Group Theory, 2016

Exercise sheet 5 (hints)

Exercise 1. You are welcome to use without proof the following properties involving the notation nx (where n is is an integer and x is a group element): $(n+m)x = nx + mx,\ n(x+y) = nx + ny,\ (nm)x = n(mx)$.

Exercise 2. (a) Argue by contradiction and assume that $\mathbb{Q}$ is generated by some rationals $a_1/b_1, \ldots, a_r/b_r$. Now find a rational that is not in the span of these.

Exercise 3. (a) Look at the example in lecturers about abelian groups of order 72.

(b) According to the Fundamental Theorem: if two abelian groups, are written as a direct sum of cyclic groups of prime order, then they are isomorphic iff for each prime power p^m the two groups have same number of summands of order p^m. Let $a(p^m)$ be the number of cyclic summands of order p^m in A and $b(p^m)$ be the number of cyclic summands of order p^m in B. Why must we have $a(p^m) = b(p^m)$?

Exercise 4. If the orders are pairwise coprime, what is the order of $x_1 + \cdots + x_n$?

What is the exponent of G in general (in terms of $o(x_1), \ldots, o(x_n)$) without the assumption that the orders are pairwise coprime?

Exercise 5. Write F^* as a direct product of cyclic groups of prime power order. If F^* is not cyclic, then (by exercise 4) there are two factors in the product that have orders that are powers of the same prime. Say that these factors are H and K of order p^r and p^s. Find now p^2 elements in HK whose order is divisble by p.