Vying for Support: Lobbying a Legislator with Uncertain Preferences*

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Abstract

We consider a model where two opposing lobbyists bid for the support of a legislator with an uncertain bias towards either lobbyist. We find that high levels of bias uncertainty leads to lobbyists offering low bids. On the other hand, low levels of bias uncertainty makes lobbyists bid aggressively. Finally, for moderate levels of bias uncertainty, we find a non-monotonic relationship between the uncertainty of the legislator's bias and the bids of the lobbyists.

Keywords: lobbying; uncertainty; integrity threshold; legislatures.

JEL Classification: D72, D80

1 Introduction

Lobbying is ubiquitous in most legislative systems, yet politicians are less likely to seek rent where there is increased scrutiny. The sectors with the highest levels of lobbying spending in the United States in the past five years do not include hot button issues such as abortion and gun laws (The Center for Responsive Politics, 2017). Low salience sectors, including finance and health care, have the highest levels of lobbyist spending for 2018 (The Center for Responsive Politics, 2018). The views of legislators are often undisclosed in these sectors, and this uncertainty over preferences provides politicians with

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opportunities for gain at the expense of the collective good. The purpose of this paper is to study lobbying in the presence of uncertainty about the legislator's preferences.

We show that at low levels of uncertainty, lobbyists bid aggressively. The potential bias of the legislator towards a lobbyist's policy is not high enough to risk losing the legislator's support and each lobbyist bids high to stay competitive. Conversely, at high levels of bias uncertainty, lobbyists take into account the possibility that the legislator has a strong preference towards a lobbyist's policy position and bid lower. Finally, for moderate levels of uncertainty, we find a non-monotonic relationship between equilibrium lobbyist's bids and uncertainty over legislator's preferences.

Our results suggest that as long as there is a possibility that a given legislator has strong preference for the lobbyist's position, it would be more cost efficient to vie for them. This is in line with empirical evidence that lobbying is often done towards legislators who already agree with them (Hojnacki & Kimball, 1998; de Figueiredo & Richter, 2013).

Preferences of legislators are often private and unknown to the lobbyists (Heberlig, 2005). Lobbyist behaviour given uncertain legislator's preferences has been explored by Buzard & Saiegh (2016); Dekel et al. (2006). In this paper, we also take into account the politician type using the integrity threshold. We argue that due to the revolving door of lobbyists phenomenon explored by Blanes i Vidal et al. (2012), the type of the politician is known by lobbyists. Ex-government staffers who enter lobbyist firms will have a clear idea of the politician's type, however some uncertainty may exist on their specific policy preferences.

Another related paper is the study on ideological uncertainty and lobbying competition by Martimort & Semenov (2008). Our paper focuses on securing legislator access: if a winning bid is accepted, the winning lobbyist will be able to secure the legislator's support. The non-policy centric approach allows us to capture the revolving door phenomenon as access to legislators may start with a single policy issue but may extend to similar causes.

2 The Model

Consider a legislator and two lobbyists. The legislator has a policy bias b distributed uniformly $b \sim U(-d, d)$, with $d > 0$, and an integrity threshold $t > 0$. The legislator's utility is given by,

$$U_L = \begin{cases} p_1 - t - b & \text{if Lobbyist 1 wins,} \\ p_2 - t + b & \text{if Lobbyist 2 wins,} \\ 0 & \text{otherwise,} \end{cases}$$

where p_i is the bid submitted by lobbyist i.

The lobbyists know the distribution of the legislator's bias and the integrity threshold. Both bid simultaneously $p_i \geq 0$ to win the legislator's support. Bids are only considered by the legislator when they are above the bias-adjusted threshold (i.e. $t \pm b$). The bid that provides the legislator with the highest utility to the legislator wins. Only the winning bid is collected. Lobbyist i gains $w \in \mathbb{R}_+$ upon winning.

The utilities of each lobbyist i are given below:

$$U_1 = \begin{cases} w - p_1 & \text{if } p_1 > t + b \text{ and } p_1 > p_2 + 2b, \\ 0 & \text{otherwise.} \end{cases}$$

$$U_2 = \begin{cases} w - p_2 & \text{if } p_2 > t - b \text{ and } p_2 > p_1 - 2b, \\ 0 & \text{otherwise.} \end{cases}$$

3 Results

Our result characterizes the equilibrium of the game, its proof is presented in the appendix.

Proposition 1. *The equilibrium bids* (p_1^*, p_2^*) *are given by:*

1. *If* $d \leq (1/2)(w - t)$ *there exists a unique equilibrium where* $p_1^* = p_2^* = w - 2d$.
2. *If* $\frac{w-t}{2} < d < w - t$ *there exists a continuum of equilibria such that* $p_1^* + p_2^* = 2t$ *and*

   2.1 *if* $(1/2)(w - t) < d < (5/7)(w - t)$ *then* $p_1^* \in \left(\frac{w-2d+2t}{3}, \frac{4t-w+2d}{3}\right)$,

   2.2 *if* $d = (5/7)(w - t)$ *then* $p_1^* \in \left(\frac{8t-w}{7}, \frac{6t+w}{7}\right)$ *and,* *2.3 if* $(5/7)(w-t) < d < w-t$ *then* $p_1^* \in \left(\frac{3t-w+d}{2}, \frac{w+t-d}{2}\right)$.

3. *If* $d \geq w-t$ *there exists a unique equilibrium* (p_1^*, p_2^*), *where* $p_1^* = p_2^* = (w+t-d)/2$.

The result of Proposition 1 is illustrated in Figure 1.

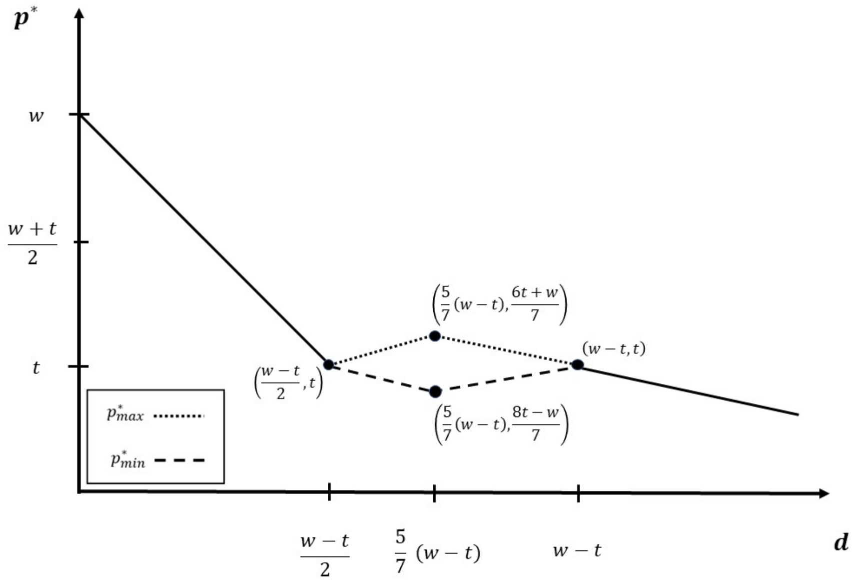

Figure 1: Summary of Equilibrium Bids over Legislator Preference Uncertainty

Figure 1 shows the relationship between the equilibrium bid, p^*, and the uncertainty over the legislator's bias, d. Note that as the possible p^* values change across the identified points in Proposition 1, we use p^*_{min} and p^*_{max} to represent the minimum and maximum possible bids under each point of the proposition. In general, we can see from Figure 1 that there is a non-monotonic relationship between the uncertainty over the legislator's bias and the equilibrium bids.

Our results indicate that as long as there is a chance that a legislator has a strong preference for a lobbyist, it would be more cost-effective for the lobbyist to approach the legislator. If this result is observed, the legislators approached by lobbyists should vary depending on the issue on the table. This supports the growing consensus in the empirical literature identified by de Figueiredo & Richter (2013) where legislators, first allied and then marginal, from both sides of the issues are approached more often than staunch opposition by lobbyists (Kollman, 1997; Holyoke, 2003; Heberlig, 2005; Hall &

Deardorff, 2006; Bertrand et al., 2014; Gawande et al., 2012).

3.1 Strategies under Low Bias Uncertainty: $d \leq (w-t)/2$

When there is low uncertainty over the legislator's bias ($d \leq (w-t)/2$), both lobbyists bid $p_1^* = p_2^* = w - 2d$, with an average bid above the legislator's bias integrity threshold (see point 1 in Proposition 1). They favour more aggressive bidding strategies. This is supported by empirical results where lobbyists and special interest groups predominantly lobby allied legislators (Hall & Deardorff, 2006; de Figueiredo & Richter, 2013; Schnakenberg, 2017). When lobbyists are certain of the legislators preferences, they are willing to spend more resources to secure their support.

3.2 Strategies under Moderate Bias Uncertainty: $(w-t)/2 < d < w-t$

When there is moderate uncertainty over the legislator's bias ($(w-t)/2 < d < w-t$), we find a continuum of equilibria, with the characteristic that the average bid is equal to the legislator's threshold . Specifically, under point 2.3 in Proposition 1, when the bias interval is $(5/7)(w-t) < d < w-t$, that is the uncertainty over the legislator's bias is quite high, lobbyists can always bid aggressively ($p_i \geq (4t - w + 2d)/3$) to secure the win. On the other hand, under point 2.1 in Proposition 1, when the uncertainty over the legislator's bias is quite low with $(1/2)(w-t) < d < (5/7)(w-t)$, bidding conservatively is risky as the interval is not long enough to ensure that the opponent does the same. We also find the special case where one lobbyist bids aggressively and another bids conservatively at $d = (5/7)(w-t)$ (see point 2.2 in Proposition 1).

3.3 Strategies under High Bias Uncertainty: $d \geq w-t$

When there is high uncertainty over the legislator's bias ($d \geq w-t$), both lobbyists maximise their expected utilities by bidding conservatively ($p_i \leq (3t - w + d)/2$).

An increase in the level of integrity of the legislator t, holding the winning valuation w constant, makes it more likely the bias interval to be sufficiently high. For highly salient issues, t is very high; the positions of legislators are heavily publicized, with high costs on reputation if the legislator's integrity is questioned. This may help explain why despite high levels of coverage on single issues in the United States (*e.g.* gun rights vs. gun control and pro-life vs. pro-choice), and the low uncertainty over legislator's preferences on the issues, often earmarked by party memberships (*e.g.* Republicans for gun rights and Democrats for gun control), lobbyist spending in the single issue sector does not reach the top five sectors with the highest lobbying expenditure in 2018 (The Center for Responsive Politics, 2018).

4 Conclusion

We explore how lobbying can proceed in an unregulated environment. A simultaneous lobbying structure is used to capture how lobbying proceeds behind closed doors. Under shadow lobbying, where lobbyist-legislator interactions are kept private, lobbyists may not be able to counteroffer. The paper focuses on lobbyist interactions over one non-strategic legislator and explores the impact of uncertainty on the lobbyist's behaviour in isolation.

The results of this paper corroborate the growing consensus that lobbyists mostly approach allied or marginal legislators and offer the additional insight that the possibility of a strong preference towards the policy lobbied for makes the legislator more attractive to lobbyists. Political agents, however, may listen to constituent opinions and adjust their preferences accordingly, which, in turn, influence lobbyist's behaviour.

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Appendix

Given that the game is symmetric for both lobbyist, we solve the game for lobby 1 only without loss of generality.

A Deriving Winning Probabilities

A lobbyist only wins if the bid is considered sufficient and provides the most payoff to the legislator. The probabilities of winning for lobby 1 is

$$\begin{aligned} P\,(1 \text{ wins}) &= P\left(p_1 > t + b \bigcap p_1 > p_2 + 2b\right) \\ &= P\left(b < \min\{p_1 - t, (p_1 - p_2)/2\}\right). \end{aligned}$$

That is,

$$P(1 \text{ wins}) = \begin{cases} P(b < p_1 - t) & \text{if } \frac{p_1+p_2}{2} < t, \\ P(b < \frac{p_1-p_2}{2}) & \text{otherwise.} \end{cases}$$

When the average bid is equal to the integrity threshold, it follows that $P(b < p_1 - t) = P(b < (p_1 - p_2)/2)$, and $P(b > t - p_2) = P(b > (p_1 - p_2)/2)$. Subsequently, given that the bias is uniformly distributed we obtain

$$P(1 \text{ wins}) = \begin{cases} \dfrac{p_1 - t + d}{2d} & \text{if } \dfrac{p_1 + p_2}{2} \leq t, \\ \dfrac{p_1 - p_2 + 2d}{4d} & \text{if } \dfrac{p_1 + p_2}{2} \geq t. \end{cases}$$

B Expected Utilities

Scenario 1: Average bid below the threshold ($\frac{p_1+p_2}{2} \leq t$)

Given expected utility:

$$EU_1(p_1, p_2) = \begin{cases} \frac{p_1-t+d}{2d}(w - p_1) & \text{if } \frac{p_1+p_2}{2} \leq t, \\ \frac{p_1-p_2+2d}{4d}(w - p_1) & \text{if } \frac{p_1+p_2}{2} \geq t. \end{cases}$$

The optimal bid is $p_1 = (w + t - d)/2$ which we refer to as $\underline{p_1}$. The minimum bid for the lobbyist to stay in the game is $t - d$. To fulfill the assumption where the average bid is below the threshold ($(p_1+p_2)/2 \leq t$), only feasible bids are considered: $p_1 \in [t-d, 2t-p_2]$, given p_2.

For the average bid to be below the threshold t, $\underline{p_1} \leq 2t - p_2$.

$$\underline{p_1} \leq 2t - p_2$$
$$\frac{w+t-d}{2} \leq 2t - p_2$$
$$p_2 \leq \frac{3t-w+d}{2}$$

when $p_2 > \frac{3t-w+d}{2}$, $\underline{p_1} = 2t - p_2$.

From the above, we obtain the following optimal bids

$$\underline{p_1} = \begin{cases} \dfrac{w+t-d}{2} & \text{if } p_2 \leq \frac{3t-w+d}{2}, \\ 2t - p_2 & \text{otherwise.} \end{cases}$$

Substituting,

$$EU_1\left(\underline{p_1}, p_2\right) = \begin{cases} \dfrac{(d-t+w)^2}{8d} & \text{if } p_2 \leq \dfrac{3t-w+d}{2}, \\ \dfrac{(d-p_2+t)(p_2-2t+w)}{2d} & \text{otherwise.} \end{cases}$$

Scenario 2: Average bid above the threshold ($\frac{p_1+p_2}{2} \geq t$)

The bid p_1 that maximizes the expected utility is Let $(w + p_2 - 2d)/2 = \overline{p_1}$.

For the average bid to be greater than or equal to the threshold lobbyist one has to bid at least $2t - p_2$, and the feasible set of bids reduce to $p_1 \in [max\{p_2 - 2d, 2t - p_2\}, w]$.

Note that the average bid has to be above the threshold t, $\overline{p_1} \geq 2t - p_2$.

$$\overline{p_1} \geq 2t - p_2$$
$$\frac{w+p_2-2d}{2} \geq 2t - p_2$$
$$p_2 \geq \frac{4t-w+2d}{3}$$

when $p_2 < \frac{4t-w+2d}{3}$, $\overline{p_1} = 2t - p_2$.

From the above, when $((p_1 + p_2)/2 \geq t)$:

$$\overline{p_1} = \begin{cases} \dfrac{w + p_2 - 2d}{2} & \text{if } p_2 \geq \frac{4t-w+2d}{3}, \\ 2t - p_2 & \text{otherwise.} \end{cases}$$

$$EU_1\left(\overline{p}_1, p_2\right) = \begin{cases} \dfrac{(2d - p_2 + w)^2}{16d} & \text{if } p_2 \geq \dfrac{4t - w + 2d}{3}, \\ \dfrac{(d - p_2 + t)(p_2 - 2t + w)}{2d} & \text{otherwise.} \end{cases}$$

C Best Responses

Case 1: $p_2 \leq \frac{3t-w+d}{2}$

Lobbyist one can choose to bid either $\underline{p_1} = (w+t-d)/2$, and keep the average bid below the threshold, or $\overline{p_1} = 2t - p_2$.

We have

$$\begin{aligned} EU_1\left(\underline{p_1}, p_2\right) = EU_1\left(\overline{p}_1, p_2\right) &\iff \\ \frac{(d-t+w)^2}{8d} = \frac{(d-p_2+t)(p_2-2t+w)}{2d} &\iff \\ p_2 = \frac{3t-w+d}{2}&. \end{aligned}$$

Whenever $p_2 \leq (3t - w + d)/2$, we have $EU_1\left(\underline{p_1}, p_2\right) \geq EU_1\left(\overline{p}_1, p_2\right)$, so $BR_1\left(p_2\right) = (w+t-d)/2$.

Case 2: $\frac{3t-w+d}{2} < p_2 < \frac{4t-w+2d}{3}$

We have $BR_1\left(p_2\right) = 2t - p_2$.

Case 3: $p_2 \geq \frac{4t-w+2d}{3}$

Lobbyist one bid either $\underline{p_1} = 2t - p_2$, and keep the average bid at the threshold, or $\overline{p}_1 = (w + p_2 - 2d)/2$.

We have

$$\begin{aligned}
EU_1\left(\underline{p}_1, p_2\right) &= EU_1\left(\overline{p}_1, p_2\right) \iff \\
\frac{(d-p_2+t)(p_2-2t+w)}{2d} &= \frac{(2d-p_2+w)^2}{16d} \iff \\
p_2 &= \frac{4t-w+2d}{3}.
\end{aligned}$$

Whenever $p_2 \geq (4t-w+2d)/3$, it follows that $EU_1\left(\overline{p}_1, p_2\right) \geq EU_1\left(\underline{p}_1, p_2\right)$, so $BR_1\left(p_2\right) = (w+p_2-2d)/2$.

To summarize, the best response of lobbyist i is given by:

$$BR_i\left(p_{-i}\right) = \begin{cases} \dfrac{w+t-d}{2} & \text{if } p_{-i} \leq \dfrac{3t-w+d}{2}, \\ 2t-p_{-i} & \text{if } \dfrac{3t-w+d}{2} < p_{-i} < \dfrac{4t-w+2d}{3}, \\ \dfrac{w-2d+p_{-i}}{2} & \text{otherwise.} \end{cases}$$

D Computation of Equilibria

Case 1: $p_2 \leq \frac{3t-w+d}{2}$

The best response of lobbyist 1 is

$$BR_1\left(p_2\right) = \frac{w+t-d}{2}.$$

Let $p_1^* = (w+t-d)/2$. An equilibrium exists if the best response of lobbyist two to p_1^* satisfies $p_2 \leq (3t-w+d)/2$. We solve for equilibria depending on where p_1^* is with respect to the lower and the upper bound.

Case 1.1: $p_1^* \leq \frac{3t-w+d}{2}$

The best response p_1^* satisfies

$$p_1^* \leq \frac{3t-w+d}{2} \iff w-t \leq d.$$

The best response of the second lobbyist is $p_2^* = (w+t-d)/2$ and satisfies $p_2^* \leq (3t-w+d)/2$ whenever $w-t \leq d$. Thus, the pair (p_1^*, p_2^*) is an equilibrium. The expected utilities of lobbyists are

$$EU_1(p_1^*, p_2^*) = EU_2(p_2^*, p_1^*) = \frac{(d - t + w)^2}{8d}.$$

Case 1.2: $\frac{3t-w+d}{2} < p_1^* < \frac{4t-w+2d}{3}$

It follows that $p_1^* > (3t - w + d)/2$ whenever $d < w - t$. The restriction on parameters so that p_1^* is below the upper bound require

$$p_1^* < \frac{4t - w + 2d}{3} \iff d > \frac{5}{7}(w - t).$$

The best response of lobbyist's two reduces to

$$BR_2(p_1^*) = 2t - p_1^* = 2t - \frac{w + t - d}{2} = \frac{3t - w + d}{2},$$

which is consistent with the initial restriction on p_2. Thus, the pair (p_1^*, p_2^*) is an equilibrium whenever $(5/7)(w - t) < d < w - t$.

Case 1.3: $p_1^* \geq \frac{4t-w+2d}{3}$

We have $p_1^* \geq (4t - w + 2d)/3$ whenever $d \leq \frac{5}{7}(w - t)$. The best response of lobbyist two is equal to

$$BR_2(p_1) = \frac{w - 2d + p_1}{2} = \frac{w - 2d + \frac{w+t-d}{2}}{2} = \frac{3w - 5d + t}{4}.$$

Let $p_2^* = (3w - 5d + t)/4$. The restriction on parameters so that p_2^* is below or equal to the lower bound require

$$p_2^* \leq \frac{3t - w + d}{2} \iff d \geq \frac{5}{7}(w - t).$$

Hence, as $p_2^* \leq (3t - w + d)/2$ whenever $d \geq \frac{5}{7}(w - t)$, and $p_1^* \geq (4t - w + 2d)/3$ whenever $d \leq \frac{5}{7}(w - t)$, the pair (p_1^*, p_2^*) is an equilibrium at $d = \frac{5}{7}(w - t)$. Substituting the latter restriction into the optimal bids, yields $p_1^* = (6t + w)/7$ and $p_2* = (8t - w)/7$. The expected utilities of lobbyists are

$$EU_1\left(p_1^*, p_2^*\right) = \frac{18(w-t)}{35}, \ EU_2\left(p_2^*, p_1^*\right) = \frac{16(w-t)}{35}.$$

Case 2: $\frac{3t-w+d}{2} < p_2 < \frac{4t-w+2d}{3}$

The best response of lobbyist one is

$$BR_1(p_2) = 2t - p_2.$$

Let $p_1^* = 2t - p_2$. As before, we analyse three cases depending on where p_1^* is with respect to the lower and upper bound.

Case 2.1: $p_1^* \leq \frac{3t-w+d}{2}$

The best response of lobbyist two is $p_2^* = (w+t-d)/2$. It follows that p_2^* is between the lower and upper bound whenever $(5/7)(w-t) < d < w-t$. Substituting for p_2^* into p_1^*, yields $p_1^* = (3t-w+d)/2$. The pair (p_1^*, p_2^*) is an equilibrium whenever $(5/7)(w-t) < d < w-t$ (compare with Case 1.2).

Case 2.2: $\frac{3t-w+d}{2} < p_1^* < \frac{4t-w+2d}{3}$

The best response of lobbyist two is $BR_2\left(p_1^*\right) = 2t - p_1^*$. Rearranging the inequality restrictions of p_1^*, yields

$$\begin{gathered} \tfrac{3t-w+d}{2} < p_1^* < \tfrac{4t-w+2d}{3} \iff \\ 2t - \tfrac{4t-w+2d}{3} < 2t - p_1^* < 2t - \tfrac{3t-w+d}{2} \iff \\ \tfrac{w-2d+2t}{3} < p_2^* < \tfrac{w+t-d}{2}. \end{gathered} \tag{1}$$

Recall that p_2^* is restricted to the Case 2 assumption:

$$p_2^* \in \left(\frac{3t-w+d}{2}, \frac{4t-w+2d}{3}\right) \tag{2}$$

There are three outcomes here:

1. The upper and lower bounds of (1) coincide with the bounds of (2).

2. The upper bound of (1) is in (2), while its lower bound is outside (2).

3. The lower bound of (1) is in (2), while its upper bound is outside (2).

Outcome 1. When all possible values of p_2^* identified in (1) lie within (2), the following conditions must be satisfied: $(3t-w+d)/2 \leq (w-2d+2t)/3$ for the lower bound, and $(4t-w+2d)/3 \geq (w+t-d)/2$ for the upper bound.

$$\begin{aligned} \frac{w-2d+2t}{3} \geq \frac{3t-w+d}{2} &\iff \frac{5}{7}(w-t) \geq d \\ \text{and } \frac{w+t-d}{2} \leq \frac{4t-w+2d}{3} &\iff \frac{5}{7}(w-t) \leq d. \end{aligned} \tag{3}$$

At $d=(5/7)(w-t)$, the bounds of (1) coincide with the bounds of (2). Hence, the pair $(p_1^*, p_2^*) \in ((3t-w+d)/2, (4t-w+2d)/3)^2$ is a continuum of equilibria. Taking into account the restriction $d=(5/7)(w-t)$, the equilibrium can be rewritten as $p_i^* \in ((8t-w)/7, (6t+w)/7)$ and $p_{-i}^* = 2t - p_i^*$. Here, we also find the symmetric equilibrium, $p_i^* = p_{-i}^* = t$.

Outcome 2. Suppose $d > (5/7)(w-t)$. From (3), the upper bound of (1) is in (2), while its lower bound is less than the lower bound of (2). Hence,

$$p_2^* \in ((3t-w+d)/2, (w+t-d)/2)\,.$$

The interval $((3t-w+d)/2, (w+t-d)/2)$ is well defined as long as $d < w-t$. Hence, the pair (p_1^*, p_2^*) where $p_2^* \in ((3t-w+d)/2, (w+t-d)/2)$ and $p_1^* = 2t - p_2^*$, is a continuum of equilibria whenever $(5/7)(w-t) < d < w-t$.

Outcome 3. Suppose $d < (5/7)(w-t)$, the lower bound of (1) is in (2), while its upper bound is greater than the upper bound of (2). The best response of lobbyist two lies in the interval $(w-2d+2t)/3 < p_2^* < (4t-w+2d)/3$. This interval is well-defined as long as $d > (1/2)(w-t)$. Hence, the pair (p_1^*, p_2^*) where $p_2^* \in ((w-2d+2t)/3, (4t-w+2d)/3)$ and $p_1^* = 2t - p_2^*$ is a continuum of equilibria whenever $(1/2)(w-t) < d < (5/7)(w-t)$.

Case 2.3: $p_1^* \geq \frac{4t-w+2d}{3}$

We have

$$BR_2\left(p_1^*\right) = \frac{w-d+p_1^*}{2} = \frac{w-2d+2t-p_2}{2} = \frac{w-2d+2t}{3}.$$

Let $p_2^* = (w-2d+2t)/3$. It must lie within the lower and upper bound. To that end, we obtain

$$p_2^* > \frac{3t-w+d}{2} \iff \frac{5}{7}(w-t) > d \text{ and } p_2^* < \frac{4t-w+2d}{3} \iff \frac{1}{2}(w-t) < d.$$

Moreover, substituting p_2^* yields $p_1^* = 2t - \frac{w-2d+2t}{3} = (4t-w+2d)/3$. The pair (p_1^*, p_2^*) is an equilibrium whenever $(w-t)/2 < d < (5/7)(w-t)$.

Case 3: $p_2 \geq \frac{4t-w+2d}{3}$

We have

$$BR_1(p_2^*) = \frac{w-2d+p_2^*}{2}. \tag{4}$$

Let $p_1^* = (w-2d+p_2)/2$. As before, we split the argument into three cases.

Case 3.1: $p_1^* \leq \frac{3t-w+d}{2}$

We have $BR_2(p_1^*) = (w+t-d)/2$. Let $p_2^* = (w+t-d)/2$. Substituting the latter into (4), yields $p_1^* = (3w-5d+t)/4$, which is below or equal to the lower bound whenever $d \geq \frac{5}{7}(w-t)$. On the other hand, p_2^* is above or equal to the upper bound, whenever $d \leq \frac{5}{7}(w-t)$.

At $d = (5/7)(w-t)$, the pair (p_1^*, p_2^*) the pair (p_1^*, p_2^*) where $p_1^* = (6t+w)/7$ and $p_2 * = (8t-w)/7$ is an equilibrium.

Case 3.2: $\frac{3t-w+d}{2} < p_1^* < \frac{4t-w+2d}{3}$

We have $BR_2\left(p_1^*\right) = 2t - p_1^*$. Let $p_2^* = 2t - p_1^*$. Substituting p_2^* into (4), yields $p_1^* = (w-2d+2t)/3$. In turn, substituting the latter into the best response of lobbyist two, yields $p_2^* = (4t-w+2d)/3$.

Similar to Case 2.3, p_1^* lies within the lower and upper bound whenever $(w-t)/2 <$

$d < (5/7)(w - t)$. Hence, the pair (p_1^*, p_2^*) is an equilibrium whenever $(w - t)/2 < d < (5/7)(w - t)$.

Case 3.3: $p_1^* \geq \frac{4t-w+2d}{3}$

We have $BR_2(p_1^*) = (w - 2d + p_1^*)/2$. Let $p_2^* = (w - 2d + p_1)/2$. Substituting p_2^* into (4), yields $p_1^* = w - 2d$. In turn, substituting p_1^* into p_2^*, yields $p_2^* = w - 2d$. Note that $w - 2d \geq (4t - w + 2d)/3$ whenever $d \leq (w - t)/2$. Therefore, $p_1^* = p_2^* = w - 2d$ is an equilibrium whenever $d \leq (w - t)/2$.