Group Theory, 2016

Exercise sheet 3 (hints)

Exercise 1. (a) Think of a homomorphism from G to G whose image is G^n and whose kernel is $G[n]$. Then apply the 1st Isomorphism Theorem.

(b) Determine what $G[n]$ is for all n. Then apply (a).

Exercise 2. (a) Recall that $a^m = 1$ iff $o(a)$ divides m.

(b) Use part (a) and Lagrange's Theorem.

Exercise 3. In $(\mathbb{Q}, +)$ one can divide by 2. What does this correspond to in $(\mathbb{Q}^+, \cdot)$?

Exercise 4. (a) For the induction step, consider two cases. If $p|o(a)$, say $o(a) = pr$, then you can find an element of order p (which one?). If p does not divide $o(a)$ then let $N = \langle a \rangle$ and consider G/N. This group is of smaller order and thus has an element bN of order p. What can you say about the order of b in G? (Use Qn 2(a)).

Exercise 5. (a) Notice that you know that the collection of all bijections from G to G is a group. You only need to show that $\mathrm{Aut}\,(G)$ is a subgroup of this larger group.

(d) Ker Ψ is a special subgroup of G that was introduced on sheet 2.