of variables and a homogenisation procedure in order to produce a chiral effective medium. (Joint work with PhD student J. Evans.)

6. *Miscellaneous smaller projects in progress:*

a. Asymptotic techniques for the description of interfacial and shock waves in nonlinear conservation laws.

b. Threshold effects for Jacobi matrices and their m-functions. (Collaborator: L. Silva)

c. Higher-order constitutive laws for systems of Maxwell equations and for non-quadratic variational problems.

d. Two-scale analysis of equations of the Navier-Stokes type with high contrast. (Collaborator: V. Zhikov)

Previous research track record:

My research has revolved around analysis of differential equations since the time of my undergraduate and MSc studies at St. Petersburg State University and Steklov Mathematical Institute, which are parts of a world-leading St. Petersburg mathematical school. My MSc thesis (Supervisor: V. M. Babich), which was concerned with the asymptotic analysis of some problems relevant to diffraction, gave a rigorous mathematical framework to a formal procedure that has been used in calculating the wavefield around a conical surface. Following this I completed a PhD at the University of Bath (Supervisor: V. P. Smyshlyaev) on the rigorous analysis of some effects observed in experiments with plastically deformed polycrystals. The ideas underpinning my approach came from the mathematical theory of homogenisation. Conversely, the research I have carried out subsequently in homogenisation has been to some extent motivated by these earlier results. In particular, it has turned out that the multi-scale phenomena in question are related to the effective behaviour of non-uniformly elliptic families of periodic PDE, which has been a subject of special interest in the last ten years or so. Among the centers for mathematical analysis involved in the related research are (excluding my current collaborators): Bath (I. Graham, R. Scheichl), Bonn (D. Peterseim), Braunschweig (R. Hempel), Dortmund (B. Schweizer), École Polytechnique (G. Allaire, A. Piatnitski), New York (R. Kohn, M. Weinstein), Rennes (M. Briane), Texas A&M (Y. Efendiev, P. Kuchment), UCL (I. Kamotsky, V. Smyshlyaev).

My postdoctoral experience consisted in the work as Junior Research Fellow at the University of Oxford (St. John's College, 4 years) and as a postdoctoral researcher at DAMTP, University of Cambridge (EPSRC-funded project, 1 year). During this time I continued developing the avenues of research I started earlier, alongside incorporating some new material into my portfolio, particularly in the subjects of calculus of variations and the dislocation theory of plasticity, through interaction with research groups of J. M. Ball, J. R. Willis, V. S. Deshpande. My work during this time has helped me consolidate my research profile, both in terms of refining earlier results and in terms of generating ideas for my subsequent work. It was also a great source of academic links, both nationally and world-wide, in several subjects: PDE theory, mechanics, spectral theory, applied analysis and applied mathematics. At this time also wrote several papers on problems in wave propagation, in particular surface and interface waves, in the spirit of the St. Petersburg school of diffraction.

During the period 2006–2014 I have held a permanent appointment at Cardiff. This has been the time of setting up my research group on the basis of my previous work, as well as bringing into my research arsenal ideas from neighbouring subjects, those in which the department has happened to develop. I was a member of two research groups within Cardiff School of Mathematics: Analysis and Applied Mathematics. In my 7 years at Cardiff, both groups have benefited from a significant influx of new research staff, with expertise in spectral theory, nonlinear PDE, calculus of variations, asymptotic methods, numerical analysis. Both groups held weekly research seminars, and I have organised a departmental colloquium, which took place about 4 times a year.

Since July 2014, I am Reader in Mathematics and EPSRC Fellow at the University of Bath. My research group currently consists of two postdoctoral research associates (Y. Ershova and M. Waurick), an EPSRC Research Fellow (S. Cooper) and a PhD student (S. D'Onofrio). I also supervise two PhD students based at Cardiff University (J. Evans, finishing in 2016, and M. Lewis, finishing in 2018).

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. I maintain strong links with S. Cooper, who was a Leverhulme-supported postdoctoral fellow at Cardiff until April 2013 and is now an ERC-supported researcher in France.

The research groups in the UK and abroad that I actively collaborate with at the moment include: Analysis of PDE at Moscow State University (V. Zhikov, S. Pastukhova), Spectral Analysis at St. Petersburg State (A. Kiselev, S. Naboko), TU Dresden (S. Neukamm), University of Montpellier II (M. Bellieud), Applied Analysis at Imperial College London (R. Craster). The interests of several UK centres of analysis and applied mathematics (Bristol, Kent, Liverpool, Oxford, Reading, UCL) are in the vicinity of my research and I expect this relationship to grow stronger in the future. I have a good track record of regular exchange of ideas with all of them, which in the long run influences the research direction pursued by the wider academic community.

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