A CRITICAL ANALYSIS OF CHARLES BRIDGE, PRAGUE

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Abstract: This paper is a detailed assessment and analysis of Charles Bridge which crosses the river Vlatva in the heart of Prague. The paper explores the history, aesthetics, design and construction of the bridge as well as the success of the bridge in terms of fulfilling its functions. The report also provides a basic structural analysis of the bridge with simplified calculations before offering possible bridge improvements based on a critical analysis.

Keywords: Charles Bridge, Arch, Masonry, Compression, Character

1 Introduction

Charles Bridge is located on the river Vlatva in the heart of Prague, the capital of the Czech Republic. The bridge dates back to around 1357 (construction was finished in 1402) when it was required to replace the previous Judith Bridge which was destroyed by flooding in 1342.

The bridge was commissioned by King Charles IV the then King of Bohemia and Holy Roman Emperor and ultimately completed by architect Peter Parler. It was originally called the Stone Bridge or Prague Bridge but became Charles Bridge in 1870.

In terms of construction the bridge is a series of 16 stone arches with a total length of around 516 meters and width of 10 meters.

The bridge was for a number of centuries the only means of crossing the Vlatva providing crucial access to Prague castle and also acting as an important European trade route. In more modern times the bridge’s function has shifted firstly towards a transport route for the city’s tram and bus networks before more recently becoming Pedestrianised to play a tourist role attracting many thousand of visitors to Prague every year. In addition to this the bridge has achieved extra fame appearing in a number of films including “Mission: Impossible.” Ref. [1]

Throughout its life the bridge has suffered a number of major floods the most recent of which coming in 2002. These have resulted in a number of restoration and protection attempts even so the bridge has undoubtedly maintained its original character and charm.

2 Aesthetics

To assess the aesthetics of the bridge I shall be using the ten rules proposed by Fritz Leonhardt to provide a framework to my discussion. This will by no means provide a definitive answer as to the bridges beauty as an inevitable amount of subjectivity will occur. The rules do however provide a good basis to investigate as many aspects of a bridge’s aesthetics as possible.

It is often said that simplicity is beauty and Charles Bridge is a great example of this. Stone arches are one of the earliest and simplest forms of construction and they really provide a sense of strength and stability making the workings and function of the bridge very clear to the observer. It is easy to understand how load is carried in

Figure 1: Charles Bridge Ref. [2]
compression down to the foundation piers and into the ground.

The bridge definitely fulfils its purpose crossing as it does one of Europe’s major rivers providing a vital link between Prague’s Old Town and the surrounding area. Obviously for a bridge of this age the functions and requirements have changed many times over the centuries and the bridge has effortlessly adapted to these new challenges providing a route for traders, trams, pedestrians and even armies at various times in it’s past.

In terms of the bridge’s appearance the proportions of the arches are crucial to the overall impression it gives to the viewer. The spans do vary slightly from 16.62m up to a maximum of 23.38m however to the naked eye they appear equally spaced. The relationship between the arch sizes and the piers and indeed between the solids and voids seems to feel right within the design. A feeling of strength and might is transmitted whilst maintaining an elegance and mystique. The pier heads are tapered to the flow adding a nice extra dimension to the appearance whilst remaining in proportion with the rest of the bridge. The depth of the bridge deck at the top of the arches is well proportioned to be thick enough to compliment the rest of the bridge but not too thick that it dominates the eye.

In addition to these proportions the bridge’s order also plays a big role in enhancing its appearance. The repeating arch structures provide a reassurance to the viewer and good symmetry ensures the bridge sits comfortably with the viewer. Within Leonhardt’s guidelines for a beautiful bridge he states that good order within a bridge is demonstrated when running your eye along its length it is able to run freely without obstruction. Ref. [3]. Each pier along Charles Bridge is shielded by an ice guard and these unfortunately do disrupt the flow of the eye somewhat but they do of course provide vital protection to the structure.

The bridge deck and main structure are kept very simple as detailed earlier so the refinements to this bridge are located elsewhere.

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The bridge is decorated with 30 statues which stand on plinths along its length. These rise up between the arch spans with each one based on a different figure from Bohemian history. The benefit of this is that it provides real intrigue throughout the length of the bridge ensuring viewing and indeed crossing the bridge doesn’t become a monotonous experience which is a possibility with such a repetitive structure.

Another area where refinements and complex design are allowed free reign over function are the bridge towers. Fig. 3 shows the tower at the old town end of the bridge. You can clearly see the elaborate gothic design which provides a very prominent and emotive entrance to the bridge as well as providing fortification to the crossing in years gone by.

Due to its age the bridge actually helped form the environment of the surrounding area rather than have to integrate into it. The fact that the bridge has stood for so many centuries with the surrounding city forming around it is a real testament to its original design and suitability to the area and culture.

The bridge has a fairly rough texture due to weathering of the stone surfaces. However this seems to compliment the character of the bridge enhancing its image as a working structure and celebrating its age and history rather than trying to hide it.

The bridge retains its natural sandstone colour with little artificial colour having been applied with the exception of a few areas of details on the statues that line the deck. This really provides a great contrast and draws the eye to the decorative details. Perhaps the best time to view the bridges is after snowfall when the blackened statues contrast beautifully with the white snow.

If there is one of the ten rules that Charles Bridge possess more than any of the others which really sums it up it has to be Character. “Magical, inspiring and transfixing” are just some of the words often used to describe it but what is it about the bridge that gives it this character? History undoubtedly plays a part with a number of legends arising from the bridge’s construction. Perhaps the most fascinating of these is that construction is said to have started at 5:31am on 09/07/1357 a time and date arranged to use only odd numbers which were thought to bring good luck giving an insight into the
superstitions that played a part in the bridge’s construction. The bridge also manages to demonstrate two very different sides to its character with a “male” side shown by its massive structure and imposing nature over the river whereas the intricate statues and towers show a softer more “female” side. All of this enhances the feelings generated upon crossing this beautiful structure.

It has to be said that there is no real link between the bridge and nature. In fact the mass of the arches appear to provide the illusion of conquering the river which was probably very reassuring to people when the bridge was first constructed. As described later in this report the bridge and the river have led a turbulent past with numerous instances of flooding providing real threats to the bridges existence. Therefore it is perhaps it’s fair to say that conquering nature rather than integrating with it was probably the driving thought of the designer.

In summary the decision as to whether a bridge is beautiful or not cannot be made simply by working through a tick list of certain criteria. It is the right combination of these criteria suitable to the location and situation that the bridge exists in that will decide on its beauty. Obviously there is a certain amount of subjectivity involved. Charles Bridge certainly possess a high number of Leonhardt’s criteria and scores particularly well on character and function so I think it’s fair to conclude that the bridge does indeed possess a high level of beauty.

3 Design and Construction

The initial construction of the bridge lasted for 45 years finishing in 1402 which was largely due to construction limitations at the time and the ever present problem of flooding.

3.1 Material selection

The bridge is constructed of blocks of Bohemian sandstone bound together with mortar. As arch bridges are the perfect way of making the best use of materials with little tensile capacity the selection of this local material fits perfectly with the design. It is also worth noting that heavy masonry tends to settle quite a lot however arch structures settle only very little so if masonry was to be the material choice then an arch design was the best solution.

3.2 Bridge design

The bridge is a multispan arch design consisting of 16 spans with varying lengths between 16 and 24 meters. The design consists of two abutments at either end of the bridge and a series of piers along the length to support the arches.

3.2.1 Abutments

The two bridge abutments act as foundations for the bridges three towers which guard the entrances to the deck. Two of these towers are found on the Lesser Quarter side of the bridge with the other situated on the old town side shown in Fig. 2. The towers are formed from sandstone blocks similar to the bridge.

3.2.2 Piers

As described earlier the piers are tapered towards the flow to allow it to pass by more smoothly. The piers are roughly 8 x 24 meters being this large allows them to protect the bridge from drifting ice during the winter in addition to resisting the horizontal thrust produced by the arches.

3.2.3 Deck

The deck of the bridge is around ten meters in width however it does vary along the length. As Fig. 4 clearly demonstrates the bridge is by no means uniform in plan as it cuts a zig – zag path across the river. This provides a nice break and an interesting feature to a bridge of this length. The deck is however uniformly flat allowing unrestricted access to all users including pram pushers and wheelchair users.

Figure 4: Aerial shot of bridge deck, Ref. [3]

3.3 Overview of masonry arch construction

During construction of multispans masonry arch bridges each individual arch produces a considerable horizontal thrust upon completion, this thrust can be dealt with in two ways. Firstly all of the arches can be constructed at the same time to ensure thrusts are equally balanced between the spans. The second option is to create very wide bridge piers designed to withstand the horizontal thrust. The choice of method depends heavily on the shape of your arch. The closer your shape is to being perfectly semi-circular the lower your horizontal thrust, this allows the bridge piers to take the bulk of this reduced force. If your arch shape is far from being semi-circular then it is likely high horizontal thrust will be present meaning simultaneous construction of the arches is the likely method of construction. Note in Fig. 1 that the piers are fairly wide in relation to the rest of the
bridge however due to the age of the bridge and the non circular shape of the arches it seems likely that simultaneous construction of the arches was the chosen technique.

### 3.4 Construction process

As is to be expected from a bridge over 600 years old there isn’t too much evidence on the exact construction process however it is possible to make educated assumptions as to how the bridge may have been built. Construction is likely to have consisted of a number of stages. Firstly the pier foundations would have been laid. In the 14th century this would have presented real problems considering the technology available.

As mentioned in the introduction Charles Bridge was actually built on the site of an existing bridge meaning foundations were already in place. However this previous bridge was of timber construction so it is unlikely that the foundations could have been used without major improvements to allow them to carry the additional loading provided by a masonry structure. In terms of the construction process it is likely that a combination of timber and rubble would have been used to divert the river flows away from the pier sites. This would have then allowed a relatively dry area for excavation which would have been carried out as deep as tools would allow until hard material was reached. Foundations consisting of rubble would then have been laid most likely in an arch formation. Conditions would have been too wet for any mortar or cement construction which would have simply washed away. The masonry blocks used to form the piers could then have been laid onto these foundations using lime mortar to bind them together. There is an urban legend surrounding the bridge which claims that eggs were used to strengthen the mortar Ref. [1]. The piers would have been constructed up to the base of the arches known as the springing point.

The final major stage of construction would have involved the construction of falsework almost certainly from timber. This falsework would have provided the platform onto which the masonry blocks could have been laid to form the arches.

Once the arches were in place rubble would most likely have been used as backfill to build up the profile of the deck. This rubble infill also increases the dead weight on the arch increasing its compressive strength. Parapet walls would then have been constructed along the edges of the deck to offer protection to the bridges users. The statues that line the bridge today were a much later addition and were constructed during the 18th century.

### 3.5 Foundations and Geotechnics

Constructing the foundations to a bridge can often be the most difficult stage of construction and certainly a strain on the resources of time and money. The foundations to Charles Bridge will have had to have been fairly large to withstand the loading of the large masonry piers which bear onto them. As mentioned in the previous section it is likely that the rubble foundations would have continued the arch formation rather than being straight vertical piles.

It is worth noting though that the wide piers do reduce the available cross section for river flow thus increasing flow velocity which has the potential to increase bed erosion and undermine the pier’s foundations. This problem is obviously heightened at times of flood and has in fact been responsible for the failure of a number of spans over the bridges lifespan.

It is unlikely that a modern day bridge built in the same location would involve as many piers and therefore foundations. This is due to the difficulty of construction in such a large and relatively fast flowing river.

### 4 Loading

Throughout its life the bridge has performed many different functions which have created a number of different loading situations with varying intensity. One of the peculiarities of this bridge is the fact that its loading has actually decreased over the years since the early 1900s when electric trams used the bridge as a route across the river through a period of bus use to the modern day where the bridge is now fully pedestrianised. The bridge has also supported an omnibus network and horse drawn tram systems as well as numerous armies throughout its life. For the purpose of loading and structural analysis in this paper I will look at the current existing situation with the bridge fully pedestrianised to get an idea of the loads that the bridge in its current state has to deal with. I will also look at the loading applied to the bridge during the period of traffic use as this is likely to have been the worst ever case so will give a good idea of the sorts of loads the bridge has been able to take.

#### 4.1 Dead loading

The dead load of the bridge consists of the masonry blocks used to form the arch ring, piers, abutments and spandrel walls. To assess the dead load of one span a number of assumptions have been made as regards bridge dimensions which are detailed in Fig. 6.

To calculate the total dead load of the bridge that is being supported by each pier I will first calculate the volume of the arch ring, spandrel walls and piers. Values

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**Figure 5:** Generic arch construction

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for these are included in Table 1 below, a standard width of 10m has been used.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Volume (m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arch ring</td>
<td>122</td>
</tr>
<tr>
<td>Spandrel wall</td>
<td>81.4</td>
</tr>
<tr>
<td>Pier</td>
<td>40</td>
</tr>
</tbody>
</table>

These values for the volumes of the various features of each span then need to be multiplied by an appropriate value for the weight of the material used as shown by Table 2.

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandstone masonry</td>
<td>2300 kg/m$^3$</td>
</tr>
<tr>
<td>Stone rubble fill</td>
<td>2240 kg/m$^3$</td>
</tr>
</tbody>
</table>

The Total dead load of one bridge span has been simplified and calculated as follows:

$$P_{DL} = (122 \times 2300) + (2 \times (81.4 \times 2300)) + (2 \times (40 \times 2300)) \approx 839040 \text{ kg} \quad (1)$$

The force of an object due to gravity is 10N/kg so multiplying the value obtained in Eq. (1) and converting to kN a total value for the loading of each span is obtained:

$$P_{DL} = 8309040 \times 10/1000 \quad (2)$$

This value represents the total dead load of each span of the bridge therefore each pier will be taking the whole of this value via two half spans.

It is important to understand that this is a very rough calculation based on estimates of sizes. It has also been heavily simplified to provide a mean average load for the span and makes no allowance for the difference in vertical load across the span as in reality the load at mid span will be a lot lower than the load at the piers. Also it is worth making the point that the spandrel walls will exert a localised vertical force on the edges of the arch rings.

4.2 Superimposed dead loading

This includes all permanent loading applied to the bridge which is in addition to the basic structure. It is separated from basic dead loading as there is a chance that at some point during the bridges life it may or may not be present. All backfill, road surfacing, street furniture and services will fall into this category. The dominant loading here will come from the backfill placed in the void between the arch ring and the spandrel walls. I will use a suitable value of 2240kg/m$^3$ as shown in table 2 for the weight of packed stone rubble fill.

Using the bridge dimensions of Figure 6 and further estimates a value for the volume of backfill used per span of 500m$^3$ has been obtained. The calculation below gives the total loading per span due to this fill:

$$P_{SDL} = 500 \times 2240 \times 10/1000 \quad (3)$$

$$= 11200 \text{ kN}$$

Again the point needs to be made that this is a best estimate based on available data. There is additional super imposed dead load provided by electrical lighting and the decorative statues however conservative values for the volume of fill were taken to allow for the omission of these factors and keep the calculations reasonably simple.

4.3 Live loading

4.3.1 Pedestrianised loading

To assess the live loading of the bridge I will first treat it as pedestrianised as this is the current situation that the structure has to deal with. Following the guidelines in BS5400 the following calculations provide a figure for live pedestrian loading:

$$\text{HA loading} = (K) \times 5 \text{kN/m}^2 \quad (4)$$

Where $K$ is a reduction factor based on the bridges overall length.

$$K = W/30 \quad (5)$$

$$W = 151 (1/L)^{0.475} \quad (6)$$

$$W = 7.77 \text{kN}$$

$$K = 7.77/30 = 0.26$$

Reduced loading = 5 x 0.26 = 1.29kN/m$^2$

Per meter length of bridge = 1.29 x 10 = 12.9 kN

It is important to note that the entrances to the bridge do allow for the possibility of either accidental or more likely intentional traffic access. Therefore as stipulated by BS 5400 four wheels of 25 units of HB loading need to be applied at a nominal location on the bridge.

4.3.2 Traffic loading

Although the traffic loading on this bridge was probably at its worst during the first half of the 20th Century it is still important to get an idea of the sort of loads that may have been present so I will use the Highways Agency system to achieve this. Firstly the road system open to traffic has to be split into notional lanes. This is achieved by looking at the widths of the carriageways. Assuming the traffic flowed in both directions across the bridge we have two carriageways both of width 5m. Therefore we have a total of two
notional lanes for each carriageway both of width 2.5m. Two types of traffic loading then need to be applied to the bridge which are HA and HB loading.

First of all HA loading consists of applying a Uniformly Distributed Load (UDL) over a notional lane. This is combined with a Knife Edge Load (KEL) positioned at the worst possible location. Using the available charts all bridges of total length greater than 380m must use a figure of 9kN/m for the unfactored HA loading. As Charles Bridge has total length of 516m this is the value I shall use. This value of 9kN/m is then divided by the width of the notional lanes to give an intensity of 3.6kN/m². The value to be used for the KEL per notional lane is 120kN which again when divide by the notional lane width gives a value of 48kN/m length.

HB loading consists of abnormal truck loads on the bridge due to the thin width of the bridge and the fact that it was only open to traffic during the first half of the 20th Century it seems highly unlikely that the bridge ever experienced loading of anything like the intensity represented by the HB case. However as mentioned in the pedestrian loading section due to the possibility of accidental loading a certain amount of HB loading must be applied.

4.4 Temperature Effects

To analyse temperature effects a simplified approach will be used. This will assume that the whole bridge cross section increases in temperature by 25°C, in reality temperature will build up non-linearly across a cross section. To calculate the strain caused by a temperature change Eq. (7) can be used:

\[ \varepsilon_T = \alpha \cdot T \]  

(7)

Where \( \alpha \) is the material temperature coefficient for masonry and has a value of 4.5 x 10⁻⁶. Using Eq. (7) we obtain:

\[ \varepsilon_T = 4.5 \times 10^{-6} 	imes 25 \]

This longitudinal strain is only the amount the bridge will feel as in reality the strain will be zero due to the fact that the bridge is restrained from changing in length due to its masonry construction. The strain will produce a longitudinal stress which is calculated as follows:

\[ \sigma = \varepsilon_T \times E \]  

(8)

\[ = 110 \times 10^{-6} \times 45 \times 10^3 \]

\[ = 4.95\text{N/mm}^2 \]

This stress will apply throughout the bridge unless expansion joints are present which allow the material to expand and contract. The force that this stress exerts can be found by multiplying by the area over which it acts.

4.5 Wind Loading

4.5.1 Horizontal wind load

The effect of wind will be to exert a horizontal force onto the spandrel walls of the bridge this will be treated as a Uniformly Distributed Load (UDL). To determine the maximum wind gust (\( V_c \)) the calculation shown in Eq. (9) is used:

\[ V_c = vK_1S_1S_2. \]  

(9)

The various factors used are detailed in Table 3 below:

<table>
<thead>
<tr>
<th>Factor</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>Mean hourly wind speed</td>
<td>13.5mph</td>
</tr>
<tr>
<td>K1</td>
<td>Wind coefficient</td>
<td>1</td>
</tr>
<tr>
<td>S1</td>
<td>Funneling factor</td>
<td>1</td>
</tr>
<tr>
<td>S2</td>
<td>Gust factor</td>
<td>1.27</td>
</tr>
</tbody>
</table>

K1 is a wind co-efficient and is dependent on the bridge height above ground level and also the length of the bridge. S1 is a funneling factor, this has been taken as one as the openness of the surrounding area of the bridge makes funneling impossible. Finally S2 is a gust factor and is dependent on the height of the bridge above ground. K1 and S2 are obtained from standard values. The mean hourly wind speed was obtained from MET office values for the Czech Republic Ref. [4]. Using Eq. (9) we obtain:

\[ V_c = 13.5 \times 1 \times 1 \times 1.27 \]

\[ = 17.145 \text{ m/s} \]

The dynamic pressure head \( q \) must then be calculated as shown by Eq. (10) and from this a value for the horizontal wind load can be reached using Eq. (11):

\[ q = 0.613v_c^2 \]  

(10)

\[ P_t = qA_tC_d \]  

(11)

From Eq. (10) we obtain a \( q \) value of 180.2 N/m and when this is substituted into Eq. (11) Along with the factors from Table 4 a value of 339.7kN is obtained for \( P_t \). Where \( A_t \) represents the solid horizontal projected area of the bridge calculated from the dimensions shown in Fig. 6 and \( C_d \) is obtained from design tables.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_t )</td>
<td>Solid horizontal area</td>
<td>1450m²</td>
</tr>
<tr>
<td>( C_d )</td>
<td>Function of b/d ratio</td>
<td>1.3</td>
</tr>
</tbody>
</table>

4.5.2 Uplift

Wind action on Charles Bridge will also cause problems of uplift acting on the underside of the arches. It is calculated using Eq. (12). This calculation is for one arch span only.

\[ P_v = q \times A_3 \times C_L \]  

(12)

Table 5: Factors for calculating uplift

<table>
<thead>
<tr>
<th>Factor</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q )</td>
<td>180.2N/m</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>150m²</td>
</tr>
<tr>
<td>( C_L )</td>
<td>0.4</td>
</tr>
</tbody>
</table>
The value of $A_3$ is the plan area which is based on the assumption of a flat underside to the deck. However as in the case of Charles Bridge we are dealing with an arched underside I have taken a conservative value for $A_3$ based on the dimensions from Fig. 6 and the width of the bridge which has been estimated at 10m. The value of $C_L$ is similar to the CD factor mentioned earlier and is dependent on span to depth ratios. Using Eq. (12) we obtain a value of 10.81kN for the uplift force which is fairly small.

### 4.6 Oscillation

The problem of vibration is a particular issue for bridges carrying pedestrians. The fact that the bridge is a massive masonry structure does mean that a fair amount of natural dampening will occur but even so a vibration check is included below. This check involves calculating the fundamental frequency which can be achieved using Eq. (13):

$$\omega_n = (\frac{\beta_n l}{2})^2 \sqrt{\frac{E I}{m l^4}} \quad (13)$$

The values required for Eq. (13) are detailed in table 5:

<table>
<thead>
<tr>
<th>Factor</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus, $E$</td>
<td>45GPa</td>
</tr>
<tr>
<td>2nd Moment of area, $I$</td>
<td>13.02$m^4$</td>
</tr>
<tr>
<td>Mass per unit length, $m$</td>
<td>2600 kg/m</td>
</tr>
<tr>
<td>Span, $l$</td>
<td>15m</td>
</tr>
</tbody>
</table>

The value $\beta_n$ is dependent on the end conditions of the bridge. For a multi span arch bridge it is reasonable to assume a clamped – clamped beam connection so a value of 22.37 will be taken for $\beta_n$. The value for $I$ has been calculated for the section at mid span.

Using Eq. (13) a value for $\omega_n$ is obtained as follows:

$$\omega_n = (22.37)^2 \sqrt{\frac{E I}{m l^4}} = 33.36 \text{ Hertz}$$

Acceptable values for oscillation range between 5 and 75 Hertz. An upper bound of 75 is set due to physiological effects induced in the pedestrians using the bridge. The lower bound of 5 is purely to prevent collapse of the bridge. The value obtained here is 33.36 Hertz meaning that the bridge is safe from oscillation effects.

It is worth noting that this method is a very basic approach and would never be used as a detailed design tool. It is meant purely to give an indication as to the bridges performance in terms of oscillation.

### 4.7 Impact loading

One further loading scenario that needs consideration for this bridge is the issue of impact loading from boats using the river Vlatva. Shields are provided to protect the piers from floating ice but it is unclear whether these would be sufficient to protect against boats.

However the type and size of boats using the river are limited by the dimensions of the bridge and it is only really smaller tourism boats that use this stretch of water. Therefore it seems reasonable to assume that any impact onto the piers won’t be a major issue particularly when you consider their stocky nature.

### 5 Structural Analysis

Masonry arch bridges can fail in a number of ways depending on the major stresses being exerted. Firstly large compression forces may crush the stone voussoirs this is the main type of strength failure. Alternatively masonry arches may experience shear failures when applied stresses cause the opening of joints. One other failure mode is the movement of the piers and abutments which is discussed further in the section 5.1 referring to the problem of flooding.

Strength assessments of masonry arches take a number of forms. For the purposes of this paper I shall be using the MEXE method. This method was originally created to assess single span arches only and of course Charles Bridge is very much a multi-span structure. However the method is still valid for bridges with stocky piers due to the fact that when a single span fails the horizontal loads on the adjacent spans are then carried solely by the piers. Stocky piers are able to equilibrate this horizontal thrust from the remaining spans whereas more slender piers become involved in the mechanism formed during collapse and there will be structural interaction between themselves and the collapsing span. Therefore they are likely to fail themselves.

The MEXE method originated as a military technique developed during World War II to provide a relatively quick and simple analysis of the structural integrity of masonry arch bridges. The first stage is to calculate the Provisional Axle Loads (PAL) using Eq. (14).

$$\text{PAL} = \left[740(d+h)^2\right] / L^{1.3} \quad (14)$$

Using estimates of the dimensions of one of the spans of Charles Bridge a value of PAL is calculated below:

$$\text{PAL} = \left[740(0.5+2)^2\right] / (20)^{1.3} = 94.1 \text{ tonnes}$$

The method states that a value for PAL must be the lesser of the value calculated and 70 so the value I shall use will be 70 tonnes. The Modified Axle Load now needs to be calculated using the PAL figure and a number of modification factors. These factors are taken from tables and again rely on estimations of the arches dimensions.

$$\text{MAL} = \text{PAL} \times F_{sr} \times F_p \times F_m \times F_j \times F_{cm} \quad (15)$$

The $F_p$ and $F_j$ can be found directly from tables. However the material factor $F_m$ requires further calculation as shown below in Eq. (16).

$$F_m = \frac{(F_s \times d) + (F_t \times h)}{(d+h)} \quad (16)$$

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Where \( F_b \) represents the barrel factor and has a value of 1. This value has been chosen due to the use of sandstone in construction and relies on the assumption that the masonry is in good condition which for a more accurate figure would need to be checked. \( F_f \) represents the fill factor and has a value of 0.7 has been chosen to represent the assumption that the rubble fill is well compacted. Inputting these values into Eq. (16) gives an \( F_m \) value of 0.76. For the Joint factor \( F_j \) Eq. (17) is used where \( F_w \) is a width factor and due to lack of information has been taken conservatively as 0.8. \( F_d \) is a depth factor which has been taken as 0.9. Finally \( F_{mo} \) is a mortar factor and a value of 0.9 has been adopted for this which represents the mortar as being loose or friable. This seems a sensible selection due to the bridges age.

\[
F_j = F_w \times F_d \times F_{mo} \tag{17}
\]

Inputting the above variables into Eq. (17) gives a value of 0.64 for \( F_j \). The final factor to calculate is the condition factor \( F_{cm} \). This relies heavily on engineering judgement to apply a value of between 0 and 1. I will take a value of 0.8 as the bridge appears to be in reasonable condition but obviously this is a very subjective estimate. Table 7 below summarises all of the factors used to calculate the MAL:

<table>
<thead>
<tr>
<th>Factor</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{sr} )</td>
<td>Span/rise ratio</td>
<td>0.95</td>
</tr>
<tr>
<td>( F_p )</td>
<td>Profile factor</td>
<td>0.9</td>
</tr>
<tr>
<td>( F_m )</td>
<td>Material factor</td>
<td>0.76</td>
</tr>
<tr>
<td>( F_j )</td>
<td>Joint factor</td>
<td>0.64</td>
</tr>
<tr>
<td>( F_{cm} )</td>
<td>Condition factor</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Using the above factors in Eq. (15) yields a result of 23.28 tonnes for the MAL. As you can see this number is less than a third of the original PAL value. This value obtained is the permissible axle load for a two axle bogie.

There are a number of issues with using the MEXE method. Firstly it is thought to be over conservative. Second the modification factors used are somewhat subjective are considered independently despite the fact that in reality they are very much inter-linked. Finally the method gives no information on the stresses in the arch ring or indeed of its deflection.

There are a number of other ways of assessing the strength capacity of masonry arch bridges. The mechanism method for example essentially involves assuming no tensile strength within the masonry which seems reasonable. The process then consists of iterating various possible hinge positions to find the lowest possible collapse load.

It is far more common nowadays to use a computer program to accurately assess the strength of a masonry arch structure and there are many packages available.

5 Susceptibility to damage

5.1 Damage caused by nature

The biggest threat to the bridge throughout its life has undoubtedly been the flooding of the river Vlatva. These flooding events have occurred on a regular basis notably in 1890 when three arches collapsed shown in Fig. 7. The most recent floods came in August 2002 which prompted a huge injection of funds for restoration and protection works on the bridge.

Essentially the floods cause three main problems to the bridge piers; angular rotation, subsidence and shift. These cause forced deformations and tension cracks will start to open up. As mentioned earlier the pier heads are tapered towards the flow to try and reduce these impacts.

![Figure 7: Devastation caused by flooding in 1890](image)

![Figure 8: Charles Bridge after snowfall](image)

One of the main problems felt by the piers of the bridge which is worsened at times of flood is scour. Scour occurs at the base of the piers and abutments as the flow spreads around the pier heads and forms eddies on the other side. These eddies erode the bed of the river causing a hole to open up which has the potential to undermine the pier’s foundation. This is a particular problem for Charles Bridge due to its age and the regular occurrence of heavy flood flows. The best way to deal with the problem is to regularly replace the material lost during erosion to keep the foundations protected.

One final problem related to flooding is the issue of buoyancy which obviously depends heavily on the depth to which the bridge is flooded. As an arch relies on the compressive force of the masonry acting downwards any reduction in this due to the uplift provided by buoyancy could require consideration.
5.2 Aesthetic damage

As one of Prague’s biggest tourist attractions the bridge attracts millions of visitors every year. This of course means it’s a big income generator for the city and this in turn creates a huge requirement to keep the bridge clean and looking its best. An inevitable amount of wear and tear will occur due to the sheer number of pedestrians using the route. The stones are also susceptible to blackening from pollution which can be a problem in a built up city like Prague. Numerous clean ups have taken place over the years to attempt to restore the original colour of the sandstone blocks. Also there have been instances of vandalism aimed at the aesthetic details of the bridge with repair being required on both the statues and the towers at various periods in time.

5.3 Intentional damage

In terms of intentional structural damage it is quite hard to damage such a solid and massive bridge. Even if one of the spans was damaged to the point of failure due to the large nature of the piers as described earlier it is unlikely that the adjoining spans would also collapse. The bridge itself is unlikely to be a major terror target due partly to the low profile held by the Czech Republic and also due to the lack of impact any attack would have. The bridge towers limit the size of vehicle that could accidentally or otherwise gain access to the bridge making the threat of any impacts from heavy goods vehicles unlikely.

Historically however it is a different story. Due to the bridge’s location in the heart of Europe it has been the witness to many key battles and was most recently under severe threat during World War II. As a major European trade route the bridge was obviously a very tempting target to armies wishing to disrupt the flow of goods.

Thankfully nowadays there are a whole host of possible crossings across the Vlatva which has relieved the strain from Charles Bridge and of course means it is no longer a strategic target.

5.3 Chemical attack

In general well constructed masonry is very durable however severe problems can be caused by changes in water content. Firstly there is the problem of freezing and thawing of water contained within cracks in the bridge. This is a particular problem in Prague which suffers quite bad winters as shown by Fig. 8. This action will aggravate the cracks exerting pressures on them and opening up larger voids. Secondly water content can enhance other chemical activity within the masonry. It is the joints and mortar that are particularly vulnerable from deterioration caused by sulphates and chlorides. These attack the chemical make up of the lime breaking it down so that there is no longer a strong bond between the masonry blocks.

One other issue that has threatened the bridge is the weathering of the stones themselves. Six hundred years of wind and rain attack have taken their toll on some of the stones which have required repair.

6 Restoration, improvements and future changes

It’s very difficult to criticise a bridge which has stood proudly for over 600 hundred years and become an integral part of one of Europe’s finest cities. It is to the huge credit of the engineer and builders that the bridge still stands today albeit with a certain amount of restoration work. It is however important to consider ways in which the bridge may be strengthened to combat the effects of age and of course flooding.

A number of options are available for strengthening a bridge of this age and type. Firstly it may be possible to apply a concrete saddle system to each span. This essentially involves excavation to the arch ring then pouring another layer of concrete on top and bonding into the abutments or piers. This new concrete layer would then take the majority of the live loading transferring it down to the piers. However this system wouldn’t necessarily be appropriate for Charles Bridge. Firstly the disruption that this type of work would cause would be huge considering there are 16 spans. Secondly due to the bridge’s age and historical importance replacing vast amounts of the original structure with modern concrete wouldn’t be acceptable.

Fig. 9 shows an alternative strengthening solution. Steel bars are placed at locations of potential hinge failure to provide extra bending capacity. This process involves drilling down into the arch ring to insert the steel. It is crucial to ensure that the bars are fully bonded to the arch structure if moments are to be carried.

Figure 9: Stainless steel anchor

This process was in fact carried out for a number of spans during the 1970s as part of major repair work.

One other possible technique that could be used to strengthen the bridge is sprayed concrete. This process essentially involves spraying a thin layer of un-reinforced concrete to the arch soffit which will take a portion of the live load applied to the bridge. Again the issue of whether this is sensitive enough to the bridges heritage is raised.

One reasonably simple way of ensuring the bridge’ durability is to ensure a reliable water proof layer is present. This should help alleviate some of the effects mentioned in section 5.3. In addition to this it is important to ensure the mortar is replaced where it has cracked or crumbled to prevent problems from worsening.

It is hard to imagine Charles Bridge going through anymore major changes to its use. It is now far too important to the tourism industry of Prague to be risked as
a route for vehicles again especially now that many other routes across the Vlatva exist. It seems that in usage terms the bridge is now in its retirement stage having served its time carrying heavy traffic it is now able to relax carrying the lightest loads of its life whilst arguably performing its most important task in attracting tourism and income.

7 Conclusions

For me Charles Bridge is one of the world’s most beautiful bridges, it comes packed with history and has become a landmark synonymous with the enchanting city of Prague. The fact that it has stood for over six hundred years albeit with a certain amount of restoration is a real testament to the original structural design and the quality of the materials used.

Bridges of this age and quality are indeed very rare and need to be delicately handled to ensure they remain for the enjoyment of future generations. Since 1993 the bridge has been a UNESCO world heritage site which really confirms its global importance.

The bridge has undergone many changes of use throughout its life and has coped admirably with all of them. It is hard to imagine any other type of bridge existing so well in this location and for me Fig. 10 really sums up what Charles Bridge has now become. It is a bridge for the people and is no longer just a means of crossing a river. It now holds a place in many people’s hearts and really demonstrates the beauty and importance that a bridge can achieve.

8 Acknowledgements

The lecture series given by Professor Tim Ibell entitled “Bridges 1” proved an invaluable resource in writing this paper. In addition to this the regular seminar sessions provided were of great use.

9 References


Figure 10: Tourists on Charles Bridge