Operators, Operator Families and Asymptotics

16–19 May 2016

Department of Mathematical Sciences, University of Bath

Programme of Talks

Monday 16 May

9:50–10:20 Arrival, collection of welcome packs, tea/coffee: **Level 1 Atrium**

10:20–10:30 Welcome **Wolfson Lecture Theatre**

10:30–11:30 Pavel Exner (Doppler Institute for Mathematical Physics and Applied Mathematics, Czech Academy of Sciences): Singular Schrödinger operators and Robin billiards: geometry, spectra and asymptotic expansions
11:30–11:45 Questions and changeover

11:45–12:45 Ian Wood (University of Kent, UK): Spectral information contained in abstract M-functions
12:45–12:55 Questions

12:55–14:00 Lunch: **Level 1 Atrium**

14:00–15:00 Rostyslav Hryniv (University of Rzeszów, Poland, and Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, National Academy of Sciences of Ukraine): Schrödinger operators with δ'-like potentials
15:00–15:10 Questions

15:10 – 15:30 Tea/coffee: **Level 1 Atrium**

15:30–16:30 Stefan Neukamm (TU Dresden, Germany): Stochastic homogenization: An estimate for the two-scale expansion for correlated coefficients
16:30–16:45 Questions and changeover

16:45–17:45 Jari Taskinen (University of Helsinki, Finland): Spectral gaps for elastic and piezoelectric waveguides
17:45–17:55 Questions

Tuesday 17 May

Wolfson Lecture Theatre:

9:40–10:40 Tanya Christinansen (University of Missouri, USA): Sharp lower bounds on a resonance counting function in even-dimensional Euclidean scattering
10:40–10:50 Questions

10:50 – 11:20 Tea/coffee: **Level 1 Atrium**

11:20–12:20 Zhongwei Shen (University of Kentucky, USA): Boundary regularity estimates in periodic homogenization
12:20–12:30 Questions

12:30–13:20 Lunch: **Level 1 Atrium**

13:20–14:20 Aaron Welters (Florida Institute of Technology, USA): Analyticity of the Dirichlet-to-Neumann map for Maxwell's equations in passive composite media
14:20–14:30 Questions

14:30 – 15:00 Tea/coffee: **Level 1 Atrium**

15:00–16:00 Tatiana Suslina (St. Petersburg State University, Russia): Spectral approach to homogenization of non-stationary Schrödinger-type equations
16:00–16:10 Questions

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Wednesday 18 May

Venue: Wolfson Lecture Theatre

9:40–10:40 Ricardo Weder (IIMAS-UNAM, Mexico): Limiting absorption principle for singular solutions to Maxwell equations and plasma heating
10:40–10:50 Questions

10:50 – 11:20 Tea/coffee: **Level 1 Atrium**

11:20–12:20 Yves Capdeboscq (University of Oxford, UK): Small volume asymptotics for Maxwell's equations
12:20–12:30 Questions

12:30–13:20 Lunch: **Level 1 Atrium**

13:20–14:20 Alexander Kiselev (National Pedagogical Dragomanov University, Ukraine): On the connections between the spectral theory of quantum graphs and homogenisation problems in low-dimensional structures
14:20–14:30 Questions

14:30 – 15:00 Tea/coffee: **Level 1 Atrium**

15:00–16:00 Giuseppe Cardone (University of Sannio, Italy): Boundary perturbations of planar waveguides: oscillating boundary, windows, non-periodic perforations
16:00–16:10 Questions and changeover

16:10–17:10 Shane Cooper (University of Bath, UK): Asymptotic analysis of stratified elastic media in the space of functions with bounded deformation
17:10–17:20 Questions

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Thursday 19 May

Venue: Room 4W1.2

10:00–11:00 Andrii Khrabustovskyi (Karlsruhe Institute of Technology, Germany): On the spectrum of a class of periodic quantum graphs
11:00–11:10 Questions

11:10 – 11:40 Tea/coffee: **Level 1 Atrium**

11:40–12:40 David Krejčiřík (Nuclear Physics Institute, Czech Academy of Sciences): Non-self-adjoint graphs
12:40–12:50 Questions

12:50–13:40 Lunch: **Level 1 Atrium**

13:40–14:40 Maxence Cassier (University of Utah, USA): On the spectral theory and limiting amplitude principle for a transmission problem between a dielectric and a metamaterial
14:40–14:50 Questions

End of Meeting

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Abstracts of Talks

Yves Capdeboscq (University of Oxford, UK): Small volume asymptotics for Maxwell's equations

In this talk, we shall discuss first-order asymptotics (in terms of volume fraction) for the behavior of solutions of the time-harmonic Maxwell system of equations in presence of inclusions (or defects). One typical difficulty is the control of the behavior of the resonances uniformly with respect to the small parameter (here, the volume fraction). This is traditionally done by means of operator theory and (collective) compact convergence. In this talk we show that new regularity estimate allow to provide a different, quantitative, approach.

This work is a collaboration with Giovanni S. Alberti (ETH Zurich)

Giuseppe Cardone (University of Sannio, Italy): Boundary perturbations of planar waveguides: oscillating boundary, windows, non-periodic perforations

We consider an elliptic operator in a planar infinite strip perturbed in different ways:

- By substituting one side of the boundary by a fast oscillating curve:

  We assume that both the period and the amplitude of the oscillations are small and impose the Dirichlet condition on the upper boundary and Dirichlet, Neumann or Robin boundary condition on the oscillating boundary. In all cases we describe the homogenized operator, establish the uniform resolvent convergence of the perturbed resolvent to the homogenized one, and prove the estimates for the rate of convergence. These results are obtained as the order of the amplitude of the oscillations is less, equal or greater than that of the period. It is shown that under homogenization the type of the boundary condition can change.

- By infinite numbers of "windows":

  We impose the Dirichlet condition on the upper boundary and frequent alternation boundary condition on the lower boundary. The alternation is introduced by the periodic partition of the boundary into small segments on which Dirichlet and Neumann (the "windows") conditions are imposed in turns. We study the cases when homogenization gives the Dirichlet or Neumann condition instead of the alternating ones, establish the uniform resolvent convergence and estimates for the rate of convergence, and analyse the spectrum of the perturbed operators.

- By a perforation by small holes along a curve:

  We impose mixed classical boundary conditions (Dirichlet, Neumann and Robin) on the holes. Assuming that the perforation is non-periodic and satisfies rather weak assumptions, we describe all possible homogenized problems. Our main result is the uniform resolvent convergence of the perturbed operator to a homogenized one in various operator norms and estimates for the rate of convergence. On the basis of the norm resolvent convergence, we prove the convergence of the spectrum.

These results are parts of joint work with D. Borisov, R. Bunoiu, T. Durante, L. Faella and C. Perugia.

Maxence Cassier (University of Utah, USA): On the spectral theory and limiting amplitude principle for a transmission problem between a dielectric and a metamaterial

In collaboration with Christophe Hazard (CNRS, France) and Patrick Joly (INRIA, France).

In this talk, we are interested in a transmission problem between a dielectric and a metamaterial. The question we consider is the following: Does the limiting amplitude principle hold in such a medium? This principle defines the stationary regime as the large-time asymptotic behavior of a system subject to a periodic excitation.

An answer is proposed here in the case of a two-layered medium composed of a dielectric and a particular metamaterial (Drude model). In this context, we reformulate the time-dependent Maxwell equations as a Schrödinger equation and perform its complete spectral analysis. This permits a quasi-explicit representation of the solution via the "generalized diagonalization" of the associated unbounded self-adjoint operator. As an application of this study, we show finally that the limiting amplitude principle holds except for a particular frequency, called the plasmonic frequency, characterised by a ratio of permittivities and permeabilities equal to -1 across the interface. This frequency is a resonance of the system and the response to this excitation blows up linearly in time.

Tanya Christinasen (University of Missouri, USA): Sharp lower bounds on a resonance counting function in even-dimensional Euclidean scattering

Mathematically, resonances may serve as a replacement for discrete spectral data for a class of operators with continuous spectrum. Physically, they correspond to decaying waves.

Motivated in part by the Weyl asymptotics of the eigenvalue counting function for the eigenvalues of the Laplacian on a compact manifold, we consider the large r behavior of a resonance counting function. Restricting ourselves to even-dimensional Euclidean scattering, we count the number of resonances in a (certain) region which grows as a parameter $r \to \infty$. Upper bounds on this resonance counting function are due to Vodev and have been known for some time. We prove sharp lower bounds for obstacle scattering without a need for trapping assumptions. Similar lower bounds are proved for some other compactly supported perturbations of $-\Delta$ on $\mathbb{R}^d$, for example, for certain metric perturbations. Some of the tools used in the proof include a Poisson formula, bounds on the trace norm of the scattering matrix on the real axis and some consequences, and the behavior of the wave trace near $t = 0$.

Shane Cooper (University of Bath, UK): Asymptotic analysis of stratified elastic media in the space of functions with bounded deformation

We consider a heterogeneous elastic structure which is stratified in some direction (say x_1). We derive the limit problem under the assumption that the Lamé coefficients and their inverses weakly* converge to Radon measures. Our method applies also to linear second-order elliptic systems of partial differential equations, and in particular to the fully anisotropic heat equation. This is joint work with Michel Bellieud (Universite de Montpellier 2).

Pavel Exner (Doppler Institute for Mathematical Physics and Applied Mathematics, Czech Academy of Sciences): Singular Schrdinger operators and Robin billiards: geometry, spectra and asymptotic expansions

The subject of this talk are spectral properties of several operator clases. They include Schrödinger operators with an attractive singular 'potential', supported by a manifold of a lower dimensionality. The simplest of them can be formally written as $-\Delta - \alpha\delta(x - \Gamma)$ with $\alpha > 0$, where Γ is a curve in $\mathbb{R}^d$, $d = 2, 3$, or a surface in $\mathbb{R}^3$; the expression can be modified to include a different singular interaction term or a regular potential bias. Another class are Hamiltonians describing quantum motion in a region with attractive Robin boundary. We discuss the ways in which spectral properties of such systems are influenced by the interaction support geometry, in particular in the situation when the coupling constant is large, with an attention to similarities and differences between the operators considered.

Rostyslav Hryniv (University of Rzeszw, Poland, and Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, National Academy of Sciences of Ukraine): Schrödinger operators with δ'-like potentials

We address the problem of a suitable definition of the one-dimensional Schrödinger operator with potential δ', where δ is the Dirac delta-function. While Schrödinger operators with δ'-*interactions* have become a standard solvable model in quantum mechanics and have been used in both physical and mathematical literature since the 1980-ies to describe e.g. various types of particle interactions, it is not clear how a δ'-potential should be defined.

For an arbitrary real-valued function V from the Faddeev-Marchenko class $L^1(\mathbb{R}; (1 + x)dx)$ and $\varepsilon > 0$, we study the operator family

$$S_\varepsilon := -\frac{d^2}{dx^2} + \frac{1}{\varepsilon^2} V\Big(\frac{x}{\varepsilon}\Big)$$

of Schrödinger operators as $\varepsilon \to 0$. If the potential V satisfies the conditions

$$\int_{\mathbb{R}} V(\xi)\, d\xi = 0, \qquad \int_{\mathbb{R}} \xi V(\xi)\, d\xi = -1, \tag{1}$$

then the functions $\varepsilon^{-2}V(x/\varepsilon)$ converge to δ' in the sense of distributions, so the limit S_0 of S_ε as $\varepsilon \to 0$, if it exists, might be taken as a mathematically motivated realization of the Schrödinger operator with *potential* δ'.

In the talk we will justify existence of the limit of S_ε as $\varepsilon \to 0$, identify the limiting operator S_0 and discuss whether this limit is the operator we are looking for. We shall also explain how the stationary scattering theory for S_ε gives physical justification of our conclusions.

The talk is based on a joint project with Yu. Golovaty (Lviv, Ukraine).

Andrii Khrabustovskyi (Karlsruhe Institute of Technology, Germany): On the spectrum of a class of periodic quantum graphs

The name "quantum graph" is usually used for a pair $(\Gamma, \mathcal{H})$, where Γ is a network-shaped structure of vertices connected by edges ("metric graph") and $\mathcal{H}$ is a second order self-adjoint differential operator ("Hamiltonian") on it, which is determined by differential operations on the edges and certain interface conditions at the vertices. We refer to the book (G. Berkolaiko, P. Kuchment, Introduction to quantum graphs, AMS, Providence, RI, 2013), which is a nice introduction to the topic.

In many applications (for example, to graphen and carbon nano-structures) periodic quantum graphs are studied. It is known that the spectrum of the corresponding Hamiltonians has a band structure, i.e. it is a locally finite union of compact intervals called *bands*. In general the neighbouring bands may overlap. A bounded open interval is called a *gap* if it has an empty intersection with the spectrum, but its edges belong to it.

In general the presence of gaps in the spectrum is not guaranteed. Existence of spectral gaps is important because of various applications, for example in physics of photonic crystals.

The current research concerns spectral properties of a class of periodic quantum graphs. The main peculiarity of the graphs under investigation is that the location of their spectral gaps can be nicely controlled via a suitable choice of the graph geometry and of coupling constants involved in interface conditions at its vertices.

This is a joint work with Diana Barseghyan (University of Ostrava). The results are published in (D. Barseghyan, A. Khrabustovskyi, Gaps in the spectrum of a periodic quantum graph with periodically distributed δ'-type interactions, J. Phys. A: Math. Theor. 48(25) (2015), 255201).

Alexander Kiselev (National Pedagogical Dragomanov University, Ukraine): On the connections between the spectral theory of quantum graphs and homogenisation problems in low-dimensional structures

We discuss two seemingly unrelated problems. The first one, coming from the spectral theory of quantum graphs, is the problem of convergence of Neumann Laplacians on "thin" domains shrinking to a graph to a corresponding second-order differential operator on this graph. The second one is the problem of critical contrast homogenisation in thin structures, which are ϵ-periodic metric graphs embedded in $\mathbb{R}^d$. We argue that in order to obtain norm-resolvent convergence to the homogenised operator in the second problem, it comes handy to use a number of techniques borrowed from the spectral theory of quantum graphs; moreover, we demonstrate that the homogenised operator itself can be deduced using some rather transparent geometrical operations on a correctly chosen graph. We then point out that the homogenised operator is in fact tightly related to the graph Laplacian solving the first problem above in a special case, that is, when the original thin domain converges to the corresponding graph in a special way, whereby the volumes of its "vertex" and "edge" parts are of the same order in ϵ. This seems to suggest an entirely new perspective for homogenisation problems, allowing to treat them in terms of resonant properties of associated "thin" structures.

David Krejčiřík (Nuclear Physics Institute, Czech Academy of Sciences): Non-self-adjoint graphs

On finite metric graphs we consider Laplace operators, subject to various classes of non-self-adjoint boundary conditions imposed at graph vertices. We investigate spectral properties, existence of a Riesz basis of projectors and similarity transforms to self-adjoint Laplacians. Among other things, we describe a simple way to relate the similarity transforms between Laplacians on certain graphs with elementary similarity transforms between matrices defining the boundary conditions.

This is joint work with Amru Hussein and Petr Siegl published in the Transactions of the AMS (2014).

Stefan Neukamm (TU Dresden, Germany): Stochastic homogenization: An estimate for the two-scale expansion for correlated coefficients

We study linear elliptic systems with rapidly oscillating, random (stationary and ergodic) coefficients. We consider the classical two-scale expansion for such systems and establish an H^1-error estimate. While estimates on the error of the two-scale expansion are well understood in the case of (deterministic) periodic homogenization, the situation for random coefficients is more subtle and it turns out that the error is highly sensitive to the mixing properties and the strength of correlations of the random coefficients. The talk is based on a joint work with Antoine Gloria and Felix Otto.

Zhongwei Shen (University of Kentucky, USA): Boundary regularity estimates in periodic homogenization

In this talk I will provide a survey on recent progress on uniform regularity estimates in periodic homogenization. We consider boundary value problems for a family of second-order elliptic systems in divergence form with rapidly oscillating periodic coefficients. We are interested in sharp regularity estimates, up to the boundary and uniform with respect to the parameter ϵ. Both Dirichlet and Neumann boundary conditions are considered. The results to be discussed include Hölder estimates, Lipschitz estimates, $W^{1,p}$ estimates, and Rellich estimates.

Tatiana Suslina (St. Petersburg State University, Russia): Spectral approach to homogenization of non stationary Schrdinger-type equations

In $L_2(\mathbb{R}^d;\mathbb{C}^n)$, we consider a selfadjoint strongly elliptic operator A_ε, $\varepsilon > 0$, given by the differential expression $b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$. Here $g(\mathbf{x})$ is a periodic bounded and positive definite matrix-valued function, and $b(\mathbf{D})$ is a first-order differential operator. We study the behavior of the operator exponential e^{-itA_ε} for small ε. We prove that, as $\varepsilon \to 0$, the operator e^{-itA_ε} converges to e^{-itA^0} in the $(H^s(\mathbb{R}^d) \to L_2(\mathbb{R}^d))$-operator norm (for a suitable s). Here $A^0 = b(\mathbf{D})^* g^0 b(\mathbf{D})$ is the effective operator. Sharp order error estimates are obtained. The results are applied to study the Schrödinger type equation $i\partial_t u_\varepsilon(x,t) = (A_\varepsilon u_\varepsilon)(x,t)$. Applications to the Schrödinger equation and the two-dimensional Pauli equation with singular potentials are given. The method is based on the scaling transformation, the Floquet-Bloch theory and the analytic perturbation theory.

Jari Taskinen (University of Helsinki, Finland): Spectral gaps for elastic and piezoelectric waveguides

We consider the band-gap structure of the essential spectrum of linear elasticity and piezoelectricity problems on periodic 3-dimensional waveguides. We consider waveguides with thin structures, which are created by thin ligaments connecting (infinitely many, translated copies of) bounded cells. We establish the existence of an arbitrary number of gaps, if the connecting ligaments of the cells are thin enough. In the case of the elasticity system we have quite precise information on the position of the spectral bands. The sharpest results are obtained using asymptotic analysis. In the case of the piezoelectricity system, the information is less precise, due to complications of the non-selfadjointness of the problem; the mere existence of the band-gap structure for the essential spectrum needs a new proof, which we able to provide. Otherwise, the methods include a self-adjoint reduction scheme, max-min-principle and weighted Sobolev estimates.

Ricardo Weder (IIMAS-UNAM, Mexico): Limiting absorption principle for singular solutions to Maxwell equations and plasma heating

There is currently interest in the heating of plasmas in tokamaks with electromagnetic waves, for example in the context of the project ITER. I will present recent results that show that it is possible to heat plasmas by means of hybrid resonances and I will give a formula for the absorbed energy.

The mathematical problem consists in the study of degenerate Maxwell equations in non-homogeneous and anisotropic media, that are not in classes that can be analyzed with the standard theories. It was necessary to consider singular integral equations of the third kind. A similar type of equations was considered by D. Hilbert and E. Picard at the beginning of the last century, and by some other authors after that. We study these equations by a limiting absorption principle that is motivated by the physics of the problem. We prove that there are locally integrable solutions and singular solutions that contain delta functions and principal values. The latter are the only ones that heat the plasma. Furthermore, our results give a method for the numerical calculation of the solutions and I will present the numerical results obtained with our method.These original contributions to the theory of integral equations are of independent interest, and will probably have applications in other areas of science and engineering. As is well known, the limiting absorption principle is a very important tool in spectral and scattering theory and there is a very extensive literature on its use in quantum mechanics and in wave propagation. On spite of this, it appears that it has not been used before for singular solutions, as in our work.

These results were obtained in collaboration with Bruno Després, Laboratoire J.L. Lions, Université de Paris 6 and Lise-Marie Imbert-Gérard, Courant Institute of Mathematical Sciences.

Aaron Welters (Florida Institute of Technology, USA): Analyticity of the Dirichlet-to-Neumann map for Maxwell's equations in passive composite media

In this talk, I will discuss the analyticity properties of the electromagnetic Dirichlet-to-Neumann (DtN) map for the time-harmonic Maxwell's equations for passive linear multicomponent media. I will also discuss the connection of this map to Herglotz functions for isotropic and anisotropic multicomponent composites. The focus of the discussion will be on two different types of geometry, namely, layered media and bounded media (with Lipschitz domains). For these geometries I will derive the analyticity properties of the associated DtN map in terms of the transfer matrix for layered media and, for bounded media, using a variational formulation of the time-harmonic Maxwell's equations. This is joint work with Graeme Milton (University of Utah) and Maxence Cassier (University of Utah).

Ian Wood (University of Kent, UK): Spectral information contained in abstract M-functions

The Weyl-Titchmarsh m-function is an important tool in the study of forward and inverse problems for ODEs, as it contains all the spectral information of the problem. In this talk we will consider the abstract operator M-function or Weyl-function which can be introduced using the abstract setting of boundary triples for an adjoint pair of operators. Our aim is to study how much spectral information is still contained in the M-function in this more general setting, in particular for the case of non-selfadjoint operators. Boundary triples allow for the study of PDEs, block operator matrices and many other problems in one framework. We will discuss properties of M-functions, their relation to the resolvent and spectrum of the associated operator and consider several examples.