Group Theory, 2016
Exercise sheet 9

Exercise 1. (a) Let p and q be distinct primes and let G be a group with pq elements. Let $n(p)$ and $n(q)$ be the number of the Sylow p-subgroups and the Sylow q-subgroups. Show that if $n(p) = n(q) = 1$, then G is the internal direct product of the Sylow p-subgroup and the Sylow q-subgroup.

(b) Show that (up to isomorphism) there is only one group of order 15, the cyclic group of order 15.

Exercise 2. Let G be a group of order 12 and suppose that there is not a normal Sylow 3-subgroup of G. Show that G is ismorphic to A_4.

Exercise 3. Let p and q be distinct primes.

(a) Show that there is no simple group of order 36.
(b) Show that there is no simple group of order p^2q^2.

Exercise 4. For a finite group G we denote by $\mathrm{Syl}_p(G)$, the set of the Sylow p-subgroups of G. Suppose that $N \trianglelefteq G$ and that $P \in \mathrm{Syl}_p(G)$.

(a) Show that $P \cap N \in \mathrm{Syl}_p(N)$.
(b) Show that $PN/N \in \mathrm{Syl}_p(G/N)$.

Exercise 5. Identify the Sylow subgroups of the following groups. How many are there for each prime? (a) $G = A_4$, (b) $G = S_4$.

Course webpage: http://people.bath.ac.uk/gt223/MA30237.html