Group Theory, 2016

Exercise sheet 7 (hints)

Exercise 1. Notice first that we can write X as a disjoint union of G orbits, say

$$x_1 \cdot G \cup x_2 \cdot G \cup \cdots \cup x_r \cdot G.$$

Now apply the Orbit Stabilizer Theorem to get information about $|x_i \cdot G|$. Use this and the decomposition of G into union of orbits to count the elements in X.

Exercise 2. Noptice that if N is a normal subgroup of S_5 then $N \cap A_5$ is a normal subgroup of A_5. Now use the fact that A_5 is simple and consider two cases that occur.

Exercise 3. (a) It may help here to look at the solution to Exercise 5 on sheet 6.

(b) If H is normal in A_5 then it must be a union of conjugacy classes. Now turn again to the solution of Exercise 5 on sheet 6.

Exercise 4. (a) Notice that a cycle of even length is an odd permutation. Now use Exercise 2 on sheet 6.

(b) There is an odd permutation that swaps the two orbits. (The result is that $x^a = x$. Now use Exercise 2 on sheet 6 again.

(c) This is a challenge!

Exercise 5. (a) Use the fact that $|x^G| = |G|/|C_G(x)|$.

(b) You need to show that the sum of $|C_G(a)|$ over all $a \in x^G$ gives you $|G|$. Use (a) for this.