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On embedded eigenvalues for second-order elliptic operators in unbounded domains

This work has been done in collaboration with Christophe Hazard, Sonia Fliss and Antoine Tonnoir [1]. We proved a general lemma whose direct corollary is the absence of positive eignvalues embedded in the essential spectrum, for all operators which coincide with the Laplace operator in a non-convex conical domain of revolution.

More precisely, combining Fourier representations and analyticity arguments, we prove the absence of nonzero L^2 functions satisfying the Helmholtz equation in a conical domain of revolution $\mathcal{C}_\alpha \subset \mathbb{R}^N$, if the vertex angle α is strictly greater than π (with no particular prescription of boundary conditions).

This result has many important consequences:

- It implies that, the Dirichlet or Neumann Laplacian in $\Omega \subset \mathbb{R}^N$ has no eigenvalues in its essential spectrum $\mathbb{R}^+$, if Ω contains a conical domain $\mathcal{C}_\alpha$ with $\alpha > \pi$. From a physical point of view, it means that there are no trapped modes for the corresponding wave equation.
- The result can also be used for complex heterogeneous media with obstacles and for other boundary conditions, like Robin conditions: we prove that there are no trapped modes if all the perturbations are contained outside a conical domain $\mathcal{C}_\alpha$ with $\alpha > \pi$.
- Finally, the same conclusion can be directly deduced for isotropic Maxwell and Navier equations.

References

[1] Bonnet-BenDhia Anne-Sophie, Fliss Sonia, Hazard Christophe and Tonnoir Antoine , *A Rellich type theorem for the Helmholtz equation in a conical domain* , C. R. Acad. Sci. Paris, Ser. I, DOI: 10.1016/j.crma.2015.10.015, vol. 354, pp 27–32, (2016)