Group Theory, 2016

Exercise sheet 6 (hints)

Exercise 2. Notice that every subgroup is normal. Let G be simple. Show first that G must be cyclic, say generated by a. Then show that G cannot be infinite. Finally show that the order must be prime.

Exercise 3. (b) If $x^u = x^{(1\,2)v}$ for some $u, v \in A_n$ then $x^w = x$ where $w = (12)vu^{-1}$.

Exercise 4. By Exercise 4(b) on sheet 3 you know that there exists a subgroup H_{n-1} of G of order $p_1 \cdots p_{n-1}$. Similarly you can find a subgroup H_{n-2} of H_{n-1} and so on.

Exercise 5. (a) A non-trivial element in S_4 can have the following possbile cycle structures $(r\ s)$, $(r\ s\ t)$, $(r\ s\ t\ u)$ and $(r\ s)(t\ u)$. If a is of one of these types the the conjugacy class a^G consists of all elements in S_4 that have the same cycle structure.

Now use Exerecise 1(b) as well as the fact that the order of any normal subgroup of S_4 must, by Lagrange, divide the order of S_4. Exercise 5 on sheet 2 will also be useful here.

(b) Notice that you are asked here for a composition series. Not all the terms need to be normal in S_4 (only normal in their successor).