MA50215 Specialist Reading Course — J.H.P. Dawes
The Dynamics of Fluids — Semester 2, 2009-10

Example Sheet 1 (Revision)

Streamlines, particle paths

1. A steady 2D 'point source' flow is given by $u = \alpha x$, $v = \alpha y$ with $\alpha > 0$ and constant.

   (i) Find the equation for a general streamline of the flow, and sketch typical streamlines.

   (ii) At $t = 0$ the fluid particles on the curve $x^2 + y^2 = a^2$ are marked with dye without disturbing the flow. Find the equation for this material fluid curve for $t > 0$.

   (iii) Does the area within the curve change in time, or not? Why?

2. Repeat question 1 for the 2D simple shear flow given by $u = \gamma y$, $v = 0$ with $\gamma > 0$ and constant. Without further calculation, consider which of this flow or the flow in question 1 stretches the curve faster at large times.

Mass conservation, Euler's equation

3. A tall cylindrical tank, having a flat base and radius a, is rotating about a vertical axis. The tank contains a volume V of inviscid fluid of constant density ρ. The tank is allowed to rotate for a long time until the flow is two-dimensional rigid-body motion with constant angular velocity Ω around the z-axis, i.e. $u = -\Omega y$, $v = \Omega x$, $w = 0$. Find the pressure p in the fluid and hence determine the height h of the free surface.

Mass conservation, Bernoulli's equation for steady flow

4. Invisicd waste water flows, at negligible velocity, into an uncovered tank at a rate of $10^{-4}\mathrm{m}^3\mathrm{s}^{-1}$ and out of a (short) exit pipe of cross-sectional area $4 \times 10^{-5}\mathrm{m}^2$ into the open air. In steady state, estimate the height (above the exit pipe) of the surface of the water in the tank.

5. A water clock is an axisymmetric vessel with a small exit pipe in the bottom. Ignoring viscosity, find the shape of the vessel for which the water level in the vessel falls equal heights in equal time intervals.

Hand in to the folder on my door by 10am on Monday 15 February.