Group Theory, 2016

Exercise sheet 2 (hints)

Exercise 1. Using the fact that both H and K are normal in G, show that the element $h^{-1}k^{-1}hk$ is both in H and K.

Exercise 2. Suppose $G/Z(G) = \langle aZ(G) \rangle$. Notice that the elements of $G/Z(G)$ are

$$\dots a^{-2}Z(G), a^{-1}Z(G), Z(G), aZ(G), a^2Z(G), \dots$$

Now G is the union of these cosets ...

Exercise 3. In the case when H is a non-trivial subgroup, let n be the smallest positive integer such that $a^n \in H$. Show that $H = \langle a^n \rangle$.

Exercise 4. Need to show that $a^*b^* = (ab)^*$ and that $a^* = b^* \Rightarrow a = b$. Notice also that $1^* = \epsilon$, the multiplicatie identity of the ring $\mathbb{Z}^G$. For the latter part show that $R = \sum_{a \in G} \mathbb{Z}a^*$.

Exercise 5. For part (a) it might be helpful to prove first the formula

$$\alpha \cdot (i_1\ i_2\ \dots\ i_r) \cdot \alpha^{-1} = (\alpha(i_1)\ \alpha(i_2)\ \dots \alpha(i_r)).$$

For example then

$$\alpha(1\ 2)(3\ 4)\alpha^{-1} = \alpha(1\ 2)\alpha^{-1} \cdot \alpha(3\ 4)\alpha^{-1} = (\alpha(1)\ \alpha(2))(\alpha(3)\ \alpha(4)).$$