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On the connections between the spectral theory of quantum graphs and homogenisation problems in low-dimensional structures

We discuss two seemingly unrelated problems. The first one, coming from the spectral theory of quantum graphs, is the problem of convergence of Neumann Laplacians on "thin" domains shrinking to a graph to a corresponding second-order differential operator on this graph. The second one is the problem of critical contrast homogenisation in thin structures, which are ϵ-periodic metric graphs embedded in $\mathbb{R}^d$. We argue that in order to obtain norm-resolvent convergence to the homogenised operator in the second problem, it comes handy to use a number of techniques borrowed from the spectral theory of quantum graphs; moreover, we demonstrate that the homogenised operator itself can be deduced using some rather transparent geometrical operations on a correctly chosen graph. We then point out that the homogenised operator is in fact tightly related to the graph Laplacian solving the first problem above in a special case, that is, when the original "thin" domain converges to the corresponding graph in a special way, whereby the volumes of its "vertex" and "edge" parts are of the same order in ϵ. This seems to suggest an entirely new perspective for homogenisation problems, allowing to treat them in terms of resonant properties of associated "thin" structures.