MA6000M: Topics in Applied Mathematics 2011 - 12

Bifurcation Theory and Applications

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This course will introduce ideas and methods from nonlinear dynamics which are widely and routinely used to understand models of a wide range of physical systems, for example fluid flows, population dynamics, chemical reactions and coupled oscillators. The 'dynamical systems viewpoint' is to concentrate on features of the dynamics that are independent of the coordinate system, for example the long-term behaviour that the system 'settles down to'.

The **first half** of the course will be concerned with the qualitative behaviour of solutions to nonlinear ordinary differential equations, with an emphasis on structural changes in response to variations in parameters (bifurcation theory). There will be a brief discussion of the generation of complicated dynamics.

The **second half** of the course will extend these ideas to the qualitative study of models for structures and instabilities in spatially-extended continuum systems. These model equations are typically collections of nonlinear parabolic partial differential equations. Throughout the course we will concentrate on understanding generic behaviours, and those parts of the dynamics that are both of physical interest and typical of a wide class of problems. Physical symmetries and asymptotic scalings play crucial roles. Examples motivated by fluid mechanics (in which context many of these ideas were first developed) will be discussed in some detail.

There will be a number of problem sheets and problem classes. The style of the course will be to develop intuition and link theory with applications. Theorems will in many cases be motivated, stated and discussed but proofs will not be given.

A perhaps over-ambitious course outline:

Part I

- Introduction (through which we will go quite fast): phase space and the qualitative description of solutions to ODEs. Topological equivalence, hyperbolicity and structural stability of flows. Stable and unstable manifolds. Codimension–one local bifurcations in flows and maps. Centre manifolds. Reduction to normal forms; normal form symmetries.
- Global bifurcations. Chaos. Lorenz and Shil'nikov mechanisms. Codimension-two bifurcations: degenerate Hopf, Takens–Bogdanov.

Part II

- Low-dimensional behaviour and bifurcations in Rayleigh–Bénard convection.
- Pattern-forming instabilities. The Ginzburg–Landau and Newell–Whitehead–Segel equations. Secondary instabilities, e.g. Eckhaus.
- Oscillatory instabilities: the complex Ginzburg–Landau equation. Benjamin–Feir instability. The Kuramoto–Sivashinsky equation.

If time:

- Phase instabilities of fully nonlinear patterns. The Cross–Newell equation.
- Localised states and the Hamiltonian-Hopf bifurcation.

Desirable previous knowledge

Some previous exposure to nonlinear dynamics would be extremely advantageous, e.g. an undergraduate course similar to the UoB course MA40045 'Dynamical Systems'.

Introductory reading

1. P. Glendinning, *Stability, Instability and Chaos.* Cambridge University Press, 1994.

   Clear, readable textbook based on lecture notes for the undergraduate courses on nonlinear dynamics at Cambridge.

2. M. Hirsch & S. Smale, *Differential Equations, Dynamical Systems and Linear Algebra.* Academic Press, New York, 1974.

3. J.D. Crawford, Introduction to bifurcation theory. *Rev. Mod. Phys.* **63**, 1991.

   Covers all the necessary background with more of the analytical details. Includes brief notes on bifurcations with symmetry towards the end.

4. S.H. Strogatz, Nonlinear Dynamics and Chaos. Perseus Books, Cambridge, MA.

   Clear, widely used.

Reading to complement course material

No single book covers both halves of the course together. In the references below, I've tried to indicate which half a book deals mostly with.

1. J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields.* Springer, **second or subsequent printing** 1986.

   An excellent general reference; dense and rewarding reading. (First half)

2. Y.A. Kuznetsov, *Elements of Applied Bifurcation Theory.* Second edition, Springer, 1998.

   Quite good for the first half of the course.

3. S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos.* Springer 1991.

   One of many general introductions to the subject. (First half)

4. R.B. Hoyle, *Pattern Formation: An Introduction to Methods.* CUP 2006.

   Based on notes for a course on very similar material, but stylistically a little different. (Second half)

September 28, 2011