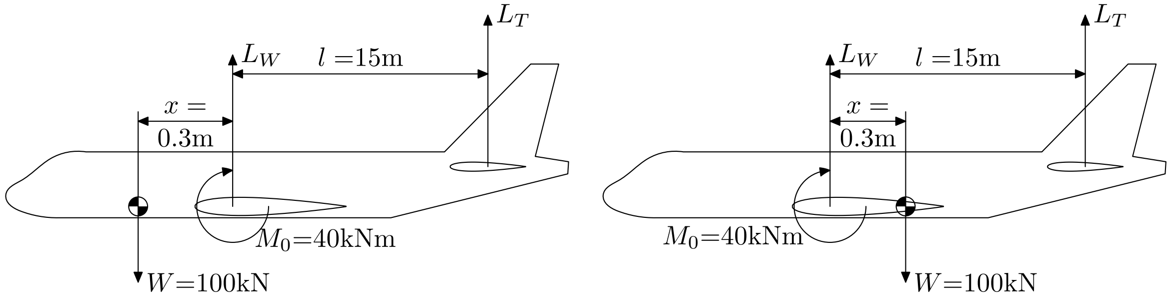


Figure 1: Aircraft with different centres of gravity

1. For the two situations shown in figure 1, calculate the values of L_W and L_T required to give both a total lift equal to the aircraft weight and give zero net moment about the aircraft c.g.

$$L_W = 99.3\text{kN},\ L_T = 0.7\text{kN},$$
$$L_W = 95.3\text{kN},\ L_T = 4.7\text{kN}.$$

2. Draw the system of forces and moments acting on a conventional aeroplane in steady straight and level flight.

   Show that the pitching moment about the centre of gravity is given by

$$C_M = C_{M_0} - (h_0 - h_n)C_L - \bar{V}C_{L_T}.$$

   For the sailplane whose details are given in Table 1, calculate the value of C_{L_T} required for trim at 50kt EAS with a pilot weighing 0.75kN, the empty weight equipped is 2.5kN, with c.g. on the mean chord $0.45\bar{c}$ aft of the leading edge of $\bar{c}$. The pilot c.g. is assumed to be 0.8m ahead of the leading edge of $\bar{c}$.

$S = 28\text{m}^2$	$S_T = 1.4\text{m}^2$	$\bar{c} = 1.15\text{m}$
$l = 5.35\text{m}$	$h_0 = 0.25$	$C_{M_0} = -0.11$

Table 1: Sailplane data

$$C_{L_T} = -0.552$$

3. Distinguish between stability and trim. Show that for an aircraft to be both stable and able to trim at positive lift coefficient the overall pitching moment about the centre of gravity must be positive at $C_L = 0$ in that configuration.

4. From first principles, show that the portion of the total lift coefficient (C_L) provided by the wing, body and nacelles (WBN) group of a conventional aircraft is given by:

$$C_{L_{WBN}} = C_L \left[1 + (h_0 - h_n)\frac{\bar{c}}{l}\right] - C_{M_0}\frac{\bar{c}}{l}.$$

If the aircraft stalls when $C_{L_{WBN}}$ reaches its maximum value, $(C_{L_{WBN}})_{\max}$ say, then obtain an expression relating the stalling speed to the c.g. position at any one given weight.

Hence calculate the c.g. shift that would increase the stalling speed by 1% if $\bar{c} = 5.6$m, $l = 15.5$m and $(h_0 - h) = 0.05$.

$$\Delta h = -0.0566,$$
$$\Delta h\bar{c} = -0.317\text{m}.$$

5. Consider the two situations shown in the diagrams below.

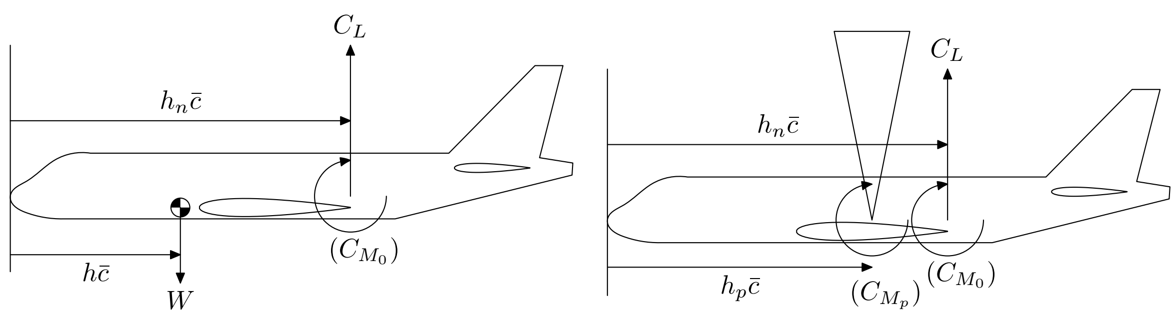


Figure 2: Full-scale and model aircraft. C_{M_p} is measured by the balance, which restrains the model in pitch.

In (a) the full scale aircraft is in steady free flight with values C_L, h, η for the lift coefficient, c.g. position and elevator angle respectively.

In (b), the model of the same aircraft is suspended from a wind tunnel balance at the same C_L and elevator setting as in (a). The balance measurement gives the pitching moment coefficient C_{MP} about the balance pivot axis which is located at h_p with respect to the same datum line as h.

(a) Write down the moment equations for situations (a) and (b), and hence derive the relationship between the balance reading C_{MP}, equivalent to the steady free flight conditions, and interrelating h_p, h and C_L.

(b) An aircraft model is found to have a zero-lift pitching moment coefficient of 0.027 for a particular elevator angle. The pitching moment is measured about the wind-tunnel axis of rotation P and has a slope:

$$\frac{\mathrm{d}C_{Mp}}{\mathrm{d}\alpha} = 0.15; \quad \text{lift curve slope } a = 5.851.$$

Determine the position of the c.g. of the full-scale aircraft relative to P if a stick-fixed margin of 0.11 is required ($\bar{c} = 3.96$m).

If the wing loading is 2.25kN/m^2 in steady level flight with the above elevator angle, what is the airspeed if the air density is 1.030kg/m^3.

0.537m forward of P, 133.3m/s TAS.

1. The data shown below apply to an aircraft in steady level flight at 200kt EAS. Calculate the elevator angle required for longitudinal trim. Also obtain the stick-fixed neutral point and static margin.

W = 30kN	$S = 23\text{m}^2$	$S_T = 3.5\text{m}^2$
$\bar{c} = 1.96$m	$l = 5.5$m	
$h_0 = 0.25$	c.g. is 0.61m aft of datum	
C_{M_0} = -0.036	η_T = -1.5°	$\epsilon = 0.48\alpha$
$a = 4.58$	$a_1 = 3.15$	$a_2 = 1.55$

[$\bar{\eta}$ = -1.658°, h_n=0.4027, K_n = 0.0915]

2. The centre of gravity range for an aircraft is found by considering that the

   (a) aft c.g. limit (h_{aft}) is determined by the minimum stability condition (minimum K_n);

   (b) forward c.g. limit (h_{fwd}) is determined by the maximum elevator angle to trim (while retaining enough elevator movement for manoeuvre).

   By considering the static forces and moments on an aircraft in symmetric flight, find an expression for the static margin stick-fixed, K_n, and show that:

$$K_n = -\bar{V}a_2\frac{\mathrm{d}\bar{\eta}}{\mathrm{d}C_L} = (h_0 - h) + \bar{V}\frac{a_1}{a}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right).$$

   An aircraft has the following values of the aerodynamic coefficients:

$$h_0\text{=}0.25,\ a\text{=}3.5,\ a_1\text{=}3.0,\ a_2\text{=}1.5,\ \partial\epsilon/\partial\alpha\text{=}0.4.$$

   Find the relationship between the c.g. position and the tail volume ratio:

   (a) for a static margin of 0.05 (h_{aft});

   (b) for the change in elevator angle to trim to 10°for a change of $C_L$of 1.0 (h_{fwd}).

   Hence find the minimum tail volume ratio such that with a c.g. shift of 0.15$\bar{c}$ the change in elevator angle to trim is not more than 10°for a change of C_L of 1.0 and the static margin is never less than 0.05.

[$\bar{V}$=0.764]

3. A transport aircraft with conventional tail is to have zero elevator angle in cruising flight at 560km/h EAS (mass 100,000kg), with the c.g. in the mid position. The landing approach, out of ground effect, is made with flaps down at 210km/h (mass 90,000kg), and the maximum elevator movement permitted for trimming is $\bar{\eta}$=±10°. Using the data below, calculate the minimum tailplane area suitable for this aircraft, and the tailplane setting η_T relative to the flaps-up wing zero lift line.

Minimum K_n= 0.05	c.g. range Δh=0.50	h_0=0.075	S=232m^2
$\bar{c}$=4.72m	l=19.5m		
a=5.7	$a_1 = 2.7$	$a_2 = 2.1$	
C_{M_0}= -0.14	ϵ=0.16α.		

The change in C_{M_0} at landing flap setting $\Delta C_{M_0} = -0.10$. Note that the wing zero lift incidence angle changes by 10° when the flaps are lowered to the landing setting.

[$S_T = 68.5\text{m}^2$, $\eta_T = -3.92°$]

1. The static margin, stick-fixed may be obtained in practice from flight tests in which the elevator angles to trim are found at certain speeds. In practice, the aeroplane is trimmed at a series of speeds by adjusting the tab setting, and both the elevator angle and tab angle are observed. Since the theory which relates the stick-fixed static margin to the elevator angles to trim implicitly assumes a constant tab angle, show that a correction must be applied to elevator angles obtained in this way such that

$$\bar{\eta}_{\text{corrected}} = \bar{\eta} + \frac{a_3}{a_2}\bar{\beta}$$

where $\bar{\eta}$ and $\bar{\beta}$ are the observed elevator and tab angles to trim at a given speed. Suggest how you would determine a_3/a_2 in flight.

2. A tailless aircraft is controlled in pitch by six elevons. Each elevon is actuated by an independent power control unit. These units are so designed that if a failure occurs the affected elevon is able to move until its hinge moment is zero.

   Assuming the failure of one such unit, calculate the elevon angles that will give longitudinal trim of the aircraft whose details are given below:

Weight =	850kN	Speed =	70m/s EAS
Wing area S =	358m^2	$(h_0 - h)$ =	0.15
C_{M_0}=	+0.02	$\partial C_{M_0}/\partial\eta$ =	-0.45
a_1 =	4.0	a_2 =	0.95
b_1 =	-0.7	b_2 =	-1.05

   Assume that each elevon contributes equally to a_2 and to $\partial C_{M_0}/\partial\eta$.

   [$\eta_{\text{failed}} = -9.61°$, $\eta_{\text{operating}} = -13.15°$]

3. A conventional aircraft flying at low speed has a flexible rear fuselage such that the tailplane setting relative to the wing zero lift line is directly proportional to the tail load. Prove that the reduction in stick fixed static margin compared with that of the rigid aircraft is given by:

$$\begin{aligned}\Delta K_n &= K_n^{\text{rigid}} - K_n^{\text{flexible}},\\ &= \bar{V}\frac{a_1}{a}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\left[1 - \frac{1}{1 + \frac{1}{2}\rho V^2 S_T a_1 f}\right]\end{aligned}$$

   For the human-powered aircraft having the characteristics given below, find the fuselage flexibility f (degrees deflection per Newton) that reduces the stick-fixed static margin by 0.05 compared to the rigid case when flying at a speed of 9.2m/s at sea level.

$$
\begin{array}{lll}
S = 28\text{m}^2 & S_T = 1.4\text{m}^2 & l = 5.34\text{m}\ \bar{c}{=}1.14\text{m} \\
a = 6.0 & a_1 = 4.5 & \epsilon = 0.20\alpha.
\end{array}
$$

[$f = 0.1°/\text{N}$]

4. The control column of a low-speed aeroplane is connected to the elevator by an arrangement of cables which suffer from considerable "stretch" when a stick force is applied. The stiffness of the circuit is given by $\mathrm{d}H_E/\mathrm{d}\eta = E\text{Nm/rad}$ where H_E is the hinge moment and η is the elevator deflection, the stick being held fixed.

   Show that the stick-fixed c.g. margin (as opposed to the "elevator fixed" c.g. margin) is given by:

$$
K_n = (h_0 - h) + \bar{V}\frac{a_1}{a}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\left[1 - \frac{a_2 b_1}{a_1 b_2}\frac{1}{1-\lambda}\right]
$$

   where

$$
\lambda = \frac{C_L S E}{b_2 S_\eta \bar{c}_\eta W}.
$$

   It should be assumed that the aircraft is initially in trim with the tab angle adjusted to give zero stick force.

   Show how this angle is related to the stick fixed and stick free c.g. margins of a rigid aeroplane.

1. What conditions define the stick-fixed and stick-free manoeuvre points of an aircraft?

   From first principles, stating your assumptions, derive an expression for the stick fixed maneouvre point of a low speed aircraft of canard layout. Show whether this is forward or aft of the corresponding neutral point and compare your expression with that for a conventional aircraft.

2. Define the maneouvre point stick-free for a conventional aircraft. How does it differ from the corresponding neutral point?

   Find the minimum stick force per G at sea level for the light aircraft whose details are given below. Comment on your result and find the c.g. position required to give 22N per g. Suggest alternative means for increasing the existing value.

   $$\begin{array}{lll} W = 2.7\text{kN} & S = 7.6\text{m}^2 & l = 2.9\text{m} \\ h_0 = 0.238 & \bar{c} = 1.2\text{m} & \bar{V} = 0.34 \\ \epsilon = 0.385\alpha & a = 3.865 & a_1 = 2.73 \\ a_2 = 2.16 & b_1 = -0.282 & b_2 = -0.536 \end{array}$$

   The permitted c.g. range is from $0.22\bar{c}$ to $0.28\bar{c}$. The stick force per g is given by

   $$\begin{aligned} Q = \frac{P_e}{n} &= -m_e S_\eta \bar{c}_\eta \frac{W}{S} \frac{b_2}{a_2 \bar{V}} H'_m, \\ &= 83.2 H'_m \text{N/g} \quad \text{for this aircraft.} \end{aligned}$$

   [$Q = 5.8$N/g, for $Q = 22$N/g, $h = 0.0853$]

3. The table below shows data for a tailless aircraft. When it performs a steady pullout at $A_N = 2.5$ at 250kt EAS at a height where the air density $\rho = 1.150\text{kg/m}^3$, the change of elevator setting compared with steady level flight under the same conditions is 3.20°.

   Calculate m_q if the static margin is known to be 0.05.

   $$\begin{array}{lll} W = 160\text{kN} & S = 50\text{m}^2 & c_0 = 10\text{m} \\ \partial C_{M_0}/\partial\eta = -0.5 & K_n = 0.05 & \end{array}$$

   [$m_q = -0.264$]

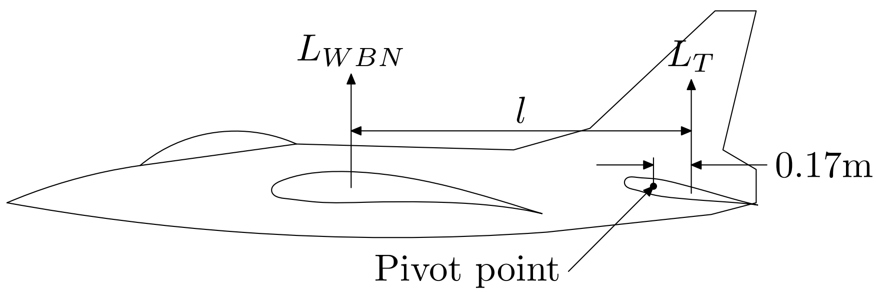
4. An aircraft of conventional layout is controlled in pitch by an all-moving tailplane, having no separate elevators (see figure and table). Show that the tail angle per g is given by

$$\frac{\Delta \eta_T}{n} = -\frac{C_L H_m}{\bar{V} a_1}$$

where the symbols have their usual meanings.

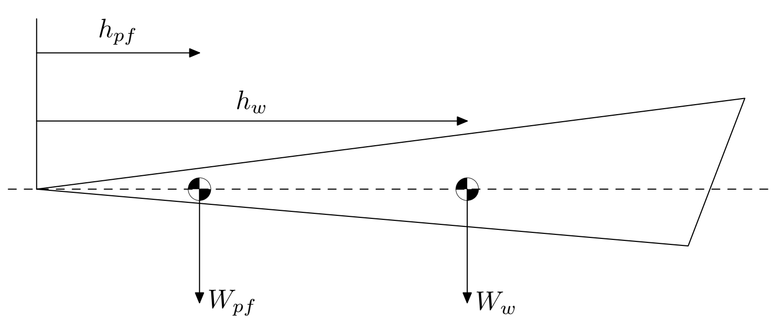
Hence calculate the tail angle, tail load and pivot moment when the aircraft is flying at 440kt EAS in an 8g pullout at a height where the relative density of the air $\sigma = 0.74$. Comment on your results.

W=175kN	V=440kt EAS	S=33.2m^2	S_T=19.1m^2
l=5.25m	$\bar{c}$=2.39m	a=3.8	a_1=2.7
$C_{M_0} = +0.03$	$\partial\epsilon/\partial\alpha$=0.38	$h_0 = 0.17$	$h = 0.50$
$\sigma = 0.74$			



$\eta_T = -4.85°$, $L_T = 224.6$kN, pivot moment: -38.18kN($C_{L_T} = 0.3739$).

5. A highly simplified model for the longitudinal control of a delta wing weight-shift microlight is shown below. The aircraft can be decomposed into a 'wing' unit and a 'pilot/fuselage' unit. These have weights W_w and W_{pf} acting at displacements h_w and h_{pf} respectively with all lengths being measured from a datum at the wing apex. The aircraft is controlled in pitch by longitudinal displacements of the pilot/fuselage unit.



(a) Show that, for trim, the pilot/fuselage centre of gravity is:

$$h_{pf} = \frac{h_0 - f_w h_w}{f_{pf}} - \frac{C_{M_0}}{C_L}\frac{1}{f_{pf}},$$

where $f_{pf} = W_{pf}/W$, $f_w = W_w/W$ and W is the total aircraft weight $W = W_w + W_{pf}$.

(b) For the microlight aircraft whose properties are listed in the table below, calculate the pilot/fuselage centre of gravity position which gives a static margin of 0.05.

W_{pf}=200kg	W_w=70kg	V_{max}=90kt EAS	$\bar{c}$=2m
h_0=0.8	h_w=0.6	$C_{M_0} = 0.1$	S=10m^2

(c) For the same aircraft, calculate the pilot/fuselage centre of gravity position for trim at the maximum flight speed.

(d) Given that the range of motion of the pilot/fuselage centre of gravity is limited to 0.5m, calculate the minimum flight speed at which the aircraft can be trimmed. Comment on your answer.

[0.803, 0.20, 70kt EAS]

1. For a conventional aircraft show that if the tab setting remains unaltered, the change of elevator hinge moment coefficient-to-trim $\Delta\bar{C_H}$ between two lift coefficients is given by

$$\bar{C_H} = -\frac{b_2}{a_2\bar{V}}\Delta C_L K_n'.$$

The aircraft described in the table below is making a zero stick force trimmed landing approach at 155kt EAS. Calculate the value to which the speed may be reduced while keeping the stick force within 150N without altering the trim tab setting, indicating clearly whether this is push or a pull force.

Weight	W	=785kN	c.g. at	h	= 0.26
Wing area	S	= 223m^2	smc	$\bar{c}$	= 5.68m
Tail area	S_T	= 46.5m^2	tail arm	l	= 15.66m
Elevator area	S_η	= 11.2m^2	elevator mean chord	$\bar{c}_\eta$	= 0.908m

h_0	= 0.16	C_{M_0}	= -0.06	ϵ	= 0.38α
a	= 4.5	a_1	= 2.75	a_2	= 1.16
b_0	= 0	b_1	= -0.133	b_2	= -0.16

The stick/elevator gearing $m_e = 1.0$m/rad.

[118 kt, pull force]

2. What are stick-fixed and stick-free manoeuvre points and what is the significance of stick force per g.

   The data in the table below refer to a conventional aircraft. Calculate the change of elevator angle required to pull 0.5g flying at 350kt EAS at an altitude where the relative density $\sigma = 0.374$.

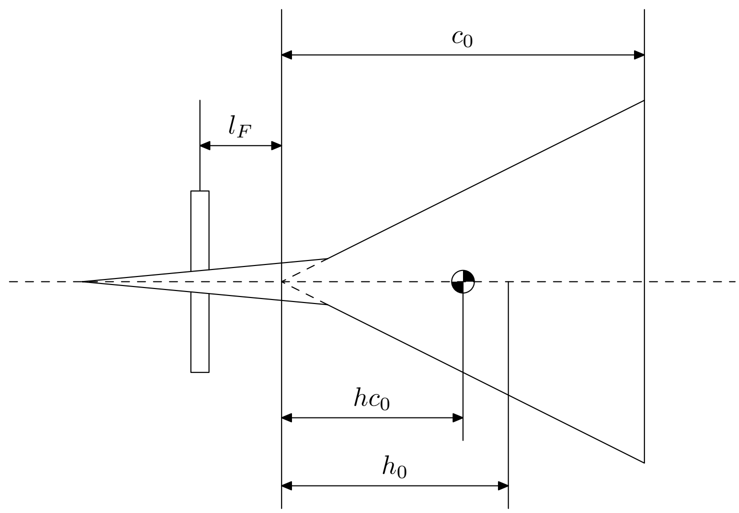
   Explain, in simple physical terms, why this change of elevator angle would be greater at a lower altitude when flying at the same lift coefficient.

[$\Delta\bar{\eta} = -1.005°$]

3. The tailless aircraft shown in the figure has been fitted with a small retractable foreplane. At low speeds this foreplane is extended and, operating in a semi-stalled condition at constant setting, it generates a constant lift coefficient $C_{LF} = 1.2$ (based on S_F).

   Use of the foreplane enables the aircraft to take off at a higher weight than the original aircraft without the foreplane. Calculate the increment in take-off weight that may be achieved when using the foreplane, by considering the trimmed lift at 200kt EAS, if the incidence is restricted to 12° by ground clearance problems, using the data in the table.

Calculate the elevon angles to trim of both versions of the aircraft. Comment on your results.



$S = 438\text{m}^2$	$S_F = 9.4\text{m}^2$	$C_{M_0} = +0.002$	$\partial C_{M_0}/\partial\eta$ = -0.25
$a_1 = 3.0$	$a_2 = 0.80$	$h_0 = 0.61$	c_0 =27.4m
$l_F = 13.26\text{m}$	$hc_0 = 15.34\text{m}$		

With foreplane: $\bar{\eta} = -0.5°$, $L = 1842$kN;
With foreplane: $\bar{\eta} = -5.81°$, $L = 1557$kN;
Increment: 285kN.

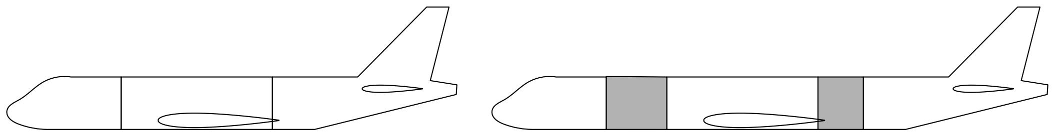
4. The aircraft described in the figure and table below is to have its capacity increased by lengthening the cylindrical portion of the fuselage by 6m. The centre section (including the wings), the nose and the tail portions are to remain unaltered.

   It is assumed that the c.g. position will be adjusted to remain unchanged with respect to the centre section unit and that, for the lengths considered, $\partial\epsilon/\partial\alpha$ is constant.

   Calculate how the additional fuselage length is to be inserted ahead of and behind the centre section, if the low speed stick-fixed static margin is to unaltered. The movement of the aerodynamic centre of the aircraft less tail is assumed to be affected only by the change of nose length Δl_N and is given by

$$\Delta h_0 = -0.037\frac{\Delta l_N}{\bar{c}}.$$

If a variant of the aircraft retains the original fuselage, but has its wing tips extended, how could the longitudinal static stability be affected?



$S = 223\text{m}^2$	$\bar{c}$= 5.6m
$S_T = 46\text{m}^2$	l=15.5m
$h_0 = 0.25$	$h = 0.20$
$\epsilon = 0.4\alpha$	C_{M_0}=-0.06
$a = 4.5$	$a_1 = 2.75$

[4.1m ahead of wing, 1.9m aft]

5. (a) The 1903 Wright Flyer was a canard configuration of conventional layout, summarized in the table below. Calculate the stick-fixed neutral point, assuming that the wing and canard have approximately equal lift curve slope, and comment on your answer.

   (b) The 1903 Flyer was stabilized in pitch by the addition of ballast to shift the centre of gravity forward. If 30% of the aircraft gross weight can be carried as ballast, where should it be placed to move the centre of gravity to the wing leading edge. What effect would this have on the aircraft performance?

$h_0 \approx 0$	$\overline{V} = 0.134$	$C_{M_0} = -0.141$
$h = 0.3\bar{c}$	$W \approx 340\text{kg}$	

The 1903 Wright Flyer. The datum is the wing leading edge.

1. The table below contains flight test data for the X-15 spaceplane. Calculate the static margin stick-fixed and estimate the zero-lift pitching moment. Estimate the dimensional and non-dimensional phugoid mode and SPO frequencies.

$S = 18.58\text{m}^2$	$s = 6.82\text{m}$	$\bar{c} = 3.13\text{m}$
$m = 7056\text{kg}$	$B = 10700\text{kgm}^2$	$V = 331\text{knot EAS}$
$a = 3.5/\text{rad}$	$\partial C_M/\partial\alpha = -0.8/\text{rad}$	
$Z_u = -332\text{Ns/m}$	$X_u = -235\text{Ns/m}$	$Z_w = -14300\text{Ns/m}$
$M_q = -158600\text{Nms}$		

2. NASA CR-2144, ‘Aircraft handling qualities data’, contains stability information for ten aircraft. Using the extract supplied for the Boeing 747:

   (a) calculate the static margin stick-fixed;

   (b) estimate the zero-lift pitching moment;

   (c) estimate the phugoid, SPO and Dutch roll periods;

   (d) estimate the time constants for rolling subsidence and the spiral mode.