Sequential expert consultation: On the optimal ordering of product reviews.

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Abstract

A decision maker consults product reviews sequentially on a platform. Reviews are either authentic or fake. Reviewers' trustworthiness is privately observed by the platform. Each review is costly to read. In each round, the decision maker chooses in a sequentially rational way whether and which review to read. The platform's ordering rule determines the order of presentation of reviews as a function of trustworthiness levels. The ordering rule influences the informativeness of reviews by affecting reviewers' incentives. We characterise the optimal ordering rule and show that it is stochastic.

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JEL classification: D81, D83.

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1 Introduction

In many contexts, a decision maker acquires information from several sources before making a decision. A potential buyer on an online shopping platform might read several product reviews before making a purchase decision, a citizen might read several online news articles before making up his mind on a political question. Consulting several sources can often be beneficial for decision makers because sources have uncertain goals or are imperfectly informed. In many instances, consultation occurs sequentially because each message that the decision maker reads requires time to process. In an online context, sources are furthermore typically made available by a platform (e.g., Google or Amazon) which determines the order in which sources are presented.

Though the above description matches a number of information search problems, our focus here is on online product reviews, where fake reviewers tied to firms coexist with authentic and benevolent reviewers. The phenomenon is widely acknowledged as economically significant. Competition authorities in the UK and US have since 2021 investigated Google and Amazon, alleging excessive leniency. A 2023 UK governmental report finds a prevalence of 10-15% of fake reviews across common goods and the rapid rise of AI language models could exacerbate the problem.[1] While focusing on reducing the number of fake reviews via legal and technological means is natural, we are witnessing an arms' race between producers and policers of fake reviews whose outcome is unpredictable.

Our approach is to optimise the design of review systems to minimise the harm caused by fake reviewers. We focus on the rule governing the presentation

[1] "Fake online reviews research: Investigating the prevalence and impact of fake online reviews", Department for Business and Trade, April 2023, Alma Economics

order of reviews, which we assume can condition on the platform's privately observed estimate of reviewers' trustworthiness. Does the ordering rule affect the informativeness of reviewers and if so, how? What is the optimal rule, in terms of maximising information transmission? Is it deterministic or stochastic, and how exactly is it conditioned on reviewers' trustworthiness? And if an optimal rule is used, is it the case that removing reviews that are likely to be fakes improves consumer welfare?

To answer these questions, we analyse the following model. A decision maker (DM) must choose an action a to match the underlying unobserved state (product quality) drawn from the unit interval, where a could for example be the amount bought. He faces n perfectly informed and indistinguishable senders whom he consults sequentially, each at an arbitrarily small cost c. He consults in a sequentially rational way: at any point in time he chooses whether and whom to consult (where senders only differ in terms of their position in the presentation order) in a way that maximises his current expected payoff. Senders have two possible (privately observed) preference types: *unbiased* or *biased*. Unbiased senders share DM's objective to match the state. Biased senders wish to maximise DM's action. Each sender's probability of being unbiased (which parameterises his trustworthiness) is privately observed by the platform. DM only knows the aggregate profile (i.e., the empirical distribution) of trustworthiness levels. Senders know the aggregate profile and their own trustworthiness. Senders communicate simultaneously via a cheap talk message, the ordering rule being commonly known.

We focus on symmetric and monotone equilibria in pure strategies. Symmetry implies that senders all use the same type-dependent communication strategy. Monotonicity stipulates that senders' messages are weakly increasing in the state and that receivers' beliefs are increasing in senders' messages.

We first show that every informative equilibrium in this class is partitional. The state space is partitioned into N intervals corresponding to N messages $t_1, .., t_N$. If sender i is unbiased, he sends t_r if the state lies in interval r. If sender i is biased, he always sends t_N. DM consults senders following the order of presentation. He continues to consult as long he has only received message t_N and stops as soon as he observes $t_r < t_N$. For each ordering rule, we focus on the most informative equilibrium within this class, which is the equilibrium with the largest number of intervals.

Our focus is on identifying ordering rules that maximise DM's expected utility. If senders are equally trustworthy, intuition suggests that the rule should not matter. If senders are unequally trustworthy, consulting more trustworthy senders first would appear beneficial. We find that both of the above intuitions are violated. Whether or not senders are equally trustworthy, using a well chosen stochastic ordering rule improves DM's expected utility by improving individual senders' informativeness.

The fundamental driving force behind the result is a preemption motive. When the state is low, the earlier a biased sender's expected position in the presentation order and the higher the trustworthiness of senders located after him, the higher his incentive to deviate to the second highest message so as to preempt further consultation which might reveal the low state. This incentive determines unbiased sender's informativeness by defining the maximum achievable number of partition intervals.

We first show that the problem of finding an ordering rule that maximises informativeness is a max-min problem, as in maximising the minimum value across biased senders from continued consultation when the state is very low. Ceteris paribus, increasing the likelihood that a very trustworthy sender is asked early improves the incentives of other senders, but hurts the incentives of the former. The ordering rule that maximises informativeness trades off these forces by making all senders' position uncertain and aligning expectations across senders. We then show that among rules that maximise informativeness, we can always find one such that it is incentive compatible for DM to consult senders following the presentation order.

We present a number of extensions. Our first extension focuses on deterministic rules. In our second extension, DM observes the trustworthiness of individual senders. It follows immediately that DM consults in a deterministic order, which in turn hurts DM by reducing informativeness. The third extension explores how DM's expected payoff is affected by the aggregate distribution of trustworthiness levels for a fixed number of senders. We find that for a fixed probability of the event that all senders are biased, the optimal distribution is obtained by maximising the trustworthiness of one sender while minimising others' trustworthiness.

Literature review Our setup builds on Morgan and Stocken (2001) who assume uncertainty about the sender's bias in the Crawford and Sobel (1982) canonical cheap talk model. Le Quement (2016) extends this problem to the case where the receiver consults several senders sequentially. Le Quement (2016) assumes that the aggregate distribution of trustworthiness levels is degenerate and exogenously imposes the fully random ordering rule. The issue of whether DM has an incentive to follow the presentation order is furthermore by definition trivial in such a homogeneous experts setup. This paper instead considers arbitrary distributions of trustworthiness levels and studies all possible ordering rules, focusing on the untouched question of the optimal ordering rule and taking into account the problem of ensuring that DM should be willing to follow the presentation order. Ottaviani and Sørensen (2001) asks who should speark first in a context where several imperfectly informed senders are consulted, the ex ante competence of these senders being different and these being motivated by reputational concerns. Krishna and Morgan (2001a) reexamine Gilligan and Krehbiel (1989) and consider both heterogeneous and homogeneous senders who all observe the state. They find that some (not all) legislative rules lead to full revelation when combined with heterogeneous preferences. In Austen-Smith (1993), the receiver faces two senders holding noisy information in a binary setup. Under some conditions, full revelation is possible with a single sender but not when two senders are consulted simultaneously. McGee and Yang (2013) study a setup where a decision maker's optimal decision is a (multiplicative) function of the uncorrelated types of two privately informed senders. In Li et al. (2016), a principal has to choose between two potential projects, information about returns being held separately by two senders who are each biased towards their own project. In both papers presented above, senders' informativeness levels are strategic complements (in contrast to our setup): informative communication by the other sender makes deviations from the truth more costly. Alonso et al. (2008) consider information transmission in a multi-division organization, where each division's profits depend on how its decision matches its privately known local conditions and the other division's decisions. One possible decision protocol is centralization, whereby division managers report to central headquarters which decide for both divisions. They find that a stronger desire to coordinate decisions worsens headquarters' ability to retrieve information from divisions. In Rantakari (2016) or Moreno de Barreda (2010), the receiver is exogeously or endogenously also in possession of some information. The main effect is that information available to the receiver can crowd out the information transmission by the sender.

This paper also relates to the literature on Bayesian-reputation building in games of information transmission (Sobel (1985), Benabou and Laroque (1992), Morris (2001) and Ely and Välimäki (2003)). Morris (2001) studies a two period advice game with a binary state space and uncertain sender preferences identical to ours. An unbiased sender has an incentive to lie in the early period in order to achieve a good reputation and be influential later. Our papers share the feature that biased senders' behavior exerts a negative externality on the informativeness of unbiased senders. In Morris (2001), an unbiased sender does not always truthfully announce a high signal because such an announcement hurts his reputation. In our paper, an unbiased sender communicates in a noisy way also when the state is not high, so as to discourage biased senders from deviating downwards.

The paper also connects to the literature on search and pricing on goods markets (see for example Baye et al (2006), Stahl (1989), Wolinsky (1986), Diamond (1971), Janssen and Parakhonyak (2014), Anderson and Renault (1999)). In these models, firms also have an endogenous preference over the consumer's search decisions and typically wish to discourage further search. The recent strand on ordered search is of particular relevance. See for example Wright et al (2019), Haan et al. (2018), Derakhshan (2018), Armstrong (2017), Arbatskaya (2007), Armstrong et al. (2009), Wilson (2010).

The paper proceeds as follows. Section 2 presents the model. Section 3 characterises equilibria for given ordering rules. Section 4 studies welfare properties and derives the optimal ordering rule. Section 5 examines extensions.

2 Model

The state of the world ω is drawn from the uniform distribution on $[0, 1]$ and captures the underlying true product quality. An uninformed receiver (DM) faces a set of n senders (reviewers) $\chi = \{A, B, ..\}$ on a shopping platform, each of whom privately observes the state and simultaneously sends a cheap talk message $m_i \in M = [0, 1]$. In the first phase of the game, the receiver can sequentially consult the senders at a cost c per sender, where c is arbitrarily small but positive. Once he stops consulting, he picks an action $a \in \Re$ and his utility is $-(\omega - a)^2 - \widetilde{n}c$, where $\widetilde{n}$ is the number of senders consulted. The optimal action after information collection is simply the conditional expected value of ω. Our assumption on c means that DM will carry on consulting as long as he expects that more consultation can generate more information.

Each sender has a privately observed type (1 or 2). Type 1, the unbiased type, has utility function $-(\omega - a)^2$. Type 2, the biased type, has utility function a. Type 1 is thus benevolent while type 2 wants to maximise DM's belief about ω. Sender i's probability of being of type 1 is p_i and thus parameterises his ex ante trustworthiness. Each sender's type is independently drawn. DM knows the empirical distribution of p_is but does not observe the identity of the sender behind each message (the identifier i). Senders know their own identity (i) and the aggregate distribution of p_is. The platform, in contrast, observes each sender's p_i directly. In other words, it knows the aggregate distribution of p_is and each sender's i. Let $\eta = \prod_{i=1}^{n}(1 - p_i)$, so η is the commonly known probability that all senders are biased.

The platform presents senders' messages in a presentation order which is generated by the commonly known ordering rule Γ. A presentation order dictates which sender's message is to be presented in which position in the sequence of the messages. The ordering rule is deterministic if one presentation order is assigned ex ante probability one. Denote by $D(\chi)$ the set of deterministic orders and denote by d any element of this set. Denote by θ_d^{Γ} the probability assigned to d under Γ. An ordering rule is given by $\Gamma = \{\theta_d^{\Gamma}\}_{d \in D(\chi)}$. Denote by p^l the unobserved trustworthiness of the sender appearing in position l of the presentation order. Denote by m^l the message of the sender in position l in the order. Denote by $p^{l,d}$ the trustworthiness of the sender appearing in position l of order d.

A sender strategy pins down how he communicates for each preference type that he might be assigned. A pure strategy for a sender $i \in \{A, B, ..\}$ is given by two functions μ_i^r, for $r \in \{1, 2\}$, where $\mu_i^r : [0, 1] \to [0, 1]$ is such that $\mu_i^r(w)$ maps the state of nature $\omega \in [0, 1]$ into a message in M. A communication strategy is monotone if $\mu_i^r(w)$, for $r \in \{1, 2\}$, is weakly increasing in ω. A profile of sender strategies induces monotonic beliefs if, to put it succintly, profiles of messages that are higher yield higher beliefs of DM. A precise definition is provided later.

A pure strategy of DM is composed of a sampling rule and an action rule. A pure sampling rule specifies, for any history of observed messages, whether or not DM continues to consult and which review he consults among the presented reviews. A pure action rule specifies the action a chosen if DM stops consulting, for any history of observed messages.

Note that given $c > 0$, there exists no fully revealing equilibrium in which all senders always truthtell and send $m = \omega$ whatever the state and their preference type. Such an equilibrium would be supported by out of equilibrium beliefs such that DM chooses a punishment action (say $a = 0$) whenever messages differ. However, the equilibrium breaks down because DM has a strict incentive to stop after one consultation given $c > 0$.

We restrict ourselves to equilibria featuring pure sender strategies that are symmetric (all senders use the same strategy), monotonic (i.e. weakly increasing in the state) and inducing monotonic beliefs (see the proof of Proposition

1 in Appendix B for a precise definition). We call such equilibria symmetric and monotone. We show that all informative equilibria within this class are partitional equilibria and thereby outcome equivalent to an equilibrium featuring the following simple strategy profile, which we shall focus on. There are thresholds $t_0 = 0 < t_1 < ... < t_{N-1} < t_N = 1$. An unbiased sender sends message $m = t_r$ if $\omega \in (t_{r-1}, t_r]$ $\forall r = 1, .., N$ and t_1 if $\omega = 0$. A biased sender always sends $m = t_N$. DM's sampling rule is a stopping rule. He stops consulting as soon as he encounters $t_r \neq t_N$. Indeed, after $t_r \neq t_N$, he acknowledges that he has now learned that $\omega \in (t_{r-1}, t_r]$ and will not learn more by consulting another review. On the other hand, he continues consulting as long as he has only encountered t_N and has not consulted all senders. In this case, he remains uncertain about ω and might gain new information by consulting the next sender. For any out of equilibrium profile of messages $\mathbf{m}$ for which beliefs cannot be derived via Bayes' rule, denoting by $\underline{m}(\mathbf{m})$ the lowest message in this set, the induced belief of DM is assumed to be $E[\omega \,|\omega \in (t_{r-1}, t_r]]$ if $\underline{m}(\mathbf{m})$ is located in the rth interval. Furthermore, DM's (sequential) consultation follows the order of presentation. For this to be incentive compatible, it must be true that after consulting the first r senders in the presentation order, the most informative sender (in expectation) is in position $r + 1$, for any $r \in \{0, .., n-1\}$.[2] We call an equilibrium of the above type a simple partitional equilibrium of size N. Finally, we define an informative equilibrium as one in which there is no single action that DM picks after any equilibrium history (senders's messaging rules being uninformative).

Our equilibrium concept is Perfect Bayesian Equilibrium (PBE). Under a

[2] Note that the equilibrium described above still exists under $c = 0$, as the DM has no strict incentive to deviate from the assumed behavior. From DM's perspective, the equilibrium is however trivially dominated by one in which he always consults all senders and all truthtell.

given ordering rule Γ, an equilibrium is given by a profile of strategies (one for each sender in χ and one for DM) as well as a system beliefs. A given profile of strategies and a system of beliefs constitute a PBE if players' strategies are sequentially rational given beliefs and other players' strategies, while beliefs are derived via Bayes'rule whenever possible. All the results stated in our analysis, whether positive or normative, are limit results in sense that there is some $\overline{c} > 0$ such that they hold true for any $c \in (0, \overline{c})$.

We focus on ordering rules that are optimal from DM's perspective. It seems reasonable to assume that platforms aim at maximising the informativeness of reviews so that instances where a buyer returns a purchased items are minimised. The DM optimal ordering is also weakly or strictly preferred by all reviewer types. Unbiased reviewers share DM's preferences. Biased reviewers are indifferent among all ordering rules, as the expected value of DM's action is constant across all possible information generating experiments by the law of iterated expectation.

3 Positive analysis

Proposition 1 *For any informative, symmetric and monotone equilibrium, there exists an outcome equivalent simple partitional equilibrium.*

Proof: See Appendix B.

The above Proposition establishes that under the imposed restriction on strategies, it is without loss of generality to focus on simple partitional equilibria. In what follows, we characterise necessary and sufficient conditions for the existence of a simple partitional equilibrium featuring partition $\{t_r\}_{r=1}^{N-1}$ under ordering rule Γ. We analyse separately the incentives of senders and

DM.

3.1 Sender incentives

We start by pinning down the beliefs of DM. Given $\{t_r\}_{r=1}^{N-1}$ and ordering rule Γ, the belief of DM when he first observes some $t_r \neq t_N$ is given by:

$$E[\omega\,|m = t_r] = \frac{t_{r-1} + t_r}{2}.$$

After he has consulted all n senders and received message t_N in total n times, his belief is given by

$$\begin{aligned} &E[\omega\,\left|m^1 = .. = m^n = t_N\right.] \\ &= \left(\frac{(1 - t_{N-1})(1-\eta)}{(1 - t_{N-1})(1-\eta) + \eta}\right)\frac{t_{N-1}+1}{2} + \left(\frac{\eta}{(1 - t_{N-1})(1-\eta) + \eta}\right)\frac{1}{2}. \end{aligned}$$

The above expected value accounts for two possible events. Either at least one of the senders is unbiased, in which case $\omega \geq t_{N-1}$, or nothing has been learned.

Given Γ and $\{t_r\}_{r=1}^{N-1}$, an important quantity is the probability assigned by sender i to the event that DM will observe $m^1 = .. = m^2 = t_N$ conditional on $\omega \leq t_{N-1}$, sender i sending t_N and taking as given that m_i will be observed. Denote this quantity by $\Psi_{i,\Gamma}$. We have

$$\Psi_{i,\Gamma} = P_{i,\Gamma}(m^1 = .. = m^n = t_N\,|\omega \leq t_{N-1}, m_i = t_N).$$

For any order $d \in D(\chi)$ and $i \in \chi$, denote by respectively $\chi_d^{i,-}$ and $\chi_d^{i,+}$ the set of senders who are presented before and after i. Let

$$Q_{i,\Gamma} := \sum_{d \in D(\chi)} \theta_d^{\Gamma}\left[\prod_{j \in \chi_d^{i,-}} (1 - p_j)\right].$$

The above is the ex ante probability that sender i will be consulted given $\omega \leq t_{N-1}$, $\{t_r\}_{r=1}^{N-1}$ and ordering rule Γ. For every $d \in D(\chi)$, we have

$$P_{i,\Gamma}(d \left| \omega \leq t_{N-1}, m^i = t_N \right.) = \frac{\theta_d^\Gamma \left[\prod_{j \in \chi_d^{i,-}} (1-p_j) \right]}{Q_{i,\Gamma}},$$

where by convention $\prod_{j \in \chi_d^{i,-}} (1-p_j) = 1$ if $\chi_d^{i,-} = \emptyset$. Next, we have

$$\begin{aligned} \Psi_{i,\Gamma} &= \sum_{d \in D(\chi)} \left(P_{i,\Gamma}(d \left| \omega \leq t_{N-1}, m^i = t_N \right.) \left[\prod_{j \in \chi_d^{i,+}} (1-p_j) \right] \right) \\ &= \frac{\prod_{j \neq i} (1-p_j)}{Q_{i,\Gamma}}. \end{aligned}$$

That is, $\Psi_{i,\Gamma}$ equals the probability that all other senders are biased divided by the probability that i is asked. Note the formula for $\Psi_{i,\Gamma}$ is independent of the assumed equilibrium partition $\{t_r\}_{r=1}^{N-1}$.

We now provide necessary and sufficient existence conditions for $\{t_r\}_{r=1}^{N-1}$ to be sender incentive compatible.

Lemma 1 *Fix Γ. Partition $\{t_r\}_{r=1}^{N-1}$ is sender incentive compatible iff:*

$$t_r - E[\omega \left| m = t_r \right.] = E[\omega \left| m = t_{r+1} \right.] - t_r \;\; \forall r < N-1, \tag{1}$$

$$t_{N-1} - E[\omega \left| m = t_{N-1} \right.] = E[\omega \left| m^1 = .. = m^n = t_N \right.] - t_{N-1}, \tag{2}$$

$$\Psi_{i,\Gamma} E[\omega \left| m^1 = .. = m^n = t_N \right.] + (1-\Psi_{i,\Gamma}) E[\omega \left| m = t_1 \right.] \geq E[\omega \left| m = t_{N-1} \right.] \;\; \forall i \in \chi. \tag{3}$$

Proof: See in Appendix A.

Conditions (1) and (2) ensure that unbiased senders have no deviation incentives, by requiring that at any threshold $\omega = t_r$, for DM$\in \{1, .., N-1\}$, an unbiased sender is indifferent between messages t_r and t_{r+1}. Condition (1) implies that $t_r = \frac{r}{N-1} t_{N-1}$ for $r < N-1$. Using this and solving (2) for t_{N-1} given N and η yields the unique solution

$$t^*_{N-1} = -\frac{\left(-2N + \sqrt{4N\eta(-1+N)+1}+1\right)}{2N(1-\eta)}. \tag{4}$$

So (1) and (2) yield a unique admissible partition

$$\left\{ t^*_1 = \frac{t^*_{N-1}}{N-1}, .., t^*_r = \frac{r t^*_{N-1}}{N-1}, .., t^*_{N-1} \right\}$$

for any $N > 1$ and $\eta \in (0,1)$. Note in particular that the above partition is independent of the assumed ordering rule as the latter does not affect (1) and (2). This will turn out to be a very useful property when we study optimal ordering rules. Note also that (1) and (2) rewrite as

$$\frac{E[\omega \,|m^1 = .. = m^n = t_N]}{t_{N-1}} = \frac{2(N-1)+1}{2(N-1)}. \tag{5}$$

Condition (3) is the incentive condition for biased senders, and thus determines whether the partition pinned down by (1) and (2) is actually sender incentive compatible. A biased sender i must prefer sending $m^i = t_N$ for any ω. To ensure this, it is sufficient to ensure no deviation incentive at $\omega = 0$. Now, consider incentives of a biased sender i at $\omega = 0$. Sending $m_i = t_N$ is risky as DM will keep on sampling and might with probability $(1 - \Psi_{i,\Gamma})$ encounter an unbiased sender and learn that $\omega \leq t_1$. Sending $m_i = t_{N-1}$ is the best deviation as it preempts any further sampling while it yields the second highest belief $E[\omega \,|t_{N-1}]$. Clearly, $\Psi_{i,\Gamma}$ determines the attractiveness of $m^i = t_N$. The higher $\Psi_{i,\Gamma}$, the better. Note that $\Psi_{i,\Gamma}$ is smaller the earlier i's expected position in the presentation order and the higher the expected trustworthiness of experts consulted after i. Using (1), condition (3) rewrites as:

$$\frac{E[\omega \,|m^1 = .. = m^n = t_N]}{t_{N-1}} \geq \frac{(1-\Psi_{i,\Gamma})+2}{2}. \tag{6}$$

Using (5) to replace the LHS in the above inequality (6), we may conclude that there exists a sender incentive compatible N-intervals partition if and only if

$$\Psi_{i,\Gamma} \geq \frac{N-2}{N-1},\ \forall i \in \chi. \tag{7}$$

Furthermore, such a partition is unique if it exists. Note that $\frac{N-2}{N-1}$ is increasing in N, so an equilibrium of larger size (larger N) requires higher $\Psi_{i,\Gamma}$. The intuition is that a larger N implies larger $E[\omega \,|t_{N-1}]$ and lower $E[\omega \,|t_1]$, so that a larger N makes it more attractive to deviate to $m^i = t_{N-1}$ given $\omega = 0$. For a given order of consultation Γ, define

$$\Psi_{\Gamma}^{\min} = \min_{i \in \chi} \Psi_{i,\Gamma}. \tag{8}$$

We summarise the above insights in the following Proposition.

Proposition 2 *a) Fix* Γ*. There exists a sender incentive compatible partition of size* N *if and only if* $\Psi_{\Gamma}^{\min} \geq \frac{N-2}{N-1}$*. If it exists, it is unique and is given by the partition* $\{t_r^*\}_{r=1}^{N-1}$ *defined in (**??**).*

b) Consider two ordering rules Γ *and* Γ'*. If a sender incentive compatible partition of size* N *exists under both orders, then it features the same partition* $\{t_r^*\}_{r=1}^{N-1}$*.*

Comparing any two ordering rules Γ and Γ', we see that either the sets of sender incentive compatible partitions under Γ and Γ' are identical; or one is a superset of the other and contains equilibria of larger size, the only deciding factor being the size of $\Psi_{\Gamma}^{\min}$ and $\Psi_{\Gamma'}^{\min}$. A two-interval equilibrium always exists as $\frac{2-2}{2-1} = 0$, while equilibria of larger size require $\Psi_{\Gamma}^{\min} \geq \frac{1}{2}$.

3.2 DM incentives

We now analyse DM's incentive to consult following the presentation order. Given that senders have identical type-dependent communication strategies, at any point in time the DM's optimal choice is to consult the sender (as pinned down by a position in the presentation order) whose expected trustworthiness is highest. The expected trustworthiness of the first sender in the presentation order is

$$E\left[p^1\right] = \sum_{d \in D(\chi)} P(\theta_d^\Gamma) p^{1,d}. \tag{9}$$

Given $\{t_r\}_{r=1}^{N-1}$ and Γ, assuming that DM has followed the presentation order in the first k rounds of consultation and observed $m^1 = .. = m^k = t_N$, the expected value of p^l for $l > k \geq 1$ is given by

$$E\left[p^l \left| m^1 = .. = m^k = t_N \right.\right] = \sum_{d \in D(\chi)} P(\theta_d^\Gamma \left| m^1 = .. = m^k = t_N \right.) p^{l,d}, \tag{10}$$

where

$$P(\theta_d^\Gamma, m^1 = .. = m^k = t_N) = \theta_d^\Gamma \left(t_{N-1} \prod_{i=1}^{k} (1 - p^{i,d}) + 1 - t_{N-1} \right). \tag{11}$$

In words, conditional on $m^1 = .. = m^k = t_N$, DM updates his prior over the set of deterministic sequences assigned positive probability under Γ, each of which assigns a specific sender to position l. He uses this to derive the implied weighted average of $p^{l,d}$s and to thus identify which sender to consult next.

Lemma 2 *Fix Γ and $\{t_r\}_{r=1}^{N-1}$. Consulting following the order of presentation* *is DM incentive compatible iff* :

$$E\left[p^1\right] \geq E\left[p^l\right] \ \ \forall l > 1, \tag{12}$$

$$E\left[p^{k+1}\left|m^1 = .. = m^k = t_N\right.\right] \geq E\left[p^l\left|m^1 = .. = m^k = t_N\right.\right], \tag{13}$$

$$\forall k, l \ s.t. \ k \in \{1, .., n-1\},\ l > k+1. \tag{14}$$

The above condition ensures that DM always wants to follow the presentation order. The first inequality ensures that he wants to consult the first sender in the presentation order when consulting first. The second condition ensures that for any $k \in \{1, .., n-1\}$, after observing $m^1 = .. = m^k = t_N$ and thus deciding to consult again, the most informative sender is located in position $k+1$ of the presentation order.

4 Normative analysis

4.1 Welfare properties of equilibria

In a partitional equilibrium featuring partition $\{t_r\}_{r=1}^{N-1}$, the expected payoff of DM is

$$\begin{aligned} &\Pi_{DM}(N, \eta) \\ &= -\left(1-\eta\right)\sum_{i=1}^{N-1}\left[\int_{t_{i-1}}^{t_i}\left(\frac{t_i + t_{i-1}}{2} - \omega\right)^2 d\omega\right] \\ &-\eta\sum_{i=1}^{N-1}\left[\int_{t_{i-1}}^{t_i}\left(E[\omega\left|m_A = m_B = t_N\right.] - \omega\right)^2 d\omega\right] \\ &-\left[\int_{t_{N-1}}^{1}\left(E[\omega\left|m_A = m_B = t_N\right.] - \omega\right)^2 d\omega\right]. \end{aligned} \tag{15}$$

Above, the first line of the RHS expression corresponds to the scenario where $\omega \leq t_{N-1}$ and there is at least one unbiased sender. The second line is the scenario $\omega \leq t_{N-1}$ and there is no unbiased sender. The third line is the scenario $\omega > t_{N-1}$ so that all senders send t_N. We ignore sampling costs which are assumed arbitrarily small. We obtain the following results.

Proposition 3 *a) If an equilibrium of size N exists under two ordering rules Γ and Γ', then DM achieves the same equilibrium expected payoff $\Pi_{DM}(N, \eta)$ under both ordering rules.*

b) $\Pi_{DM}(N+1, \eta) > \Pi_{DM}(N, \eta)$ *for any* $N \geq 1$ *and* $\eta \in (0,1)$.

c) $\frac{\partial \Pi_{DM}(N,\eta)}{\partial \eta} < 0$ *for any* $N \geq 1$ *and* $\eta \in (0,1)$.

Proof: See in Appendix A.

The proof of Point a) is as follows. Recall first that by point b) of Proposition 2, if an equilibrium of size N exists under two ordering rules Γ and Γ', then it features the same partition $\{t_r^*\}_{r=1}^{N-1}$. Next, simply note that for a fixed partition, DM's expected utility fundamentally depends only on one aspect, namely whether or not at least one of the senders is unbiased. If all senders are biased, he will end up consulting n times and receive message t_N n times regardless of the consultation order. If at least one of the senders is unbiased, then given any fixed state $\widetilde{\omega}$ he will end up with the same final belief regardless of the consultation order. Specifically, if $\omega \leq t_{N-1}$, he will learn the interval in which $\widetilde{\omega}$ is located while if instead $\omega > t_{N-1}$, he will observe n times message t_N.

Point b) states that among two partitional equilibria, the equilibrium with a larger number of intervals yields a higher expected utility of DM. This reflects the fact that a less coarse partition allows unbiased senders to communicate more informatively. Point c) captures the fact that a lower probability of all senders being biased implies a higher probability of learning ω accurately.

4.2 Optimal ordering rules

We now identify an optimal ordering rule, i.e. a rule that maximises the achievable expected payoff of DM.[3] By Proposition 3, an ordering rule Γ is optimal if it yields the equilibrium of largest size among all ordering rules. By Proposition 2, an ordering rule Γ yields the largest achievable sender incentive compatible partition if it satisfies:

$$\Gamma = \arg\max_{\Gamma} \min_{i \in \chi} \Psi_{i,\Gamma}. \tag{16}$$

Denote by $N^{\max}$ the size of the largest achievable sender incentive compatible partition.

In principle, the incentive compatibility constraint of DM could complicate the search for an optimal ordering rule as some partitions that are sender incentive compatible under a given Γ might not be part of an equilibrium as DM's incentive compatibility conditions are not satisfied. To account for this potential issue, we take a two steps approach in our search for an optimal ordering rule. We first ignore DM's incentive compatibility condition and find a necessary and sufficient condition for an ordering rule to solve (16). As we will show in Lemma 3 shortly, all of these ordering rules yield the same value of $\Psi_{\Gamma}^{\min}$ and the same largest sender incentive compatible partition $\{t_r^*\}_{r=1}^{N^{\max}-1}$. Next, we show that among these ordering rules, there is guaranteed to exist at least one such that under this largest partition $\{t_r^*\}_{r=1}^{N^{\max}-1}$, DM's incentive compatibility constraint is also satisfied. This second step is accomplished in two substeps, by first identifying a simple class of rules that satisfy (16) and then searching within this class.

[3] Recall that a rule typically yields a set of simple partitional equilibria, and we focus on the largest equilibrium under each rule.

Lemma 3 *An order* Γ *satisfies (16) if and only if* $\Psi_{i,\Gamma} = \Psi_{j,\Gamma} = \frac{\eta}{1-\eta} \sum_j \frac{p_j}{1-p_j}$ *for all* i, j.

Proof: See in Appendix A.

The above Lemma features two aspects. First, in all rules satisfying (16), it is true that $\Psi_{i,\Gamma} = \Psi_{j,\Gamma}$ for any i, j. To see this, assume there exists exists exactly one sender k such that $\Psi_{k,\Gamma} \leq \Psi_{j,\Gamma}$ for all j, with at least one strict inequality say for sender l. Then by continuity of the functions $\{\Psi_{i,\Gamma}\}_i$, which are linear equations on the probabilities of all deterministic orders with coefficients that are polynomials in $\{p_i\}_i$, it is possible to find an order Γ' where $\Psi_{k,\Gamma'} > \Psi_{k,\Gamma}$ and $\Psi_{k,\Gamma'} < \Psi_{l,\Gamma'} < \Psi_{l,\Gamma}$. This means that $\min_{i \in \chi} \Psi_{i,\Gamma'}$ is larger than $\min_{i \in \chi} \Psi_{i,\Gamma}$, which leads to a contradiction. A second insight is that if the senders' beliefs are such that $\Psi_{i,\Gamma} = \Psi_{j,\Gamma}$ for all i, j, they pin down a unique admissible set of beliefs for all senders, namely $\Psi_{i,\Gamma} = \frac{\eta}{1-\eta} \sum_j \frac{p_j}{1-p_j}$ for all i. This means that given any ordering rule, we can easily check whether or not it satisfies (16).

While Lemma 3 gives necessary and suffient conditions for an order to satisfy (16), one faces an issue of dimensionality when explicitly constructing ordering rules that satisfy (16). The Lemma yields n equations whereas an ordering rule is pinned down by $n! - 1$ unkowns, as there are $n!$ possible deterministic orders while the probabilities of all deterministic orders must add up to 1. We take a constructive approach and identify a class of ordering rules that satisfy (16). The class builds on the concept of latin squares, first studied in the 18th century by Korean and Swiss mathematicians Choi Seok-jeong and Leonhard Euler.

Definition *a) A latin square ordering rule is an ordering rule such that exactly* n *deterministic orders* $\{d_1, ..., d_n\}$ *have positive probability and for every*

$i \in \chi$ and $l \in \{1,..,n\}$, there is a unique $d \in \{d_1,...,d_n\}$ for which sender i occupies position l. b) A proportional latin square ordering rule is a latin square ordering rule such that for any $d \in \{d_1,...,d_n\}$, $\theta_d = \frac{p_i/(1-p_i)}{\sum_j p_j/(1-p_j)}$, where i is the sender who occupies position 1 in d.

For example, for $\chi = \{A, B, C\}$ there are two possible latin square ordering rules. One is such that only $\{\theta_{ABC}, \theta_{BCA}, \theta_{CAB},\}$ are positive and one is such that only $\{\theta_{ACB}, \theta_{BAC}, \theta_{CBA}\}$ are positive. One can represent each of these as a square, where each row corresponds to a different deterministic order assigned positive probability. Each of the obtained squares is a latin square. The first of these latin square rules yields the following proportional latin square ordering rule:

$$\left\{\theta_{ABC} = \frac{p_A/(1-p_A)}{\sum_{i\in\chi} p_i/(1-p_i)}, \theta_{BCA} = \frac{p_B/(1-p_B)}{\sum_{i\in\chi} p_i/(1-p_i)}, \theta_{CAB} = \frac{p_C/(1-p_C)}{\sum_{i\in\chi} p_i/(1-p_i)}\right\}.$$

Note that in any proportional latin square ordering rule, the probability that a sender appears first in the presentation order is increasing in the sender's trustworthiness and decreasing in others' trustworthiness.

We obtain the following result.

Lemma 4 *All proportional latin square ordering rules satisfy (16).*

Proof: See in Appendix A.

The above Lemma establishes that proportional latin square rules achieve the maximal partition size if we ignore DM's incentive constraint. There is no known way to characterize all latin squares of a general order n. Moreover, it is not known how many latin squares of a particular order exist, although this number is exponentially increasing in n. Thus, since proportional latin squares are a subset of all possible optimal ordering rules, there is no known way to characterize all optimal rules.

The next question is whether we can find any proportional latin square ordering rule that is furthermore DM incentive compatible. The answer is positive.

Lemma 5 *Among the set of proportional latin square rule, there is a rule* Γ *such that given* Γ *and* $\{t_r^*\}_{r=1}^{N^{\max}-1}$*, consulting according to the order of presentation is incentive compatible for DM.*

The proof of the above Lemma is as follows. We have a simple $(n-1)$-step algorithm for identifying a proportional latin square rule Γ^* such that given Γ^* and $\{t_r^*\}_{r=1}^{N^{\max}-1}$, DM's incentive conditions are satisfied. To see this, start from an arbitrary proportional latin square ordering rule Γ and assume that the equilibrium partition is $\{t_r^*\}_{r=1}^{N^{\max}-1}$. It is easy to show using the rearrangement inequality that in the first consultation, the sender appearing in position 1 of the presentation order is the most trustworthy sender in expectation.

Consider now consultation round 2 assuming that DM observed message t_N^* in the first consultation. Assess the relative trustworthiness of senders located in positions 2 to n of the presentation order. If sender $r = 2$ is the most trustworthy sender, then keep the ordering rule Γ and proceed to sender $r = 3$. If instead some $r > 2$ is the most trustworthy sender, then construct a new ordering rule Γ' by permutating senders 2 and r in all of the n sequences that have positive probability under Γ. Note that Γ' is also a proportional latin square rule and it is such that in round 2, sender 2 is the most trustworthy sender. The reason is that in round 2, the expected trustworthinesses of senders 2 and r have now been interchanged, as is immediately clear from (10) and (11).

Repeat the procedure for round 3, by checking who is the most trustworthy sender after t_N^* has been observed in rounds 1 and 2. Iterate the procedure until consultation round n is reached. At this point, one has constructed a proportional latin square rule Γ^* such that given Γ^* and $\{t_r^*\}_{r=1}^{N^{\max}-1}$, DM's incentive conditions are satisfied.

To conclude, the following Proposition follows immediately from our previous three Lemmas:

Proposition 4 *Some proportional latin square ordering rule is an optimal rule. It can be identified in no more than $n-1$ steps.*

We conclude with an example featuring two senders. Let $\chi = \{A, B\}$. $\Psi_{A,\Gamma}$ is decreasing in θ_{AB} and $\Psi_{B,\Gamma}$ is increasing in θ_{AB}. $\Psi_{A,\Gamma}$ and $\Psi_{B,\Gamma}$ cross once. The θ_{AB} that maximises $\min_{i \in A,B}\{\Psi_{A,\Gamma}, \Psi_{B,\Gamma}\}$ is thus such that $\Psi_{A,\Gamma} = \Psi_{B,\Gamma}$.

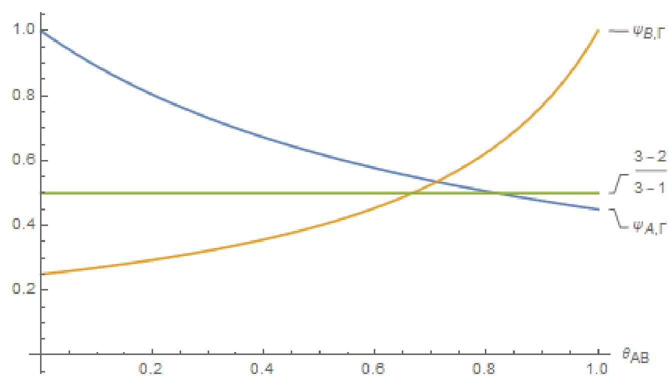

Figure 1 ($p_A = .75$, $p_B = .55$).

Solving $\Psi_{A,\Gamma} = \Psi_{B,\Gamma}$ yields $\theta_{AB} = 0.71$ and gives $\Psi_{A,\Gamma} = \Psi_{B,\Gamma} = 0.5305$. A size-3 equilibrium requires $\min\{\Psi_{A,\Gamma}, \Psi_{B,\Gamma}\} \geq \frac{1}{2}$. The choice of θ_{AB} is consequential. Setting $\theta_{AB} = 0.71$, a size-3 equilibrium exists. Setting θ_{AB} too high or low, only size 2 is achievable. In general, if $p_A \gtrless p_B$ then $\theta^*_{AB} \gtrless \theta^*_{BA}$, i.e. the more trustworthy sender is more likely to be asked first. Recall that neither A nor B should want to deviate to t_{N-1} if biased and $\omega = 0$. The key is that the more trustworthy expert, if biased, is less threatened by the other expert than vice versa. To equalise $\Psi_{i,\Gamma}$s he should thus be more likely to be first in the presentation order.

5 Extensions

5.1 Deterministic ordering rules

Note first that any deterministic ordering rule is trivially suboptimal. For a deterministic ordering rule Γ pinned down by $d \in D(\chi)$, recalling that $\chi_d^{i,+}$ denotes the set of senders who are consulted after sender i, we have $\Psi_{i,d} = \prod_{j\in\chi_d^{i,+}} (1-p_j)$ and $\Psi_d^i \neq \Psi_d^j$ for any i, j, which violates a necessary condition for optimality.

Remark 1 *The only deterministic ordering rule that can be part of an equilibrium is such that i appears before j if $p_i > p_j$.*

The proof is as follows. Clearly, under any deterministic rule, knowing the ordering rule allows DM to perfectly infer the identity i of each sender and thus his trustworthiness p_i on the basis of his position in the presentation order. In consequence, he consults senders in hierarchical order, starting from more trustworthy senders.

On the positive side, this ordering rule yields the highest $\underset{i\in\chi}{min}\{\Psi_{A,d}, \Psi_{B,d}, ..\}$ and thus the largest equilibrium size among all deterministic ordering rules. For any deterministic order pinned down by d, it is immediate that

$$\underset{i\in\chi}{min}\{\Psi_{A,d}, \Psi_{B,d}, ..\} = \Psi_{i,\Gamma}$$

if i is the first sender consulted. It follows immediately that the most attractive deterministic ordering rule, in terms of inducing the largest equilibrium size, is such that the first sender consulted is the sender with the highest p_i. Indeed, for $i, i' \in \chi$, it holds true that $\prod\limits_{j \in \chi_{-i}} (1 - p_j) > \prod\limits_{j \in \chi_{-i'}} (1 - p_j)$ iff $p_i > p_{i'}$.

5.2 Observable trustworthiness levels

We here consider the case where the platform shares its estimate of experts' likelihood of being biased with consumers. Yelp.com shares information about reviewers with customers and Amazon used to do so. Our model predicts that this is not beneficial to consumers. Suppose that DM now knows the identity i of each sender and thus observes p_i directly for each sender. Note that the DM's equilibrium beliefs are as in the main model. $E[\omega \,| m^1 = .. = m^n = t_N]$ is affected by senders' trustworthiness levels only via η, which is independent of how exactly the entries in $\{p_A, .., p_Z\}$ are allocated among individual senders.

Clearly, in any partitional equilibrium, DM consults more trustworthy senders first, which means that in equilibrium we must have $\Psi_{\Gamma}^{\min} = \prod\limits_{j \in \chi_{-i}} (1 - p_j)$, where i is the most trustworthy expert. This is strictly less than the value of $\Psi_{\Gamma'}^{\min}$ achieved by a proportional latin square ordering rule Γ'.

5.3 Varying the pool of senders

We here investigate the role of the distribution of trustworthiness levels, assuming that an optimal ordering rule is used by the platform, thus ensuring that $\Psi_{i,\Gamma} = \frac{\eta}{1-\eta} \sum_{k \in \chi} \frac{p_k}{1-p_k}$ for all i. We restrict ourselves to comparing distributions that yield the same value of η, the probability that all experts are biased. We look for the optimal profile of p_is conditional on this constraint and, furthermore, assuming a lower bound on the trustworthiness of any individual.

We thus solve

$$\max_{\{p_i\}_i} \frac{\eta}{1-\eta} \sum_i \frac{p_i}{1-p_i} \tag{17}$$

$$\text{s.t.} \quad \prod_i (1-p_i) = \eta, \tag{18}$$

$$\min_{i \in \chi} p_i \geq \varepsilon, \;\; \text{for } \varepsilon \in [0, 1-\eta^{\frac{1}{n}}). \tag{19}$$

Define in what follows $\mathbf{p} = \{p_i\}_{i=A}^{Z}$ and define $\eta(\mathbf{p})$ as the corresponding value of $\prod_i (1-p_i)$. Let $\Lambda(\eta, n)$ be the set of all distributions involving n senders and that yield the same value of η.

Proposition 5 *a) The solution to problem (17)-(19) is given by* $p_i = 1 - \frac{\eta}{(1-\varepsilon)^{n-1}}$ *for some* $i \in \{1, \ldots, n\}$ *and* $p_j = \varepsilon$ *for all* $j \neq i$.

b) Consider two profiles $\mathbf{p}, \mathbf{p}' \in \Lambda(\eta, n)$ *such that for some* i, j *we have* $p'_i > p_i$ *and* $p'_j < p_j$ *while instead* $p_k = p'_k$ *for all* $k \notin \{i, j\}$*. If* $p_i > p_j$*, then* $\mathbf{p}'$ *yields a weakly higher expected payoff of DM and vice versa if instead* $p_i < p_j$.

Proof: See Appendix A.

Concerning Point a). If $\varepsilon = 0$, then the solution to the problem is trivial. The objective function can always be made equal to 1 by setting $p_i = 1 - \eta$ for any one $i \in \{1, \ldots, n\}$ and $p_j = 0$ for all $j \neq i$. In this case $\prod_i (1-p_i) = \eta$ and $\frac{\eta}{1-\eta} \sum_i \frac{p_i}{1-p_i} = \frac{\eta}{1-\eta} \frac{1-\eta}{\eta} = 1$. This is enough to guarantee the existence of an equilibrium of any size (and recall that larger equilibria yield a higher DM expected payoff). In general, assuming a lower bound $\varepsilon > 0$, the optimal distribution is one where all probabilities take the lowest possible value but one of them, which takes the highest. Point b) compares pools of senders in which we shift trustworthiness levels between two senders by making the more trustworthy sender even more trustworthy and the less trustworthy sender even less trustworthy, in a way that keeps η fixed. We see that such an polarising shift is beneficial to DM, in a way that echoes a).

6 Conclusion

The model that we present in this paper can also be applied to the case of user generated commenting on general issues, such a Quora or newspaper comment sections. The pool of senders providing opinions is a mixture of honest citizens and agenda driven partisans possibly tied to organisations. Senders are as such ex ante idenitcal from readers' perspective, but the platform may have access to data that allows it to estimate the trustworthiness of individuals. The platform is free to decide in which order responses are shown and might condition the order on these estimates. Google's search page offers another instance of the ordering problem. For any given search query, the PageRank algorithm provides an ordered set of results. In this particular case, however, different sources typically have different trustworthiness levels in the eyes of readers.

A main result of this paper is to identify how to optimally order experts in a sequential consultation problem. We find that that the order should be stochastic, which implies that less trustworthy experts might sometimes be asked earlier. Experimental work would be called upon to qualitatively test our predictions, in order to see whether senders' behaviour is driven by the preemption motive that drives our findings.

Further theoretical work should aim at generalising our findings to related setups. We assume that senders all perfectly observe the state, whereas in reality sender information is noisy and thus diverse. One could also consider different bias structures, such as non-maximal bias or known bias. Another direction would be to endogenise reviewers' bias. There, the question would be whether some review systems ensure more trustworthy reviewers on average, by attracting more good reviewers or less biased reviewers.

7 Bibliography

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8 Appendix A

8.1 Proof of Lemma 1

Step 1 From the incentives of unbiased senders, it must be true that

$$E[\omega \left| m^1 = .. = m^n = t_N \right] - t_{N-1} = t_{N-1} - \frac{t_{N-1} + t_{N-2}}{2} \tag{20}$$

and it must also be true that all thresholds between $t_0 = 0$ and t_{N-1} are equally spaced, which means that for any $K < N-1$, we have $t_r = \left(\frac{K}{N-1}\right) t_{N-1}$. Using $t_{N-2} = \left(\frac{N-2}{N-1}\right) t_{N-1}$, (20) is equivalent to:

$$\frac{E[\omega \,|m^1 = .. = m^n = t_N]}{t_{N-1}} = \frac{2(N-1)+1}{2(N-1)}.$$

Inserting the closed form expression for $E[\omega \,|m^1 = .. = m^n = t_N]$, we obtain for any given N and η, the unique solution value of t_{N-1} which is given by (4).

Step 2 In an equilibrium featuring the partition $\{t_r\}_{r=1}^{N-1}$, let $m(\omega^*)$ denote the message sent by an unbiased sender if the state is ω^* and $\omega^* < t_{N-1}$. Denote by $E[\omega \,|m(\omega^*)]$ K's expected value of the state if he encounters the equilibrium message $m(\omega^*)$. From the incentives of biased senders, we need that for every sender $i \in \chi$ and for every $\omega^* \leq t_{N-1}$, it holds true that:

$$\Psi_{i,\Gamma} E[\omega \left| m^1 = .. = m^n = t_N \right] + (1-\Psi_{i,\Gamma}) E[\omega \,|m(\omega^*)] \geq \frac{t_{N-1}+t_{N-2}}{2}. \quad (21)$$

This condition ensures that any biased sender is willing to send m_N rather than deviate to m_{N-1}, whatever the realised state. Clearly, $E[\omega \,|m(\omega^*)]$ is increasing in ω^*, so the condition is most difficult to satisfy for $\omega^* = 0$. Thus, (21) is satisfied if and only if it is satisfied at $\omega^* = 0$. We thus need that for every sender $i \in \chi$ it holds true that:

$$\Psi_{i,\Gamma} E[\omega \left| m^1 = .. = m^n = t_N \right] + (1-\Psi_{i,\Gamma}) \frac{t_1}{2} \geq \frac{t_{N-1}+t_{N-2}}{2}. \quad (22)$$

Recall that the size of every interval to the left of t_{N-1} is identical and given by:

$$2\left(E[\omega \left| m^1 = .. = m^n = t_N \right] - t_{N-1}\right).$$

We may thus rewrite the constraint (22) as

$$\Psi_{i,\Gamma}E[\omega\left|m^1=..=m^n=t_N\right]+(1-\Psi_{i,\Gamma})\left[E[\omega\left|m^1=..=m^n=t_N\right]-t_{N-1}\right]$$
$$\geq 2t_{N-1}-E[\omega\left|m^1=..=m^n=t_N\right]$$

which is equivalent to

$$\frac{E[\omega\left|m^1=..=m^n=t_N\right]}{t_{N-1}}\geq\frac{(1-\Psi_{i,\Gamma})+2}{2}.$$

Now, bringing together the two conditions derived from the incentives of biased and unbiased senders, an equilibrium with N intervals exists if and only if:

$$\frac{2(N-1)+1}{2(N-1)}\geq\frac{(1-\Psi_{i,\Gamma})+2}{2}.$$

8.2 Proof of Proposition 3

The proof analyses the general case of $n\geq 2$ senders.

8.2.1 Point b): Effect of N

Step 1 Recall that in equilibrium, we have:

$$t_{N-1}\left(\frac{2(N-1)+1}{2(N-1)}\right)=E[\omega\left|m^1=..=m^n=t_N\right].$$

In what follows, define $f(N,\eta)=t^*_{N-1}$, where t^*_{N-1} is given as in (4).

Step 2 $\Pi_{DM}(N,\eta)$ is given by minus the following sum:

$$(1-\eta)(f(N,\eta))\frac{1}{12}\left(\frac{f(N,\eta)}{N-1}\right)^2 \tag{23}$$
$$+(1-\eta)\int_{f(N,\eta)}^{1}\left(\omega-f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega$$
$$+\eta\int_0^1\left(\omega-f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega.$$

This can be further decomposed into the following elements:

$$
\begin{aligned}
&(1-\eta)(f(N,\eta))\frac{1}{12}\left(\frac{f(N,\eta)}{N-1}\right)^2 \\
&+\eta\int_0^{f(N,\eta)}\left(\omega - f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega \\
&+\int_{f(N,\eta)}^{f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)}\left(\omega - f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega \\
&+\int_{f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)}^{1}\left(\omega - f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega.
\end{aligned} \tag{24}
$$

Consider the four lines that constitute expression (24) above. The expression in the last line is decreasing in N, as we shall show in next step. In the subsequent step, we prove that the sum of the three expressions appearing in the first, second and third line is also decreasing in N. This proves point a).

Step 3 Consider:

$$\int_{f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)}^{1}\left(\omega - f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)\right)^2 d\omega.$$

Note first that $\int_t^1 (\omega - t)^2 \, d\omega = -\frac{1}{3}(t-1)^3$ is trivially decreasing in t. Now, we need to show that $f(N,\eta)\left(\frac{2(N-1)+1}{2(N-1)}\right)$ is increasing in N. Note that:

$$
\begin{aligned}
&\frac{\partial\left(\left(\frac{2(N-1)+1}{2(N-1)}\right) f(N,\eta)\right)}{\partial N} \\
&= \frac{1}{4N^2(1-\eta)(N-1)^2\sqrt{4\eta N^2 - 4\eta N + 1}} G_0(\eta, N),
\end{aligned}
$$

where

$$
\begin{aligned}
G_0(N,\eta) = {} & \sqrt{4\eta N^2 - 4\eta N + 1} - 2N\eta - 2N \\
& - 2N\sqrt{4\eta N^2 - 4\eta N + 1} + 2N^2\eta + 2N^2 + 1.
\end{aligned}
$$

We simply need to show that $G_0(N,\eta) > 0$. Simple algebraic manipulation shows that this is equivalent to proving that $-4N^2(\eta-1)^2(N-1)^2 < 0$, which is always true.

Step 4 Consider the three expressions appearing in the first, second and third line of (24). The sum of these equals:

$$T(N,\eta) = \frac{1}{192N^3(\eta-1)^3}\frac{2N-1}{(N-1)^3}\left(\sqrt{4\eta N^2-4\eta N+1}-2N+1\right)^3\left(4\eta N^2-4\eta N+1\right).$$

We want to prove that $T(N,\eta)$ is always decreasing in N. Note that:

$$\frac{\partial T(N,\eta)}{\partial N} = \frac{1}{(\eta-1)^3}\frac{1}{192N^4(N-1)^4}\left(\sqrt{4\eta N^2-4\eta N+1}-2N+1\right)^2 G_1(\eta,N),$$

where

$$\begin{aligned} G_1(N,\eta) &= 10N-10N^2\left(4\eta N^2-4\eta N+1\right)^{\frac{3}{2}}+8N\eta-3\left(4\eta N^2-4\eta N+1\right)^{\frac{3}{2}} \\ &+10N\left(4\eta N^2-4\eta N+1\right)^{\frac{3}{2}}-24N^2\eta+16N^3\eta-12N^2+8N^3 \\ &+50N^2\eta\sqrt{4\eta N^2-4\eta N+1}-80N^3\eta\sqrt{4\eta N^2-4\eta N+1} \\ &+40N^4\eta\sqrt{4\eta N^2-4\eta N+1}-10N\eta\sqrt{4\eta N^2-4\eta N+1}-3. \end{aligned}$$

To show that $\frac{\partial T(N,\eta)}{\partial N} < 0$, we simply need to show that $G_1(N,\eta) > 0$. Simple algebraic manipulation shows that this in turn equivalent to proving that:

$$-4N^2(\eta-1)^2(N-1)^2\left(4N\eta-16N-4N^2\eta+16N^2+3\right) < 0,$$

which is always true.

8.2.2 Point c): Effect of η

Step 1 Consider expression (24). The expression appearing in the last line is trivially increasing in η, as proved now. We have:

$$\begin{aligned}&\frac{\partial\left(f(N,\eta)\right)}{\partial\eta}\\&=-\frac{1}{2N\left(\eta-1\right)^{2}\sqrt{4\eta N^{2}-4\eta N+1}}G_{0}(\eta,N),\end{aligned}$$

where $G_0(N,\eta)$ was defined earlier in our analysis of comparative statics with respect to N. We wish to prove that the above is negative. This is equivalent to showing that $G_0(N,\eta) > 0$, which we already proved is true.

Step 2 Consider the three expressions appearing in the first, second and third line of (24). We now show that the sum of these three expressions (denoted $T(N,\eta)$) is increasing in η. Note that:

$$\frac{\partial T(N,\eta)}{\partial\eta}=\frac{1}{192N^{3}}\frac{2N-1}{\left(N-1\right)^{3}}\frac{1}{\left(\eta-1\right)^{4}}\left(\sqrt{4\eta N^{2}-4\eta N+1}-2N+1\right)^{2}G_{2}(\eta,N),$$

where

$$\begin{aligned}G_{2}(N,\eta)=&\ 10N-10N^{2}\sqrt{4\eta N^{2}-4\eta N+1}+8N\eta-3\left(4\eta N^{2}-4\eta N+1\right)^{\frac{3}{2}}\\&+10N\sqrt{4\eta N^{2}-4\eta N+1}-24N^{2}\eta+16N^{3}\eta-12N^{2}+8N^{3}\\&+10N^{2}\eta\sqrt{4\eta N^{2}-4\eta N+1}-10N\eta\sqrt{4\eta N^{2}-4\eta N+1}-3.\end{aligned}$$

To show that $\frac{\partial T(\eta,N)}{\partial\eta} > 0$, we simply need to show that $G_2(\eta,N) > 0$. Simple algebraic manipulation shows that this in turn equivalent to proving that $(4N\eta - 16N - 4N^2\eta + 16N^2 + 3) > 0$, which is always true.

8.3 Proof of Lemma 3

The proof is decomposed into the statement of four Lemmas which together yield the result. Recall that we focus on ordering rules that satisfy:

$$\Gamma = \arg\max_{\Gamma} \min_{i \in \chi} \Psi_{i,\Gamma}. \tag{25}$$

The first Lemma below shows that any ordering rule Γ that solves (25) is such that $\Psi_{i,\Gamma}$ is constant across senders. We then show that the achieved value of $\Psi_{i,\Gamma}$ is the same across all ordering rules that solve (25). Finally, we explicitly pin down the achieved value of (25).

Lemma 6 *An ordering rule* Γ *solves (25) if and only if* $(1 - p_i)Q_{i,\Gamma} = (1 - p_j)Q_{j,\Gamma}$ *for all* i, j *and consequently* $\Psi_{i,\Gamma} = \Psi_{j,\Gamma}$*for all* i, j.

Proof. Notice first that an ordering rule solves $\max_\Gamma \min_i \Psi_{i,\Gamma}$ if and only if it solves $\min_\Gamma \max_i (1 - p_i)Q_{i,\Gamma}$

We proceed by contradiction. Let $\widetilde{\chi}$ be the set of senders such that $\widetilde{\chi} = \arg\min_i \Psi_{i,\Gamma} = \arg\min_i \frac{\eta}{(1-p_i)Q_{i,\Gamma}} = \arg\max_i (1 - p_i)Q_{i,\Gamma}$. If the statment of the Lemma is not true then there exists a sender $k \notin \widetilde{\chi}$ Assume first that $\widetilde{\chi}$ contains only one sender, say A. Then all senders who are not A do not belong to $\widetilde{\chi}$.

Take any deterministic order assigned positive probability in the ordering rule Γ such that A acts before a sender not in $\widetilde{\chi}$, call the deterministic order d and that sender k. Such order always exists as otherwise A is last in all deterministic orders with positive probability, which implies that $A \notin \widetilde{\chi}$. Create a new ordering rule $\widehat{\Gamma}$ identical to Γ but such that order d has probability $\widehat{\theta}_d = \theta_d - \varepsilon$ for some small $\varepsilon > 0$, and order d', which is the same as d but where the positions of a and k are swapped, has probability $\widehat{\theta}_{d'} = \theta_{d'} + \varepsilon$.

Notice that Γ and $\widehat{\Gamma}$ generate $\{Q_{j,\Gamma}\}_j$ and $\{Q_{j,\widehat{\Gamma}}\}_j$ respectively such that $Q_{j,\Gamma} = Q_{j,\widehat{\Gamma}}$ for all $j \neq a, k$ and $Q_{k,\widehat{\Gamma}} > Q_{k,\Gamma}$ and $Q_{a,\widehat{\Gamma}} < Q_{a,\Gamma}$. Since $Q_{i,\Gamma}$ for all sender i is linear in the probabilities of ordering rule Γ, by continuity ε can be chosen such that $(1-p_a)Q_{a,\widehat{\Gamma}} > (1-p_k)Q_{k,\widehat{\Gamma}}$.

We have just proven that there exists an ordering rule $\widehat{\Gamma}$ with $\max_i(1-p_i)Q_{i,\widehat{\Gamma}} < \max_i(1-p_i)Q_{i,\Gamma}$, which implies that Γ does not solve $\min_\Gamma \max_i(1-p_i)Q_{i,\Gamma}$, a contradiction.

If $\widetilde{\chi}$ instead has more than one sender, repeat the reasoning in this proof to arrive at a contradiction. Thus proving the Lemma. ■

An ordering rule solving (25) thus requires $(1-p_i)Q_{i,\Gamma} = (1-p_j)Q_{j,\Gamma}$ for all i, j which implies $\Psi_{i,\Gamma} = \Psi^j_\Gamma$ for all senders i, j. Next, we show that all ordering rules solving (25) lead to the same value for $\Psi_{i,\Gamma}$ for all i.

Lemma 7 *Take any two ordering rules Γ and Γ' that solve (25), with their respective $Q_\Gamma = \{Q_{j,\Gamma}\}_j$ and $Q_{\Gamma'} = \{Q_{j,\Gamma}\}_j$. Then, $(1-p_j)Q_{j,\Gamma} = (1-p_j)Q_{j,\Gamma}$ and consequently $\Psi^j_\Gamma = \Psi^j_{\Gamma'}$ for all j.*

Proof. Notice first that for any ordering Γ with its respective $\{Q_{j,\Gamma}\}_j$ we have

$$\sum_j p_j Q_{j,\Gamma} = 1 - \eta. \tag{26}$$

The left hand side is the probability that the receiver learns the truth; the sum for every sender of the probability that this sender is asked and tells the truth (notice that when $w = 0$ it is not possible for two senders to be asked and both tell the truth, as when one does so consultation stops). The right hand side is the same but expressed differently; it is the probability that at least one sender is honest (i.e. not true that all senders are biased). Algebraically, if d is any deterministic order and d_i is the sender who occupies the i-th position in this order then

$$\begin{aligned}
&\sum_j p_j Q_{j,\Gamma} \\
&= \sum_d \theta_d (p_{d_1} + p_{d_2}(1-p_{d_1}) + p_{d_3}(1-p_{d_2})(1-p_{d_1}) + \cdots + p_{d_n}(1-p_{d_{n-1}})\dots(1-p_{d_1})) \\
&= \sum_d \theta_d (1-\eta) = (1-\eta)\sum_d \theta_d = 1-\eta.
\end{aligned}$$

Since (25) implies $(1-p_i)Q_{i,\Gamma} = (1-p_j)Q_{j,\Gamma}$ for any i, j, we have $Q_{j,\Gamma} = Q_{i,\Gamma}\frac{1-p_i}{1-p_j}$. This leads to

$$\sum_j Q_{j,\Gamma} = (1-p_i)Q_{i,\Gamma}\sum_j \frac{1}{1-p_j}. \tag{27}$$

On top of that, (25) implies $\sum_j (1-p_j)Q_{j,\Gamma} = n(1-p_i)Q_{i,\Gamma}$ for any i. If we combine (26) and (27) with this observation we obtain

$$\begin{aligned}
(1-p_i)Q_{i,\Gamma}\sum_j \frac{1}{1-p_j} - (1-\eta) &= n(1-p_i)Q_{i,\Gamma} \\
(1-p_i)Q_{i,\Gamma} &= \frac{1-\eta}{\sum_j \frac{p_j}{1-p_j}}.
\end{aligned}$$

That is, the value of $(1-p_i)Q_{i,\Gamma}$ and consequently of $\Psi_{i,\Gamma}$ is the same under all odering rules solving (25) and for all i. ■

Lemma 8 *Let Γ be an ordering rule that solves (25). Then $\Psi_{i,\Gamma} = \Psi_{j,\Gamma} = \frac{\eta}{1-\eta}\sum_j \frac{p_j}{1-p_j}$ for all i, j.*

Proof. In every ordering rule Γ solving (25), since for any i, j we have $\Psi_{i,\Gamma} = \Psi_{j,\Gamma}$ and $\Psi_{i,\Gamma} = \frac{\prod_{j\neq i}(1-p_j)}{Q_{i,\Gamma}} = \frac{\eta}{Q_{i,\Gamma}(1-p_i)}$, it follows immediately that for every sender i

$$\Psi_{i,\Gamma} = \frac{\eta}{1-\eta}\sum_{k\in\chi}\frac{p_k}{1-p_k}.$$

■

8.4 Proof of Lemma 4

For a given latin square odering rule, given sender i and position l, let d_l be the unique deterministic order such that $\theta_{d_l} > 0$ and sender i occupies position l. Furthermore, let $\{1_{d_l}, 2_{d_l}, \ldots, (l-1)_{d_l}\}$ be the senders who occupy positions $\{1, 2, \ldots, l-1\}$ respectively in deterministic order d_l. We have,

$$\begin{aligned}
&(1-p_i)Q_{i,\Gamma} \\
&= (1-p_i)\left(\theta_{d_1} + \theta_{d_2}(1-p_{1_{d_2}}) + \ldots + \theta_{d_n}\prod_{j\neq i}(1-p_j)\right) \\
&= (1-p_i)\left(\begin{array}{c} \frac{p_i/(1-p_i)}{\sum_j p_j/(1-p_j)} + \frac{p_{1_{d_2}}/(1-p_{1_{d_2}})}{\sum_j p_j/(1-p_j)}(1-p_{1_{d_2}}) \\ + \ldots + \frac{p_{1_{d_n}}/(1-p_{1_{d_n}})}{\sum_j p_j/(1-p_j)}\prod_{j\neq i}(1-p_j) \end{array}\right) \\
&= \frac{1}{\sum_j \frac{p_j}{1-p_j}}\left(p_i + p_{1_{d_2}}(1-p_i) + \ldots + p_{1_{d_n}}\prod_{j\neq 1_{d_n}}(1-p_j)\right) \\
&= \frac{1-\eta}{\sum_j \frac{p_j}{1-p_j}}.
\end{aligned}$$

That is,

$$(1-p_i)Q_{i,\Gamma} = (1-p_j)Q_{j,\Gamma} = \frac{1-\eta}{\sum_j \frac{p_j}{1-p_j}}$$

for all i, j, which is the requirement for an optimal random order.

8.5 Proof of Proposition 5

8.5.1 Point a)

Consider the constrained optimisation problem defined in (17), (18) and (19). We use Kuhn-Tucker:

$$L(\{p_i\}, \lambda, \{\mu_i\}) = \frac{\eta}{1-\eta}\sum_i \frac{p_i}{1-p_i} + \lambda(\prod_i(1-p_i) - \eta) - \sum_i \mu_i(p_i - \varepsilon)$$

with $\lambda \geq 0$ and $\mu_i \geq 0$ for all i. Since the problem is symmetric for $\{p_i\}$ we can assume without loss of generality that the first $k \in \{1, 2, \ldots, n\}$ probabilities are stritly greater than ε and the last $n-k$ probabilities are equal to ε. In other words, $\{\mu_i\}_{i=1}^{k} = 0$ and $\{\mu_i\}_{i=k+1}^{n} > 0$ for some k. The problem is then to solve the K-T conditions for any k, and then choose the k that maximizes the objective function. Note we cannot have $k = 0$ as this would mean $\prod_i (1-p) = (1-\varepsilon)^n$, which is not in general equal to η. We have for all i

$$\begin{aligned} \frac{\partial L}{\partial p_i} &= \frac{\eta}{1-\eta} \frac{1}{(1-p)^2} - \lambda \frac{\eta}{1-p_i} - \mu_i \\ &= 0 \\ \frac{\partial^2 L}{\partial^2 p_i} &= \frac{\eta}{1-\eta} \frac{-2}{(1-p)^3} - \lambda \frac{\eta}{(1-p_i)^2} \\ &< 0. \end{aligned}$$

Note that for those i for which $\mu_i = 0$ we have $\frac{\eta}{1-\eta} - \lambda\eta(1-p_i) = 0$, which implies $p_i = 1 - \frac{1}{\lambda(1-\eta)}$. That is, at the optimum those p_i which are not ε all are equal to some value p given by $p = 1 - \frac{1}{\lambda(1-\eta)}$.

Therefore, we have $\eta = (1-p)^k(1-\varepsilon)^{n-k}$. This means

$$p = 1 - \left(\frac{\eta}{(1-\varepsilon)^{n-k}}\right)^{\frac{1}{k}}.$$

Thus, at the optimum we have that the first k probabilities are equal to p and the rest are equal to ε. We are left to calculate what is the optimal k. For given k we have that the objective function is equal to

$$\psi = \frac{\eta}{1-\eta}\left[k\left(\left(\frac{(1-\varepsilon)^{n-k}}{\eta}\right)^{\frac{1}{k}} - 1\right) + (n-k)\frac{\varepsilon}{1-\varepsilon}\right]$$

Let us study the behaviour of this expression as a function of k. Taking the derivative with respect to k and using the fact that $p = 1 - \left(\frac{\eta}{(1-\varepsilon)^{n-k}}\right)^{\frac{1}{k}}$ we obtain

$$\frac{\partial \psi}{\partial k} \propto \frac{p}{1-p} - \frac{\varepsilon}{1-\varepsilon} - \frac{1}{1-p} \log \frac{1-\varepsilon}{1-p},$$

where p depends on k.

Notice that $\frac{\partial \psi}{\partial k}_{p=\varepsilon} = 0$ and that $\frac{\partial^2 \psi}{\partial k \partial p} = -\frac{1}{(1-p)^2} \log \frac{1-\varepsilon}{1-p} < 0$. Hence, we have that $\frac{\partial \psi}{\partial k}$ is decreasing in p and equal to 0 at the lowest possible value for p. Therefore, it is negative. This means that the k that maximizes Ψ is the minimum possible. That is, $k = 1$. Therefore, the optimal solution is $p_i = 1 - \frac{\eta}{(1-\varepsilon)^{n-1}}$ for any one $i \in \{1, \dots, n\}$ and $p_j = \varepsilon$ for all $j \neq i$.

8.5.2 Point b)

Assume that for some $\varepsilon > 0$ we have that p_i increases to $p_i' = p_i + \varepsilon$ and that p_j decreases to $p_j' = p_j - \rho(\varepsilon)$, where $\rho(\varepsilon)$ solves

$$(1 - p_i - \varepsilon)(1 - p_j + \rho(\varepsilon)) = (1 - p_i)(1 - p_j).$$

which is equivalent to

$$\rho(\varepsilon) = \left(\frac{(1-p_i)}{(1-p_i-\varepsilon)} - 1\right)(1 - p_j)$$

It is easy to show that

$$\frac{p_i'}{1-p_i'} + \frac{p_j'}{1-p_j'} > \frac{p_i}{1-p_i} + \frac{p_j}{1-p_j}.$$

Therefore, $\Psi^{*'} > \Psi^*$.

9 Appendix B

9.1 Proof of Proposition 1

9.1.1 Preliminary definitions

Recall that m^l denotes the message appearing in position l of the presentation order. Consider an observed history h in which DM consulted k senders, first consulting the sender located in position l, then the sender in position l', then the sender in position l'', etc. We denote such a history by the k-entries vector $h = \{m^l, m^{l'}, m^{l''}, ..\}$. We denote the rth entry of h by h_r. We say that a history has length k if DM consulted k times. We say that two histories h and h' are *comparable* if, across these two histories, DM faced the same presentation order, followed the same order of consultation, and has consulted the same number of times (so the histories have the same length). The action rule α pins down the action $\alpha(h)$ taken by DM if he stops consulting and chooses an action after h.

Definition 1 *Senders' strategies induce monotonic beliefs when for any two comparable histories h and h' of lengh $k \in \{1, .., n\}$, if it holds true that there is some $i \in \{1, ...k\}$ such that $h_j = h'_j$ for all $j \neq i$ and $h_i > h'_i$ then it holds true that $E[\omega|h] \geq E[\omega|h']$, assuming that DM's beliefs are formed via Bayes rule and the senders' strategy profile.*

The following examples illustrate the above definition.

Example 1 *Given $n = 5$, if $h = \{m, m', m''\}$ and $h' = \{m, \widetilde{m}', m''\}$ with $m' > \widetilde{m}'$ then if senders' strategies induce monotonic beliefs it must be that $E[\omega|h] \geq E[\omega|h']$.*

Example 2 *Given* $n = 5$, *if* $h = \{m, m', m''\}$ *and* $h' = \{m, m', m'', m'''\}$ *then even if senders' strategies induce monotonic beliefs we cannot establish an ordinal relation between* $E[\omega|h]$ *and* $E[\omega|h']$.

Example 3 *Given* $n = 5$, *if* $h = \{m, m', m''\}$ *and* $h' = \{m, \widetilde{m}', \widetilde{m}''\}$ *with* $m' > \widetilde{m}'$ *and* $m'' > \widetilde{m}''$ *then if senders' strategies induce monotonic beliefs it must be that* $E[\omega|h] \geq E[\omega|h']$.

Example 4 *Given* $n = 5$, *if* $h = \{m, m', m''\}$ *and* $h' = \{m, \widetilde{m}', \widetilde{m}''\}$ *with* $m' < \widetilde{m}'$ *and* $m'' > \widetilde{m}''$ *then even if senders' strategies induce monotonic beliefs we cannot establish an ordinal relation between* $E[\omega|h]$ *and* $E[\omega|h']$.

Definition 2 *An equilibrium is monotone if sender strategies are monotonic and induce monotonic beliefs.*

Definition 3 *An equilibrium is partitional if it satisfies the following description. There is a sequence of strictly increasing thresholds* $\{t_0, t_1, \ldots, t_N\}$ *with* $N > 1$, $t_0 = 0$ *and* $t_N = 1$ *such that the following holds true. For any two comparable histories* h *and* h'*of lengh* k, *if it holds true that there is some* $i \in \{1, ...k\}$ *such that* $h_i = h'_i$ *for* $i \neq j$ *and* $h_j > h'_j$ *where either* $h_j, h'_j \in [t_k, t_{k+1})$ *for some* $k \in \{0, \ldots, N-1\}$ *or* $h_j, h'_j \in [t_{N-1}, 1]$, *then we have* $\alpha(h) = \alpha(h')$.

9.1.2 Proof

In what follows, as stated in the main text, we restrict ourselves to symmetric and monotone equilibria. The proof is decomposed into three Lemmas. The first Lemma establishes that there can never be a subset of the state space for which DM learns the state perfectly if he meets an unbiased sender. The second Lemma uses this property to show that any equilibrium must be partitional. The third Lemma, building on this, shows that for any informative equilibrium that satisfies our restrictions there exist an an outcome equivalent simple partitional equilibrium.

Lemma 9 *(No Perfect Communication on an Interval) There exists no symmetric and monotone equilibrium where there is a non-degenerate interval* $\widetilde{A}$ *such that if* $\omega \in \widetilde{A}$ *then if the receiver consults an unbiased sender he stops consultation and plays* $\alpha = \omega$*.*

Proof. Step 1 (assume the contrary and define $\sup \widetilde{A}$ **as the supremum of the state for which perfect communication is possible)** Assume the contrary, then there is a possibly uncountable collection of disjoint non-degenerate sets $\{\widetilde{A}_i\}_i$ such that if $\omega \in \widetilde{A}_i$ for some i then the receiver stops consultation and plays m. For all i let $\sup \widetilde{A}_i$ be the supremum of set $\widetilde{A}_i$. Create an increasing sequence in $[0, 1]$ by ordering increasingly the set of all suprema $\{\sup \widetilde{A}_i\}_i$. Since such sequence is bounded by 1, by the monotone convergence theorem it converges to its supremum. Let $\sup \widetilde{A}$ be such supremum.

Step 2 (if state is $\omega{=}\sup \widetilde{A}$ **then unbiased sender believes action must be** $\sup \widetilde{A}$**)** If the state is $\omega = \sup \widetilde{A}$ then an unbiased sender believes with probability 1 in equilibrium that the action of the receiver must be $\sup \widetilde{A}$ once he stops consultation. To see this notice first that by monotonicity this action is greater or equal than $\sup \widetilde{A} - \varepsilon$ for all small enough $\varepsilon > 0$. If $\sup \widetilde{A} = 1$ then the action is $\sup \widetilde{A}$ with certainty, and if $\sup \widetilde{A} < 1$ but the action played is not $\sup \widetilde{A}$ with some probability then the expected action must by strictly higher than $\sup \widetilde{A}$. That is, there exists an $\varepsilon > 0$ such that the action played is $\sup \widetilde{A} + \varepsilon$ with some probability p, in which case the unbiased sender's best response is in $m \in ((1-p)\sup \widetilde{A} + p(\sup \widetilde{A} - \varepsilon), \sup \widetilde{A}) \subset \widetilde{A}$. Thus, there is a deviation incentive of unbiased senders, a contradiction.

Step 3 (define $m_{\widetilde{A}}$ as the message that induces action arbitrarily close to $\sup \widetilde{A}$) For small $\varepsilon > 0$ there exist a message $m_{\widetilde{A}}(\varepsilon)$ such that if sent by a sender the receiver stops consultation and plays $\sup \widetilde{A}(\varepsilon) - \varepsilon$. By monotonicity $m_{\widetilde{A}}(\varepsilon)$ is increasing in ε and, furthermore, it is bounded above by 1. Thus, it converges to its supremum. Let $m_{\widetilde{A}}$ be such supremum.

Step 4 (action is strictly higher than $\sup \widetilde{A}$ when all messages are greater or equal to $\sup \widetilde{A}$) By monotonicity, if the state is $\omega = \sup \widetilde{A}$ then unbiased senders send message $m \geq m_{\widetilde{A}}$. On top of that, for any state by monotonicity biased senders always send message $m \geq m_{\widetilde{A}}$. A biased sender believes that if consulted then for any state the expected action played by the receiver in equilibrium, say α, is at least $\sup \widetilde{A}$, as otherwise he can guarantee any action arbitrarily close to $\sup \widetilde{A}$ by sending message $m_{\widetilde{A}}$. In equilibrium, by monotonicity, if the state is $\omega \in [0, \sup \widetilde{A})$ the receiver only plays an action of at least $\sup \widetilde{A}$ if he consults biased senders only, and less than $\sup \widetilde{A}$ otherwise, say $\hat{\alpha} < \sup \widetilde{A}$ at most. Using incentive compatibility for biased senders we must have $\delta\alpha + (1-\delta)\hat{\alpha} \geq \sup \widetilde{A}$, which since $\delta \in (0,1)$ and $\hat{\alpha} < \sup \widetilde{A}$ means $\alpha > \sup \widetilde{A}$. That is, the equilibrium action of the receiver must be strictly higher than $\sup \widetilde{A}$ when all messages he receives are $m_{\widetilde{A}}$ or above regardless of the state of nature.

Step 5 (if $\omega = \sup \widetilde{A}$ all messages are greater or equal to $\sup \widetilde{A}$) Assume $\omega = \sup \widetilde{A}$, then the receiver receives all messages equal to or above $\sup \widetilde{A}$, which by the paragraph above means he plays an action strictly greater than $\sup T$, but this is incompatible with unbiased senders' equilibrium beliefs. As we showed previously if the state is $\omega = \sup \widetilde{A}$ then an unbiased sender believes with probability 1 that the action played by the receiver is $\sup \widetilde{A}$.

■

Lemma 10 *All informative, symmetric and monotone equilibria are partitional.*

Proof. Step 1 (by contradiction, exists two histories with increasing actions in some interval $\widetilde{A}$ for some sender j) Assume there is an equilibrium that is not partitional. This means that there exists a non-degenerate interval $\widetilde{A}$ and a pair of comparable equilibrium histories h_{-j}, h'_{-j} for all senders but j with $h_i = h'_i$ for senders $i \neq j$, such that for all $h_j, h'_j \in \widetilde{A}$ we have $h_j \neq h'_j$ implies $\alpha(h) \neq \alpha(h')$.

By monotonicity for all $h_j, h'_j \in \widetilde{A}$ with $h_j > h'_j$ we have $\alpha(h) > \alpha(h')$. Assume henceforth without loss of generality that $h_j > h'_j$.

Step 2 (biased senders always send at least $\sup \widetilde{A}$) By monotonicity in equilibrium for any state biased senders always send message of at least $\sup \widetilde{A}$. This is because in at least one equilibrium history (the one given in step one of the proof) it leads to a strictly higher action than anything below $\sup \widetilde{A}$, and for any other equilibrium history it leads, by monotonicity, to an action at least as high as any other message below $\sup \widetilde{A}$.

Step 3 (define function $\hat{\alpha}(h_j)$ as the increasing action in history $h_{-j} \times h_j$ as a function of $h_j \in \widetilde{A}$) For any history h_{-j} where some sender j is not consulted, define the function $\hat{\alpha} : \widetilde{A} \to I$ as a strictly increasing map between h_j and the action played in history $h = h_{-j} \times h_j$. We have that $\hat{\alpha}$ is increasing and that $I \subseteq [\hat{\alpha}(\inf \widetilde{A}), \hat{\alpha}(\sup \widetilde{A})]$.

Step 4 (if the image set of $\hat{\alpha}$, i.e. I, contains an interval, we contradict Lemma 9) If there exists an $x \in \mathring{I}$ and $\hat{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \varepsilon)$ we have $x + \varepsilon \in I$ then I contains intervals. That is, there exists an $x \in \mathring{I}$ and an $\varepsilon > 0$ such that for all state $\omega \in (x, x + \varepsilon)$ there exists a message

$m \in \widetilde{A}$ such that $\hat{\alpha}(m) = \omega$. Since biased senders always send message $\sup \widetilde{A}$, after observing m the receiver learns the that sender is unbiased and his strict best response is to stop consultation and play $\hat{\alpha}(m)$. This contradicts Theorem 1.

Step 5 (if the image set of $\hat{\alpha}$, i.e. I, does not contain any interval, we still contradict Lemma 9 because I must be dense in some subset of $I \cap R$) Assume instead that for all $x \in \mathring{I}$ there exists no $\hat{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \varepsilon)$ we have $x + \varepsilon \in I$. That is, I contains no intervals.

Step 5.1 (I contains no intervals but for at least one point in I there is aonther one in I infinitesimally close by) Note that for all $\delta > 0$ there exists an $x, x' \in I$ with $x < x' < x + \delta$. This is because otherwise there exists a $\delta > 0$ such that for all $x \in I$ we have $(x, x + \delta) \not\subset I$. This means that I has at most $\frac{\sup I - \inf I}{\delta}$ elements. This is a contradiction as $\hat{\alpha}$ is a strictly increasing mapping from a set with infinitely many elements so its domain I must also have infinitely many elements.

Step 5.2 (for all error $\varepsilon > 0$ and for some states not in I sender can induce an action ε-close to the state) We have that for all $\varepsilon > 0$ if we take $\delta \in (0, 2\varepsilon)$ and $x, x' \in I$ such that $x < x' < x + \delta$ then for all $\hat{x} \in (x, x')$ with $\hat{x} \notin I$ either $|x - \hat{x}| < \frac{\delta}{2} < \varepsilon$ or $|x' - \hat{x}| < \frac{\delta}{2} < \varepsilon$.

Step 5.3 (there is then an interval with full communication, a contradiction) Notice that since $x, x' \in I$ there exists $m, m' \in \widetilde{A}$ respectively such that $\hat{\alpha}(m) = x$ and $\hat{\alpha}(m') = x'$. That is, we have found that for any $\varepsilon > 0$ and any state ω in the interval $[x, x']$ we can find a message that induces an action at least ε-close to ω. This means that again we have found an interval where there is full communication of the state, a contradiction to theorem 1.

■

Lemma 11 *In all informative, symmetric and monotone equilibria, there exists a sequence of strictly increasing thresholds* $\{t_0, t_1, \ldots, t_{m-1}, t_m\}$ *with* $t_0 = 0$ *and* $t_m = 1$ *such that:*

1. *Biased senders always send a message in* $[t_{N-1}, t_N]$*,*

2. *unbiased senders all send the same message,*

3. *DM keeps consulting for as long as he has received messages in* $[t_{N-1}, t_N]$*, and stops consulting either once he has received a message not in* $[t_{N-1}, t_N]$*, or when he has consulted all senders,*

4. *if DM observes a message not in* $[t_{N-1}, t_N]$*, say it belongs to* $[t_{k-1}, t_k]$ *with* $k \in \{1, \ldots, N-1\}$*, he then plays an action* $\alpha(k)$ *that is strictly increasing in* k*. If DM only observes messages in* $[t_{N-1}, t_N]$*, he then plays an action* $\alpha(N) > \alpha(k)$ *for all* $k \in \{1, \ldots, N-1\}$*.*

Proof. Step 1 (Eliminate redundant partitions) Take any informative partitional equilibria. If for any two comparable equilibrium histories h and h' such that there is some sender i where for all $i \neq j$ and $h_i, h'_i \in [t_k, t_{k+1})$ for some $k \in \{0, \ldots, N-1\}$ (or $h_i = h'_i = t_N$) and $h_j > h'_j$ with $h_j \in [t_r, t_{r+1})$ and $h'_j \in [t_s, t_{s+1})$ for some $r < s$ we have $\alpha(h) = \alpha(h')$, then we can redefine the partitions as $\{t_0, \ldots, t_r, t_s, t_{s+1}, t_N\}$ without-loss of generality. If for any two comparable equilibrium histories $\hat{h}$ and $\hat{h}'$ where again all messages but one are in the same interval and the action is not increasing, we can again redefine the partitions eliminating the cut-offs where the action of the receiver is non-increasing.

Continuing in this fashion we eventually get to a partition $t_0, t_1, \ldots, t_{m-1}, t_m$ with $t_0 = 0$ and $t_m = 1$ such that there exists two comprable equilibrium histories h and h' where there is some sender i if for all $i \neq j$ we have $h_i, h'_i \in [t_k, t_{k+1})$ for some $k \in \{0, \ldots, N-1\}$ and $h_j > h'_j$ with $h_j \in [t_r, t_{r+1})$

and $h'_j \in [t_s, t_{s+1})$ for some $r < s$ we have $\alpha(h) > \alpha(h')$.

Step 2 (Only two different messages are ever observed in equilibrium) By monotonicty, we have then that biased senders always send message $m \in [t_{N-1}, t_N]$ for a given state. Therefore, in equilibrium, any message that is not in $[t_{N-1}, t_N]$ was sent by an unbiased sender with probability 1. Moreover, since strategies are symmetric and senders do not observe other senders' messages, i.e. the history of observed messages, all unbiased senders send the same message for given state of the world. This means that in a partitional equilibrium there are only two messages ever observed by the receiver, the one sent by biased senders, and the one sent by unbiased senders.

Step 3 (Perfect information if message received is not in the top partition) Therefore, since messages not in $[t_{N-1}, t_N]$ are only ever sent by unbiased senders, once the receiver observes a message not in $[t_{N-1}, t_N]$ he does not have incentives to keep consulting experts, as he has learned as much as can be learnt in equilibrium. Thus, he stops consultation.

Step 4 (Summing up) Altoghether steps 1-3 in this proof lead to the result in the Lemma. Note finally that it is immediate that for any given equilibrium of the form described in the above Lemma, there exists a unique outcome equivalent simple partitional equilibrium. ■