Some notes on aircraft stability and control

I: Static stability

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1 The basics

1.1 Equilibrium and stability

A system is in equilibrium if the sum of all the forces and moments acting on it is zero.

A system is statically stable if, when disturbed from equilibrium, it tends to return to the equilibrium position.

A system is dynamically stable if, given enough time, it actually returns to the equilibrium position.

The distinction between the two cases is shown in figure 1.

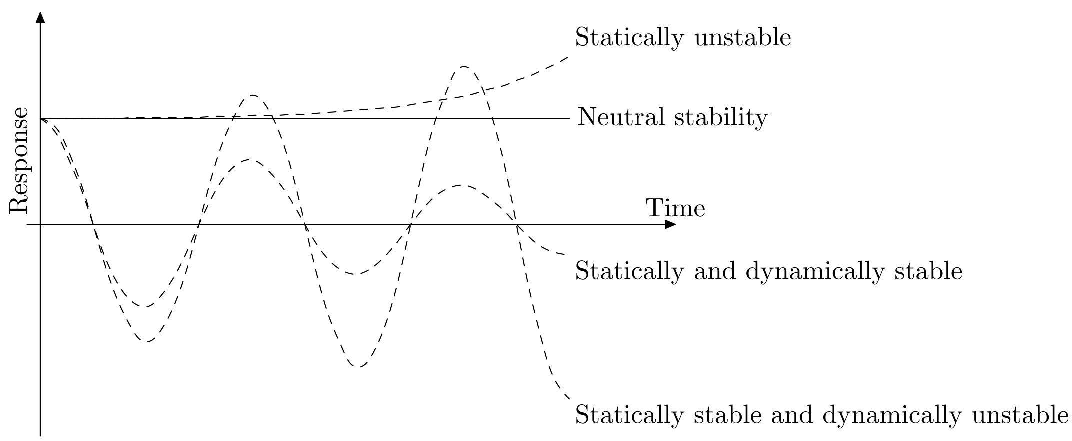


Figure 1: Definitions of stability

1.2 Functions of aircraft controls

The *elevator* provides pitch control. It is used to provide pitching moment equilibrium and enables adjustment of incidence.

Ailerons give roll control. They are used to bank the aircraft to initiate turns and to oppose disturbances in roll due to crosswinds or gusts.

The *rudder* gives yaw control. Its most important functions are in balancing engine-failed yawing moments, turn co-ordination, spin recovery and crosswind landing.

The three controls are used to enable moment equilibrium in all three axes simultaneously.

Controls may be operated mechanically by the pilot, or there might be aerodynamic assistance, power assistance or full power operation. Full power operation can be mechanically or electronically signalled (fly by wire).

The sign conventions for the controls and motions are shown in figure 2.

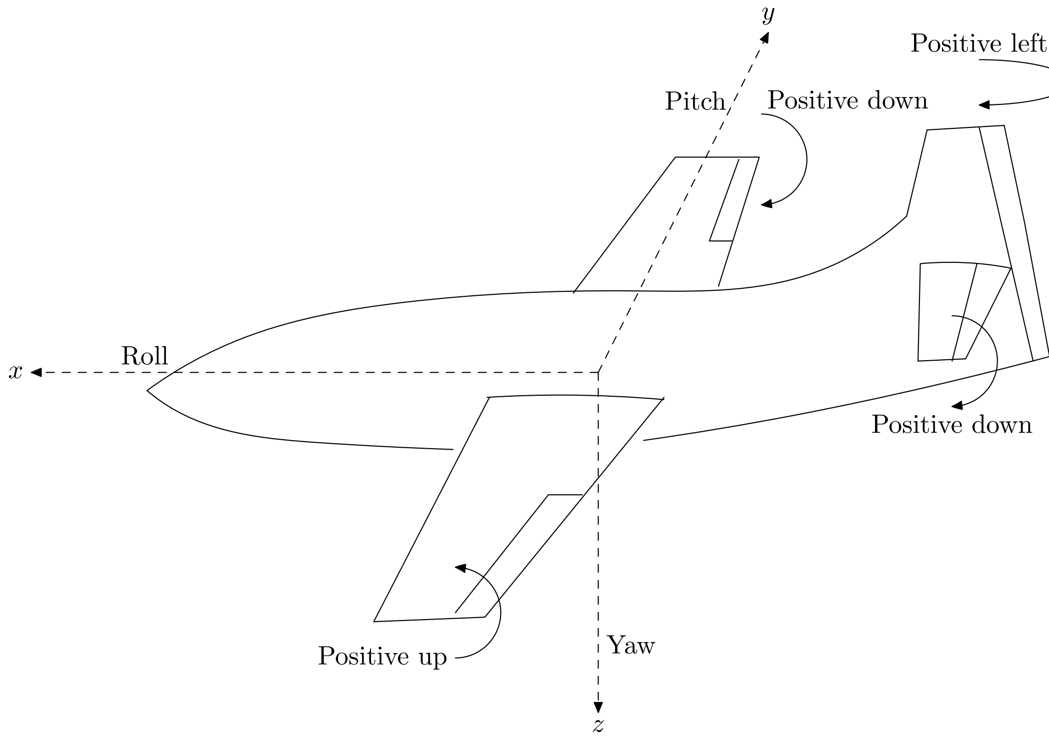


Figure 2: Axes and sign conventions

1.3 Forces and moments

The forces and moments on an aircraft arise from two sources: the *mass* of the aircraft and its *aerodynamics*.

The mass and mass distribution give rise to inertial forces (mass×linear acceleration, moment of inertia×angular acceleration) and gravitational forces (mass×acceleration due to gravity).

The aerodynamic contributions are:

1. static forces and moments due to linear velocities;
2. damping forces and moments due to angular velocities;
3. control forces and moments produced by rudder, ailerons, elevators etc.

1.4 Effects of symmetry

Since most aircraft are laterally symmetric (i.e. the left-hand side is a mirror image of the right-hand side) it follows that in forward flight with wings level and with no roll or yaw the resultant of the aerodynamic forces must lie in the plane of symmetry. Hence, in straight flight any symmetric disturbance will result in only horizontal and vertical motion of the centre of gravity (c.g.) and pitching about the c.g. This is longitudinal symmetric motion.

1.5 Trim and stability

The sign conventions for examining equilibrium and stability in free flight are shown in figure 3.

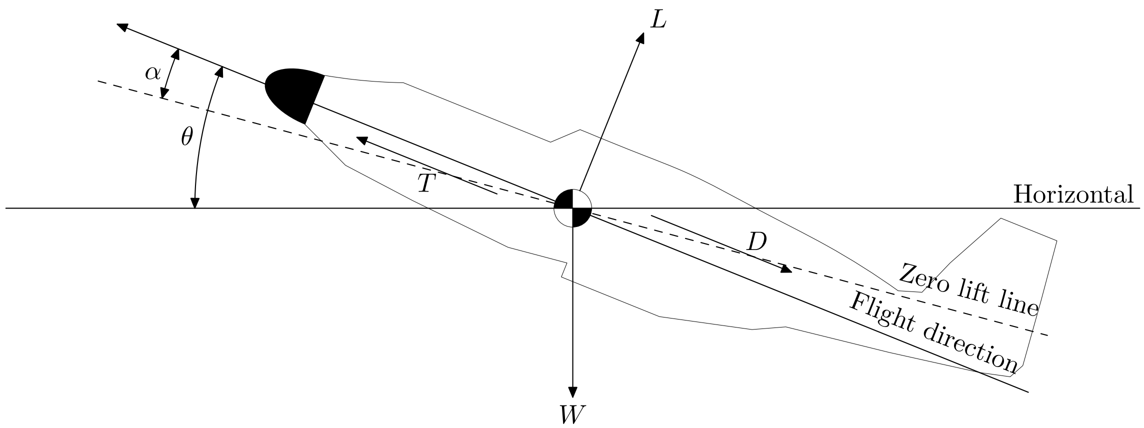


Figure 3: Sign conventions for longitudinal stability

For equilibrium there must be no net forces or moments acting on the aircraft. Hence, resolving parallel and perpendicular to the aircraft at equilibrium:

$$\text{Perpendicular:} L - W\cos\theta = 0, \tag{1}$$

$$\text{Parallel:} T - D - W\sin\theta = 0, \tag{2}$$

$$\text{Moments about the c.g.:} M_{\text{cg}} = 0. \tag{3}$$

M_{cg} cannot come from the weight of the aircraft since, by definition, the gravitational moments about the c.g. are zero. Therefore, the aerodynamic moments coming from the wing/fuselage and tail must all be in balance. If $M_{\text{cg}} = 0$ the aircraft is said to be *trimmed*.

For stability all that is necessary is that if, say, the aircraft pitches nose up then the resulting incremental aerodynamic moment about the c.g., ΔM_{cg}, should be negative in order to push the nose down again, i.e. $\partial M_{\text{cg}}/\partial\alpha$ must be negative.

It is quite possible to achieve trim without stability. It is also possible to achieve stability without being able to trim at a useful incidence. These states are shown in figure 4.

1.6 Aerodynamic centre and neutral point

The aerodynamic forces and moments acting on an aircraft of a given shape are independent of the position of the centre of gravity. Therefore, the aerodynamic loads can be considered separately from the gravitational loads.

The loads around an aircraft in trim conditions are sketched in Figure 5. The aerodynamic loads can be considered to act through a point at which the aerodynamic moments are zero. This location is known as the centre of pressure. However, as the incidence of the aircraft

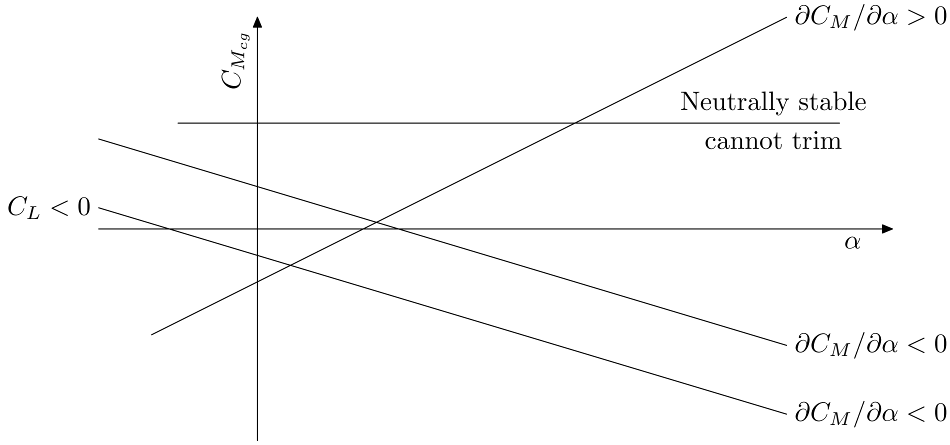


Figure 4: Trim and stability

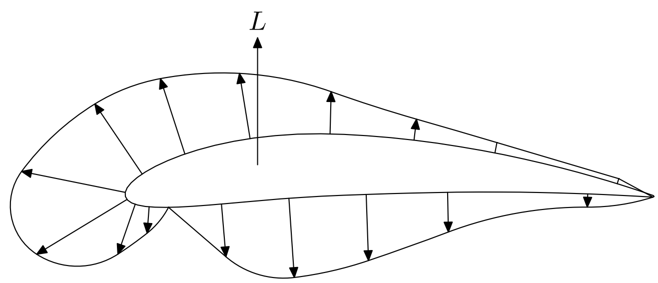


Figure 5: Centre of pressure

varies the location of the centre of pressure changes, it is therefore not very useful for calculations or consideration of stability.

A more useful concept is that of *aerodynamic centres*. The aerodynamic centre is generally defined as the location at which the pitching moment is unaffected by the incidence (this assumes linear aerodynamics). One way of thinking of the aerodynamic centre is by considering the *incremental* aerodynamic loads. These are sketched in figure 6.

When the incidence is increased slightly the incremental aerodynamic load distributions can be considered to act through a certain point—their combined centre of pressure. This point is the aerodynamic centre of the whole aircraft, and is the point where $\mathrm{d}M/\mathrm{d}\alpha = 0$.

The relationship between the *aerodynamic centre of the whole aircraft* and the location of the centre of gravity of the aircraft defines its static stability. There are three possible cases, shown in Figure 7.

1. If the c.g. is forward of the aerodynamic centre, $\mathrm{d}M_{\mathrm{cg}}/\mathrm{d}\alpha$ will be negative and the aircraft will therefore be statically stable.

2. If the c.g. is aft of the aerodynamic centre, $\mathrm{d}M_{\mathrm{cg}}/\mathrm{d}\alpha$ will be positive and the aircraft will therefore be statically unstable.

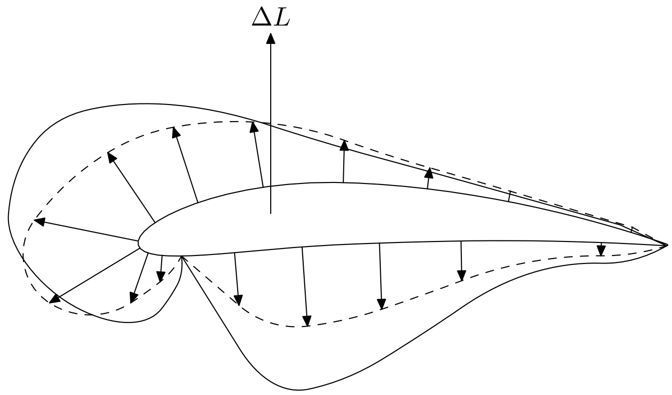


Figure 6: Incremental loads

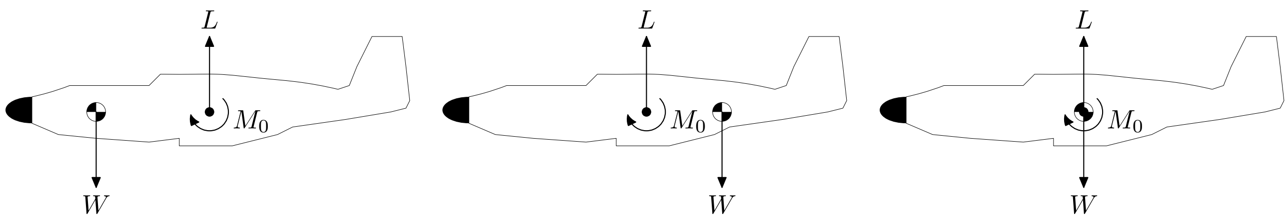


Figure 7: c.g and aerodynamic centre relationships

3. If the c.g. is at the aerodynamic centre, $\mathrm{d}M_{\text{cg}}/\mathrm{d}\alpha$ will be zero and the aircraft will therefore be neutrally stable.

The *neutral point* of an aircraft is defined as the position of the centre of gravity for which the static stability is neutral. A neutral point is a purely aerodynamic property of a particular configuration (neglecting distortion and aeroelastic effects). For a wing alone $\mathrm{d}M/\mathrm{d}\alpha = 0$ is the condition defining the aerodynamic centre (i.e. for a wing alone the neutral point is the aerodynamic centre. Similarly, the neutral point of a complete aircraft may be regarded as the aerodynamic centre of the *complete aircraft*.

The term “aerodynamic centre” is usually reserved for aerofoils only, and “neutral point” is applied to complete aircraft.

1.7 Definitions of static and c.g. margins

The gradient of the $C_M$vs α graph obviously indicates the degree of static stability (see figure 4), but the parameters used to communicate the degree of stability and their precise definitions are the *static margin*:

$$K_n = -\frac{\mathrm{d}C_{M_{\text{cg}}}}{\mathrm{d}C_R} \tag{4}$$

where $C_R = (C_L^2 + C_D^2)^{1/2}$, the resultant force.

The c.g. margin, H_n, is the distance of the neutral point aft of the c.g. (measured in "mean chord", $\overline{c}$). It can easily be shown that:

$$H_n = -\frac{\mathrm{d}C_M}{\mathrm{d}C_L} = h_n - h \tag{5}$$

where $h\overline{c}$ is the distance of the c.g. aft of a reference point and $h_n\overline{c}$ is the distance of the neutral point aft of a reference point.

At low speeds and for small flight path inclinations $C_R \approx C_L$ and C_L, C_M, C_R are not influenced by Mach number or aeroelastic effects. Hence $H_n \approx K_n$ under these conditions.

1.8 Basic aerofoil and control characteristics

Aerofoils and wings

To enable comparisons to be made between different flight conditions, and between wind-tunnel testing and full scale flight the lift, drag and pitching moments are generally non-dimensionalised:

$$\text{Lift coefficient}\, C_L = \frac{L}{\rho V^2 S/2} \tag{6}$$

$$\text{Drag coefficient}\, C_D = \frac{D}{\rho V^2 S/2} \tag{7}$$

$$\text{Moment coefficient}\, C_M = \frac{M}{\rho V^2 S\overline{c}/2} \tag{8}$$

The root chord length, c_0, is often used as the characteristic length for tailless aircraft. Note that for each of these coefficients, the subscript is *upper* case. This indicates that the coefficients relate to three dimensional planforms.

A typical lift curve for a wing is shown in figure 8.

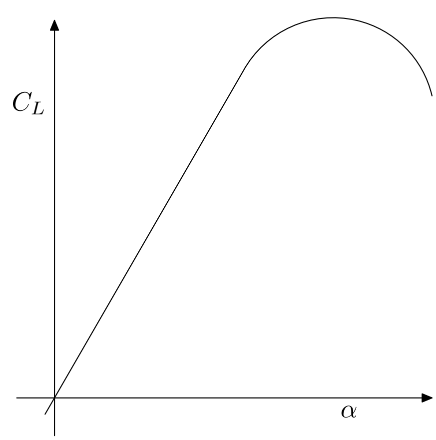


Figure 8: Lift curve

For convenience, in stability work we usually use incidence, α, measured from the zero lift line. *This is different to most other aerodynamic work, and should be checked for each case*. All of the stability analysis in this course will examine the linear region of the lift curve, and hence would need to be significantly modified for high angle of attack regimes and/or manoeuvring combat aircraft.

Control forces and moments

If the geometric incidence of a wing/tailplane is held constant the effect of a positive deflection of a trailing edge control is to generate extra lift along with a pitching moment. This pitching moment is generally in the nose down sense due to the form of the additional loading distribution (see figure 9).

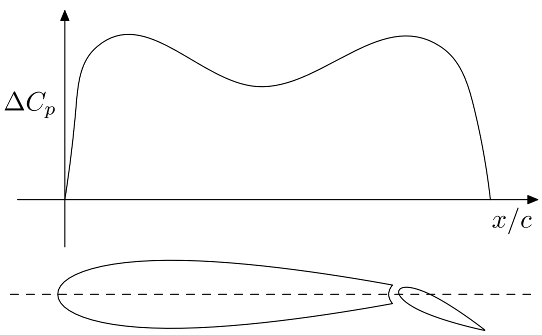


Figure 9: Loading due to control deflection

A positive control deflection therefore gives a positive contribution to lift and a negative pitching moment. Similarly, if we have a tab at the trailing edge of the control it will generate lift together with a negative pitching moment. The deflections of controls and tabs are generally given the symbols η and β and they are measured as shown in figure 10.

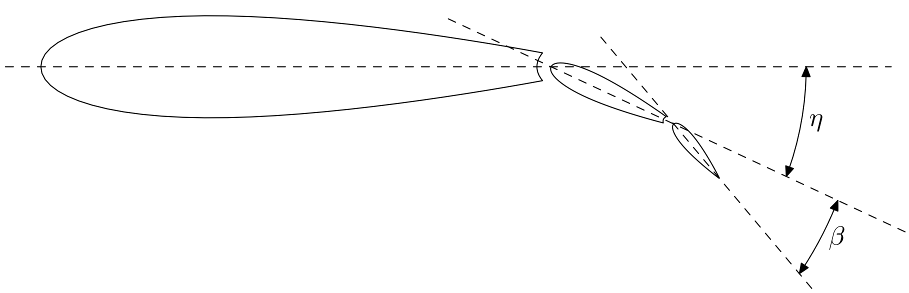


Figure 10: Measurement of control/tab deflections

The contributions of control surfaces and tabs to lift and pitching moment are generally assumed to be linear. To simplify future derivations and expressions the following definitions are used. These refer to the lift generated due to tailplane incidence, α_T, elevator deflection,

η, and tab deflection, β:

$$a_1 = \frac{\partial C_{L_T}}{\partial \alpha_T}, \quad a_2 = \frac{\partial C_{L_T}}{\partial \eta}, \quad a_3 = \frac{\partial C_{L_T}}{\partial \beta}. \tag{9}$$

The pressure distributions associated with each of these deflections are shown in figure 11.

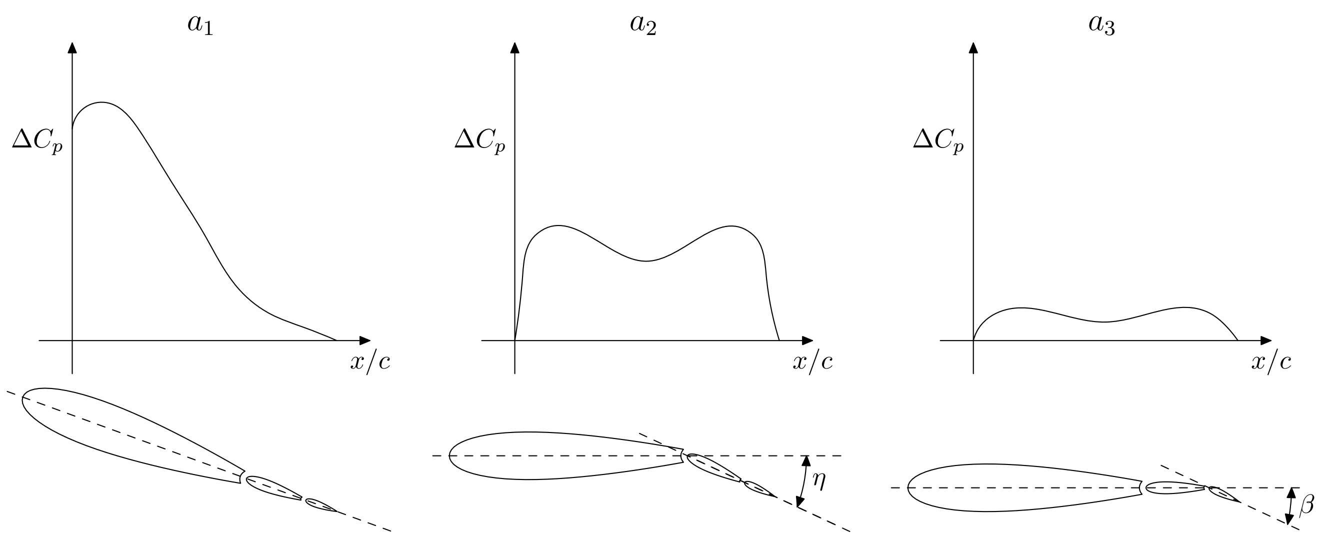


Figure 11: Pressure distributions due to deflections, $a_1 > a_2 > a_3$.

It is assumed that the lift and pitching moments can also be added together. Hence

$$C_{L_T} = a_1\alpha_\mathrm{T} + a_2\eta + a_3\beta.$$

Similarly,

$$C_{Mac} = C_{M_0} + \frac{\partial C_{M_0}}{\partial \eta}\eta + \frac{\partial C_{M_0}}{\partial \beta}\beta.$$

Control hinge moments

To enable controls to be designed such that the forces that the pilot must exert can be estimated the moments acting on the control itself (rather than on the whole aircraft) need to be calculated. Similarly to the forces and moments acting on the aircraft described in the preceding section the contributions from tailplane incidence, elevator and tab deflection are considered separately and summed to give the total control hinge moment. The hinge moment coefficient, C_H, is defined as:

$$C_H = \frac{\text{Hinge moment}}{\rho V^2 S_\eta c_\eta / 2} \tag{10}$$

Where S_η is the control surface area and c_η is the control surface mean chord. Both of these values are measured aft of the hinge line, as shown in figure 12.

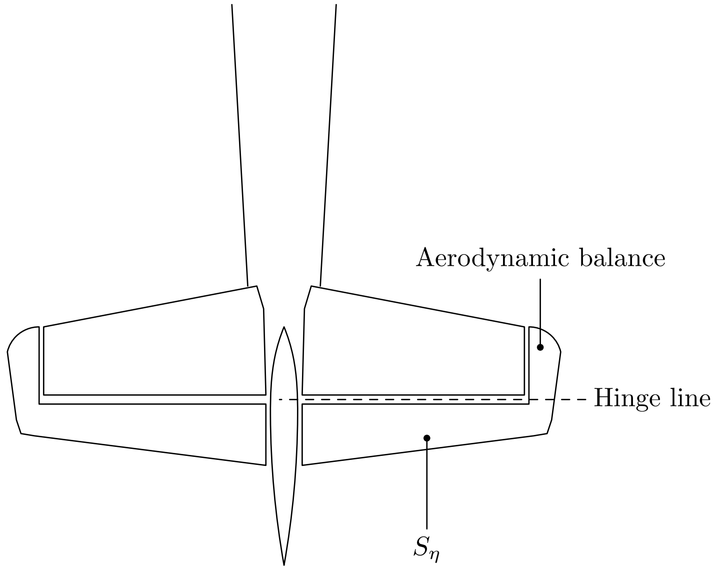


Figure 12: Measurement of control surface area

Again, to simplify future derivations the following definitions are used to represent the control hinge moments due to tailplane incidence, elevator and tab deflection respectively:

$$b_1 = \frac{\partial C_H}{\partial \alpha_T}, \quad b_2 = \frac{\partial C_H}{\partial \eta}, \quad b_3 = \frac{\partial C_H}{\partial \beta}, \tag{11}$$

For *non-symmetric* tailplane cross-sections there is usually a hinge moment when all other deflections are zero. This is given the symbol b_0. The pressure distributions associated with each of these terms are shown in figure 13.

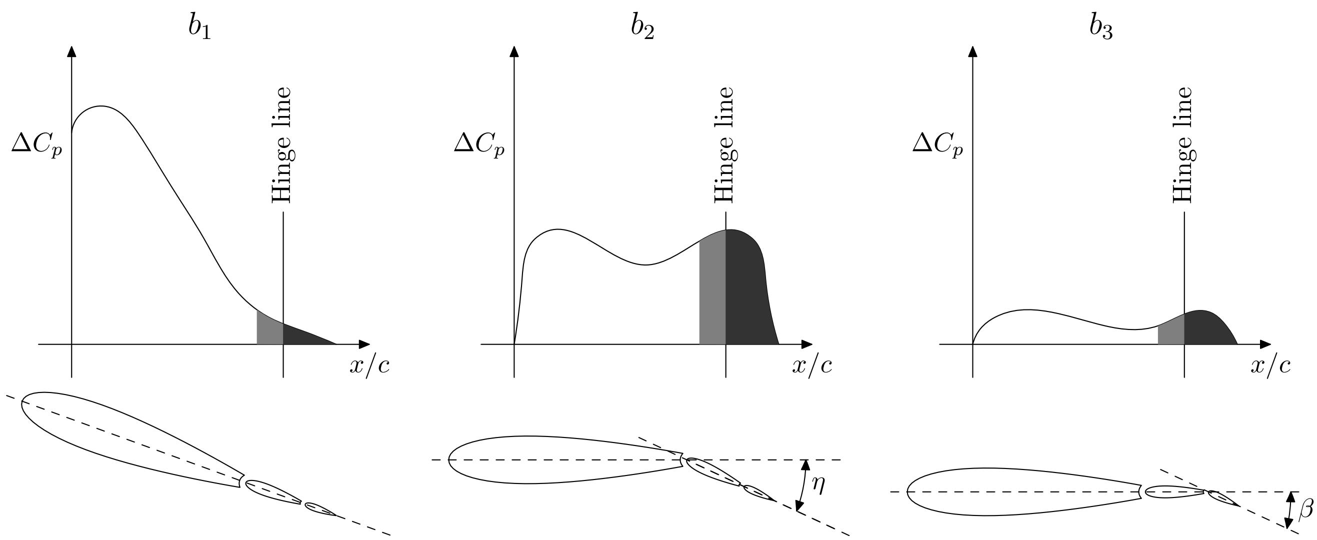


Figure 13: Control hinge moments

The hinge moments acting on a given control can therefore be defined as:

$$C_H = b_0 + b_1\alpha_{\mathrm{T}} + b_2\eta + b_3\beta. \tag{12}$$

The characteristics of any given control will depend on a number of factors including the relative size of the control (c_f/c), the hinge position and the shape of the balance portion, as well as the aerofoil cross-section. These factors will be discussed when stick forces for manoeuvres are examined.

2 Longitudinal static stability—stick fixed

2.1 Elementary longitudinal stability

"Stick fixed" stability is concerned with calculating the trim angles and stability of an aircraft with the control surfaces held at a constant location. Hence, no account is made of the control forces that the pilot (or power assist etc.) must provide. This simplifies the analysis such that only the forces and moments acting on the whole aircraft need to be in equilibrium for a trim condition. The simplest way to consider the trim and stability of a conventional aircraft is shown in figure 14.

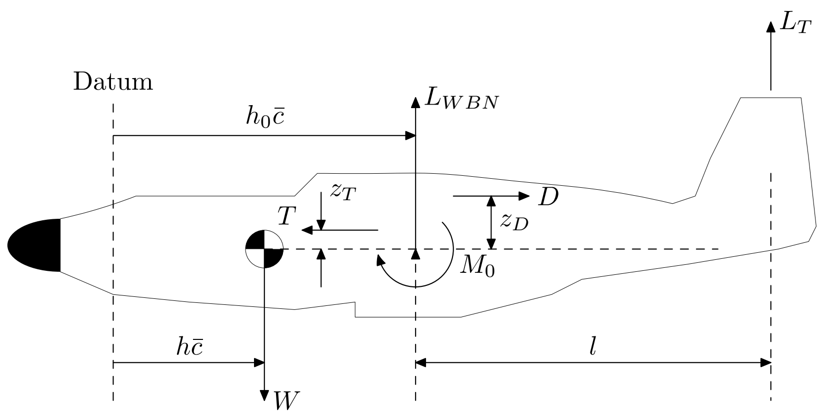


Figure 14: Stick fixed stability (conventional aircraft)

It is assumed that the aerodynamics are linear, that the angles α and η_{T} are small and that there is no wake effect on the tailplane (§2.2). Note that the total lift of the aircraft has now been divided into a "Wing/Body/Nacelle" (WBN) component and a Tailplane (T) component. Taking moments about the c.g.:

$$\begin{aligned} M_{\mathrm{cg}} &= M_0 - L_{WBN}(h_0 - h)\bar{c} - L_T((h_0 - h)\bar{c} + l) - Tz_T + Dz_D \\ &= M_0 - (h_0 - h)\bar{c}(L_{\mathrm{WBN}} + L_T) - L_T l - Tz_T + Dz_D \\ &= M_0 - (h_0 - h)\bar{c}L - L_T l - Tz_T + Dz_D. \end{aligned}$$

If we assume that Tz_T and Dz_D are small, which is reasonable since lift is normally much larger than drag (and hence thrust):

$$M_{\text{cg}} = M_0 - (h_0 - h)\overline{c}L - L_T l.$$

As is usual for aerospace calculations, non-dimensional parameters are used. C_L, C_D and C_M have been defined previously (§1.8). However, a new definition is required for the lift coefficient of the tailplane, C_{L_T}:

$$C_{L_T} = \frac{L_T}{\rho V^2 S_T/2} \tag{13}$$

where S_T is the area of the tailplane.

Hence, if the pitching moment M_{cg} is divided through by $S(\rho V^2/2)$ we get:

$$C_{M_{\text{cg}}} = C_{M_0} - (h_0 - h)C_L - C_{L_T}\frac{S_T}{S}\frac{l}{\overline{c}}.$$

If we define the *tail volume coefficient*, $\overline{V}$, as:

$$\overline{V} = \frac{S_T}{S}\frac{l}{\overline{c}} \tag{14}$$

$$C_{M_{\text{cg}}} = C_{M_0} - (h_0 - h)C_L - \overline{V}C_{L_T}. \tag{15}$$

This is an extremely important equation and is at the root of all stick-fixed stability calculations. If you're stuck when attempting a question that asks, "What is the lift required at the tailplane for trim." or, "Calculate the elevator angle required for trim." this is always a good starting point!

Remember, for trim $M_{\text{cg}} = 0$ hence $C_{M_{\text{cg}}} = 0$.

2.2 Effect of downwash on tailplanes

Downwash is present at the tailplane of a conventional aircraft due to the trailing vortices generated by the wing, as shown in figure 15.

This has a significant effect on the lift generated by the tailplane, since it alters the effective angle of attack as shown in figure 16.

Hence:

$$\alpha_{\text{T}} = \alpha + \eta_{\text{T}} - \epsilon$$

where η_{T} is the tailplane setting relative to the wing zero lift line.

For an untwisted wing, the downwash velocity (and hence the downwash angle, ϵ) at the tailplane is proportional to the lift generated by the wing. Therefore, since we are only interested in the linear region of the lift curve, the downwash angle is also proportional to α. Hence:

$$\epsilon = \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\alpha + \epsilon_0.$$

Figure 15: Trailing vortices

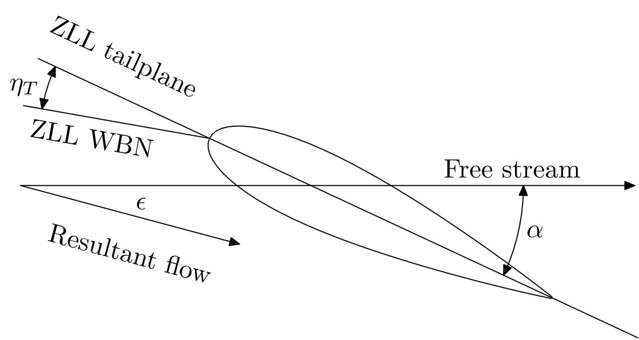


Figure 16: Effect of downwash on tailplane

ϵ_0 is only present for a wing where the zero lift angle of attack varies along its length (i.e. a wing with a varying cross-section/camber along its length or with twist). Putting these two equations together results in:

$$\alpha_{\mathrm{T}} = \alpha + \eta_{\mathrm{T}} - \left(\epsilon_0 + \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\alpha\right) = \alpha\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + (\eta_{\mathrm{T}} - \epsilon_0).$$

By definition (§1.8):

$$C_{L_T} = a_1\alpha_{\mathrm{T}} + a_2\eta + a_3\beta.$$

So,

$$C_{L_T} = a_1(\alpha + \eta_{\mathrm{T}} - \epsilon) + a_2\eta + a_3\beta$$

and therefore:

$$C_{L_T} = a_1\alpha\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + a_1(\eta_{\mathrm{T}} - \epsilon_0) + a_2\eta + a_3\beta.$$

To be able to solve this equation, and calculate the lift acting on the tailplane, we require a relationship between this and C_L.

Since we are looking at the linear region of aerodynamics and we have defined that there is zero lift at zero incidence (figure 8), we can therefore say that:

$$C_L = a\alpha$$

and hence

$$\alpha = C_L/a$$

where a is the overall lift curve slope of the aircraft. We can substitute this into the previous equation to get:

$$C_{L_T} = \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_{\mathrm{T}} - \epsilon_0) + a_2\eta + a_3\beta. \tag{16}$$

Equation 15 made no assumptions about where the lift at the tailplane came from, or whether there was a downwash effect etc. Therefore, this equation can be substituted into Equation 15, resulting in the rather torturous, but extremely useful equation below:

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_{\mathrm{T}} - \epsilon_0) + a_2\eta + a_3\beta\right].$$

This equation is so useful because it allows us to calculate the elevator angles needed for trim at given flight conditions. The elevator angle to trim is given the symbol, $\overline{\eta}$. This is often a design criterion for aircraft. For example, the c.g. range available for an aircraft is generally limited by the elevator angle required to trim on approach to a runway at low speed with flaps and undercarriage extended. To calculate the elevator angle to trim, $\overline{\eta}$, the previous equation is rearranged to make η the subject and, since we are calculating the conditions for trim, C_M=0.

This results in:

$$\overline{\eta} = \frac{1}{\overline{V}a_2}\left[C_{M_0} - (h_0 - h)C_L - \overline{V}\left(\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_\mathrm{T} - \epsilon_0) + a_3\beta\right)\right].$$

This equation is not really worth remembering, since it is easily derived by rearranging Equation 15.

2.3 Stick fixed stability and c.g. margins

By definition,

$$K_n \approx H_n = -\frac{\mathrm{d}C_M}{\mathrm{d}C_L}.$$

Hence, to examine stability we need to differentiate equation 15 with respect to C_L:

$$-\frac{\mathrm{d}C_M}{\mathrm{d}C_L} = (h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)$$

By definition, the neutral point of the aircraft is the c.g. position giving zero $\mathrm{d}C_M/\mathrm{d}C_L$. Hence,

$$h_n = h_0 + \overline{V}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right).$$

h_0 is the "neutral point" of the wings, body and nacelle of the aircraft. Therefore, the addition of a tail has moved the neutral point of the aircraft aft by an amount $\overline{V}(a_1/a)(1 - \mathrm{d}\epsilon/\mathrm{d}\alpha)$.

3 Longitudinal static stability—stick free

3.1 Introduction

So far, we have examined the elevator angles required to trim, and the effect of the tailplane on the overall longitudinal stability of the aircraft. However, the amount of force that the pilot (or hydraulic system etc.) needs to exert has been ignored. Clearly, for an aircraft to be as effective as possible these factors must be considered.

The first step in this analysis is to examine the "stick free" stability of the aircraft. The phrase "stick free" indicates that the pilot has released the controls. Therefore, the elevator may move to any location. In the equilibrium condition, the controls will settle in a location where the hinge moments are zero (ignoring frictional effects).

Note that "stick-free" conditions only affect the elevator. Typically the tailplane, η_T, and the tab, β, are locked into position and will not move if the pilot releases the stick.

3.2 Analysis

The basic hinge moment equation is:

$$C_H = b_0 + b_1\alpha_\mathrm{T} + b_2\eta + b_3\beta.$$

If the elevator is free ("stick free"), then C_H=0. We can therefore rearrange to calculate the elevator angle, η, for the stick free case.

$$\eta = -\frac{b_0 + b_1\alpha_\mathrm{T} + b_3\beta}{b_2}.$$

From our earlier analysis (and from the equation sheet),

$$C_{L_T} = \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_\mathrm{T} - \epsilon_0) + a_2\eta + a_3\beta.$$

So, substituting for η results in:

$$C_{L_T} = \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_\mathrm{T} - \epsilon_0) - \frac{a_2}{b_2}(b_0 + b_1\alpha_\mathrm{T} + b_3\beta) + a_3\beta.$$

For a given flight condition we know everything on the right hand side of this expression except α_T. Fortunately, we've already worked out an expression for α (§2.2, page 13)

$$\alpha_\mathrm{T} = \frac{C_L}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + (\eta_\mathrm{T} - \epsilon_0).$$

Collecting terms and rearranging results in:

$$C_{L_T} = \left(a_1 - \frac{a_2 b_1}{b_2}\right)\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)\frac{C_L}{a} + \left(a_1 - \frac{a_2 b_1}{b_2}\right)(\eta_\mathrm{T} - \epsilon_0) + \left(a_3 - \frac{a_2 b_3}{b_2}\right)\beta - \frac{a_2 b_0}{b_2}.$$

This is obviously quite a complex expression. However, we can make it more straightforward by defining two new variables, $\overline{a_1}$ and $\overline{a_3}$ where:

$$\overline{a_1} = a_1\left(1 - \frac{a_2 b_1}{a_1 b_2}\right),$$
$$\overline{a_3} = a_3\left(1 - \frac{a_2 b_1}{a_3 b_2}\right)$$

(both of these expressions are given on the equation sheet).

If we substitute these expressions back into the equation for C_{L_T} we get:

$$C_{L_T} = \overline{a_1}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)\frac{C_L}{a} + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta - \frac{a_2 b_0}{b_2}.$$

We now have an expression that allows us to calculate the lift generated by the tailplane if the stick is released and the elevator stabilises such that the elevator hinge moments are zero.

We already have equation 15 which enables us to calculate the lift required at the tailplane. Therefore, since:

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V} C_{L_T},$$

we can substitute for C_{L_T} to get:

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta - \frac{a_2 b_0}{b_2}\right].$$

This expression allows us to calculate the tab angle required to trim with zero stick force, $\overline{\beta}$. Remember, for trim C_M=0.

3.3 Stick free stability

As previously, to examine stick free stability we differentiate with respect to C_L.

$$\frac{\mathrm{d}C_M}{\mathrm{d}C_L} = -(h_0 - h) - \overline{V}\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right).$$

The static margin stick free,

$$K_n' = -\frac{\mathrm{d}C_M}{\mathrm{d}C_L} = (h_0 - h) + \overline{V}\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right).$$

Since the static margin (stick fixed), K_n, is:

$$K_n = (h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)$$

the two static margins will be the same if:

$$a_1 = \overline{a_1} = a_1\left(1 - \frac{a_2 b_1}{a_1 b_2}\right)$$

that is, if

$$\frac{a_2 b_1}{a_1 b_2} = 0.$$

a_1 and a_2 are both positive, since increasing either the incidence of the tailplane or the elevator deflection increases the lift generated by the tailplane. b_2 must be negative for correct 'feel' of elevator (i.e. when the stick is pulled, it pushes back at you). b_1 can be made zero by using aerodynamic balances or by moving the hinge line of the elevator (which also affects b_2).

There are 3 possible cases for the stick free case. These are shown in figure 17.

1. If $a_2 b_1 / a_1 b_2 > 0$ (i.e. b_1 is negative), the aircraft becomes less stable in the stick free case. This is known as having a "convergent" elevator.

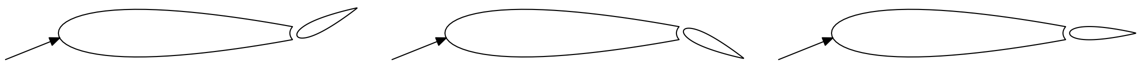


Figure 17: Stick free elevator conditions

2. If $a_2b_1/a_1b_2 < 0$ (i.e. b_1 is positive), the aircraft becomes more stable on freeing the stick. This is known as having a “divergent” elevator.

3. If $a_2b_1/a_1b_2 = 0$ (i.e. b_1 = 0), the aircraft is equally stable in in both stick- fixed and stick-free cases. This is known as having a “null” elevator.

The neutral point stick free h'_n and static and c.g. margins stick free H'_n are precisely analogous to the stick fixed cases.

4 Flight test measurement of static margins

4.1 Introduction

So far, we have used analytical techniques to examine the static stability of aircraft, and it is possible to use semi-empirical techniques to estimate parameters such as a_1, a_2, b_1, b_2 (see supplementary sheet). However, when we have designed an aircraft, how do we check that its stability characteristics are as we have calculated?

For most loadings or flight conditions our estimates will be adequate, but we need to know what the allowable loads and c.g. range of the aircraft are. If our estimates are too conservative, we lose potential revenue due to limitations on how the aircraft can be loaded. Conversely, if we have estimated that the aircraft is more stable than it actually is, then the aircraft may at some time be loaded in such a way that it is unstable or unsafe.

It is therefore necessary to be able to measure the stability of an aircraft in flight. This forms one part of the flight test course that will be undertaken in Semester 2, during the Group Design and Business Project.

4.2 K_n—elevator angle to trim

In trim conditions, the pitching moment acting on the aircraft, and hence C_M, are zero. We have already shown that (on equation sheet):

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V}C_{L_T}$$

and,

$$C_{L_T} = \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_\mathrm{T} - \epsilon_0) + a_2\eta + a_3\beta.$$

Hence,

$$C_M = 0 = C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_\mathrm{T} - \epsilon_0) + a_2\eta + a_3\beta\right].$$

We have previously shown (§2.3) that this can be differentiated to examine the stick-fixed stability of an aircraft such that:

$$K_n \approx H_n = -\frac{\partial C_M}{\partial C_L} = (h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right).$$

We have also shown (Example Question 2.3) that the pitching moment equations can be rearranged to get the elevator angle to trim, $\overline{\eta}$:

$$\overline{\eta} = \frac{1}{\overline{V}a_2}\left[C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_\mathrm{T} - \epsilon_0) + a_3\beta\right]\right]$$

and that there is a relationship between $\overline{\eta}$ and K_n because:

$$\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}C_L} = -\frac{1}{\overline{V}a_2}\left[(h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)\right] = -\frac{K_n}{\overline{V}a_2}.$$

But what do these relationships look like? $\overline{\eta}$ is plotted against C_L in figure 18 and the relationship between $\mathrm{d}\overline{\eta}/\mathrm{d}C_L$ and the location of the centre of gravity is shown in figure 19.

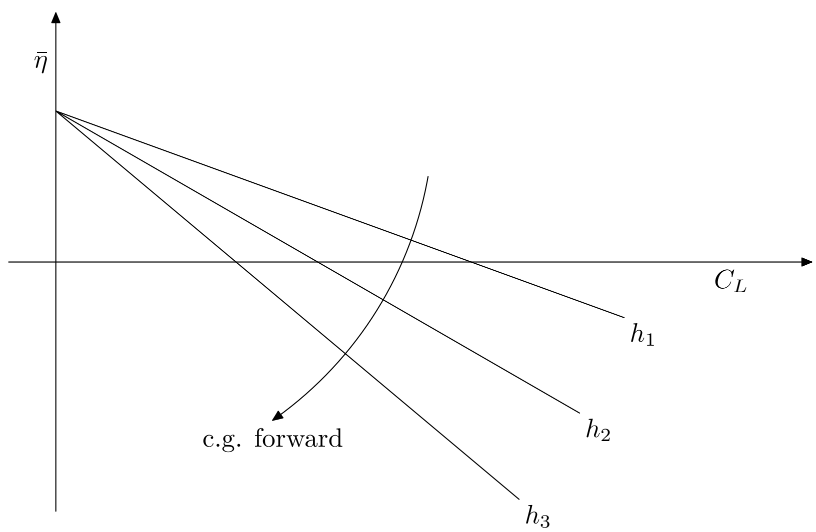


Figure 18: Elevator angle to trim at varying lift coefficients

Therefore, if the aircraft is flown straight, level and trimmed, and the elevator deflection is measured, a point on one of the lines on Figure 18 can be plotted. If the incidence and speed of the aircraft are then changed such that the aircraft is trimmed at a different lift coefficient another point at the same centre of gravity location can be measured. This can be repeated further to measure more points for the same c.g. location.

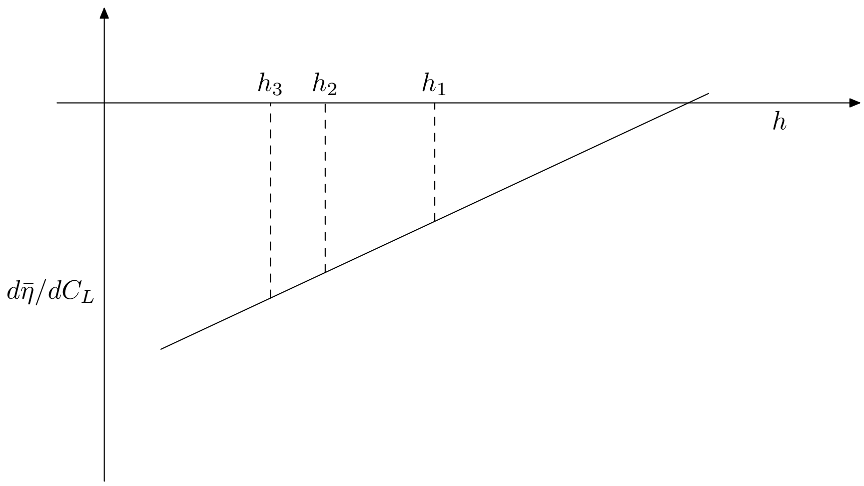


Figure 19: Measurement of neutral point location

If the aircraft is now landed, the location of the centre of gravity can be moved by a number of means; the fuel might be added to a different fuel tank, passengers might be loaded to different parts of the aircraft or luggage/freight might be loaded to different areas of the aircraft. The aircraft can now be trimmed at a number of speeds for this centre of gravity location, thus resulting in a second line (of different gradient) on the C_L vs $\overline{\eta}$ graph. If the gradients of these lines (i.e. $\mathrm{d}\overline{\eta}/\mathrm{d}C_L$) are now plotted against the c.g. location, as in figure 19, the stick fixed neutral point location can be found since it is the c.g. location where $\mathrm{d}\overline{\eta}/\mathrm{d}C_L$, and hence K_n, is zero.

4.3 What does the pilot feel?

Something that is worth noting is that this is not what the pilot will experience. Although the pilot will have a good feel for the elevator deflection (it will generally be directly proportional to the distance he/she has pushed/pulled the stick) the concept of a lift coefficient is somewhat contrived. The pilot might, however, note how the elevator angle varies with flight speed. Let's have a look at what the pilot will experience, then.

The variation of elevator angle with speed can easily be found from the derivation that we have already done, since:

$$\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}V} = \frac{\mathrm{d}\overline{\eta}}{\mathrm{d}C_L}\frac{\mathrm{d}C_L}{\mathrm{d}V}.$$

By definition,

$$C_L = \frac{L}{\rho V^2 S/2}.$$

Hence,

$$\frac{\mathrm{d}C_L}{\mathrm{d}V} = -\frac{2L}{\rho V^3 S/2} = -\frac{2C_L}{V}.$$

Therefore,

$$\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}V} = -\frac{2C_L}{V}\frac{\mathrm{d}\overline{\eta}}{\mathrm{d}C_L} = \frac{2C_L}{V}\frac{K_n}{\overline{V}a_2}.$$

which is sketched in figure 20.

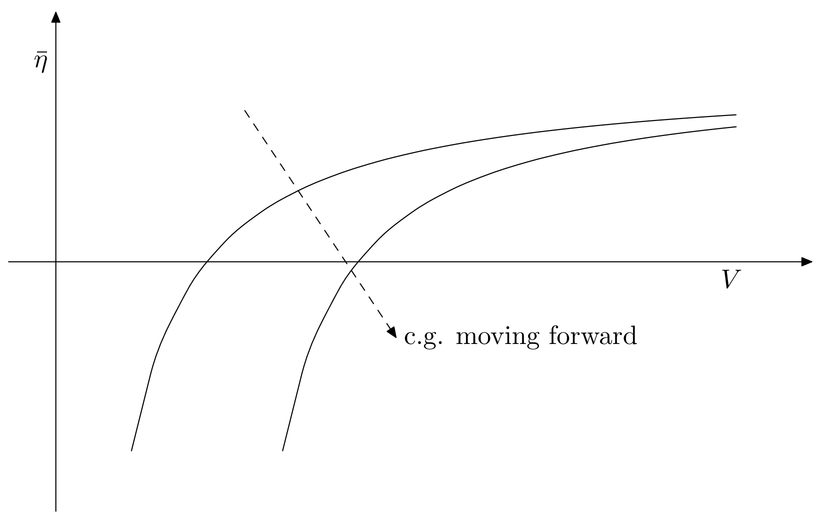


Figure 20: What the pilot experiences

4.4 K_n'—tab angle to trim

As normal, we start with the standard pitching moment equation and we are looking at the case where we are trimmed, hence:

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V}C_{L_T} = 0$$

and

$$C_{L_T} = \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_\mathrm{T} - \epsilon_0) + a_2\eta + a_3\beta.$$

From §3.2:

$$\eta = -\frac{b_0 + b_1\alpha_\mathrm{T} + b_3\beta}{b_2}$$

$$\text{and} \quad C_{L_T} = \frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta - \frac{a_2 b_0}{b_2}$$

Hence

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) - \frac{a_2 b_0}{b_2}\right]$$

If we rearrange to find the tab angle to trim, $\overline{\beta}$, where C_M=0 we get:

$$\overline{\beta} = \frac{1}{\overline{V}\overline{a_3}}\left\{C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_\mathrm{T} - \epsilon_0) + \overline{a_3}\beta - \frac{a_2 b_0}{b_2}\right]\right\}$$

Differentiating with respect to C_L results in:

$$\frac{\mathrm{d}\overline{\beta}}{\mathrm{d}C_L} = -\frac{1}{\overline{V}\overline{a_3}}\left[h_0 - h + \overline{V}\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)\right].$$

But from §3.3:

$$K'_n = (h_0 - h) + \overline{V}\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right).$$

Hence:

$$\frac{\mathrm{d}\overline{\beta}}{\mathrm{d}C_L} = -\frac{K'_n}{\overline{V}\overline{a_3}}.$$

Therefore, there is a relationship between the stick free static margin, K'_n, and the tab angle to trim, $\overline{\beta}$, in exactly the same way as there is a relationship between the stick fixed static margin and the elevator angle to trim. So, we can use a very similar technique to measure the stick free neutral point, h'_n, as we would to measure the stick fixed manoeuvre point, h_n.

Sketches of the variation of $\overline{\beta}$ with lift coefficient and the variation of $\mathrm{d}\overline{\beta}/\mathrm{d}C_L$ with c.g. location are shown in figure 21 and figure 22 respectively.

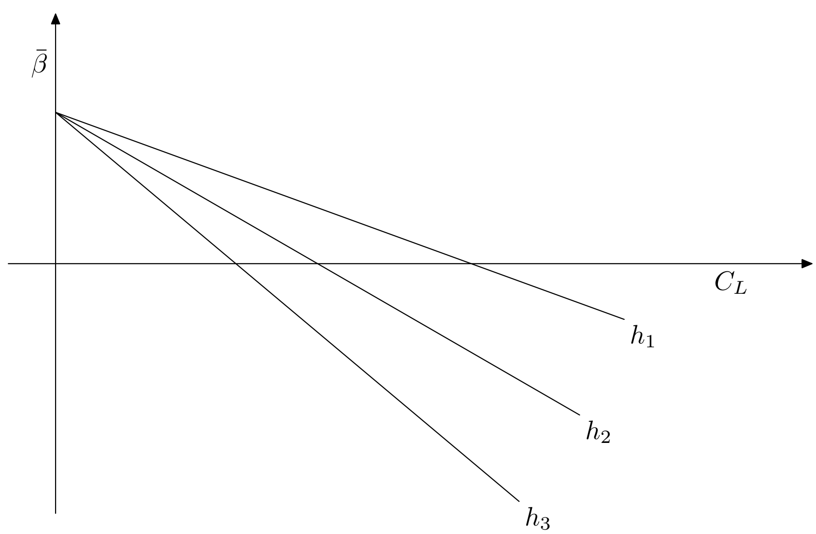


Figure 21: Tab angle to trim at varying lift coefficients

The flight tests to measure the stick free static margin are therefore undertaken in exactly the same way as those to measure the stick fixed static margin, except the tab angle to trim with zero stick force is measured instead of the elevator angle.

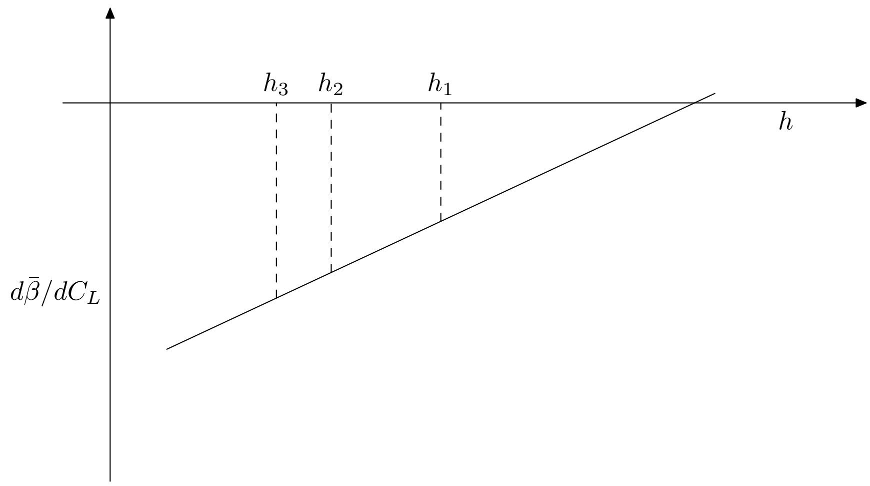


Figure 22: Measurement of stick free neutral point location

5 Tailless aircraft

5.1 Introduction

So far, we have only examined ‘conventional’ aircraft—those with a tailplane aft of the main wing. This analysis can easily be adapted to examine ‘canard’ configured aircraft, where the balancing lifting surface is known as a ‘foreplane’ and is positioned forward of the main wing. For such configurations, the basic equations are the same, but the distance, l, from the aerodynamic centre of the wing/body/nacelle to the foreplane is negative, as shown in figure 23.

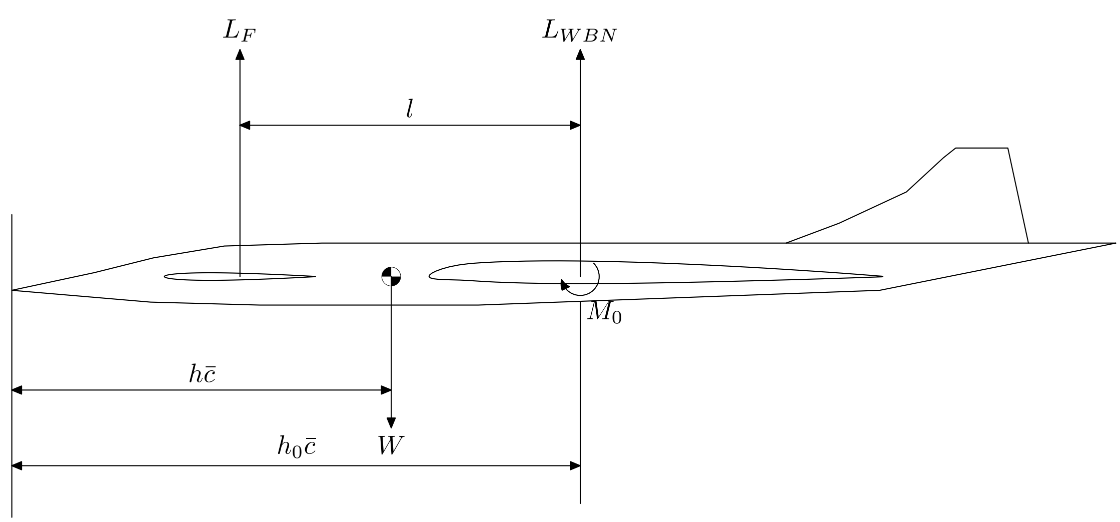


Figure 23: Canard configuration

The ‘tailless’ configuration has no lifting surface other than the main wing. An obvious example is Concorde. A number of military aircraft are tailless, such as the Avro Vulcan, although in recent years 'close coupled canard-deltas' have become popular. Examples of these aircraft are Eurofighter Typhoon and the Saab Gripen. The control surfaces used for tailless aircraft are shown in figure 24.

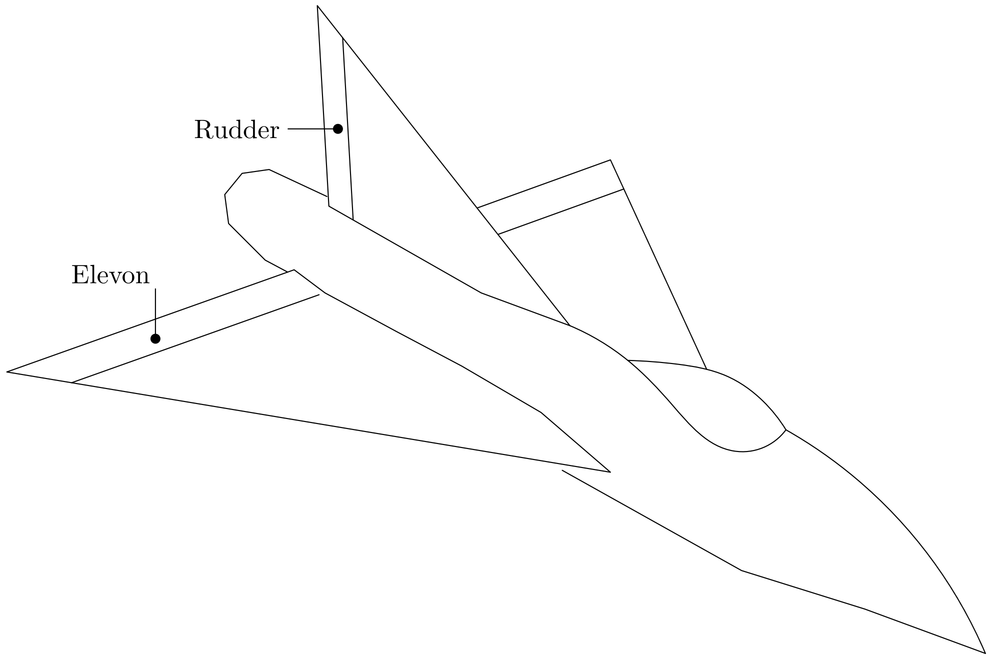


Figure 24: Control surfaces for tailless aircraft: the elevons operate together for pitch control and differentially for roll

5.2 Stick fixed stability

There are two key differences between the analysis of conventional and tailless aircraft. The first, and most obvious, is that there is no tailplane and thus no terms relating to tailplane lift. Secondly, in addition to deflections of the elevons causing a change in lift, they also generate a large pitching moment. This can cause complications, especially on landing.

The representation of a tailless aircraft used for examining static stability is given in figure 25

The total lift generated by a tailless aircraft is broken down in the same way as for the tailplane of a conventional aircraft so that:

$$C_L = a_1\alpha + a_2\eta.$$

Tailless aircraft very rarely have tabs, since the controls are almost always powered, hence the reduction of stick forces by using a tab is rarely necessary. The analysis of tailless aircraft is

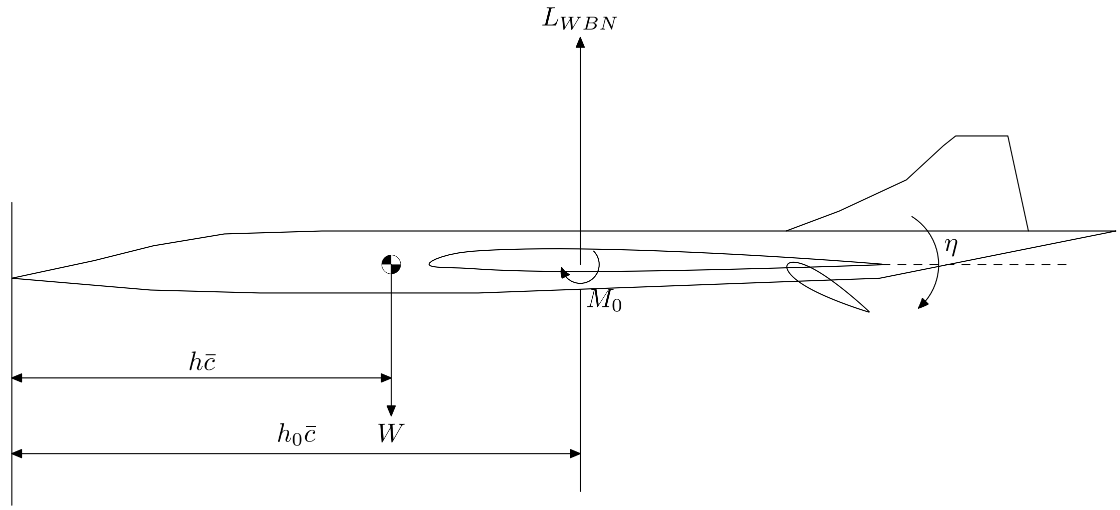


Figure 25: Representation of tailless aircraft

straightforward. If we take moments about the centre of gravity:

$$M_{\text{cg}} = M_0 + \frac{\partial M_0}{\partial \eta}\eta - (h_0 - h)c_0 L$$

Non-dimensionalizing results in:

$$C_M = C_{M_0} + \frac{\partial C_{M_0}}{\partial \eta}\eta - (h_0 - h)C_L.$$

As for conventional aircraft, this expression can be rearranged to calculate the elevon angle required to trim.

5.3 Static margin

As for conventional aircraft, we differentiate the pitching moment equation with respect to C_L to calculate the static margin:

$$\frac{\mathrm{d}C_M}{\mathrm{d}C_L} = -(h_0 - h).$$

Therefore,

$$K_n = h_0 - h.$$

5.4 Stick free stability

The stick free case for tailless aircraft is even more straightforward—since in practice there isn't a stick free case!

Due to the size of the elevons, and the size of typical aircraft, tailless aircraft almost always have powered controls, which do not have a ‘stick free’ condition because they are held in place by hydraulic systems. Therefore, within the scope of this course it is not necessary to consider the stick free case.

6 Stick forces

6.1 Introduction

All the analysis that we have undertaken to date has ignored the stick force required to trim an aircraft. Obviously, this is an important criterion since it will directly affect whether a pilot can hold the aircraft in trim for any length of time. Also, when we consider the stick forces required for manoeuvring, later in the course, we need to ensure that the forces required are realistic. But what is a reasonable force? Table 1 indicates the maximum forces, in Newtons, that can be applied to the controls for different lengths of time.

		Aileron		Elevator		Rudder	
		Stick	Wheel	Stick	Wheel	(Push)	
Maximum all-out effort	2 hands	400	530	800	980	1780	N
Maximum permissible effort	2 hands	—	360	440	440	890	N
	1 hand	220	220	310	310		N
Maximum comfortable effort	2 hands	—	130	—	180	270	N
	1 hand	90	90	130	130		N
Largest full travel		± 254	± 508	± 230	± 230	± 126	mm

Table 1: Maximum control forces

It is worth noting that the elevator push/pull force is always higher than the maximum aileron force and that the maximum rudder pedal force is higher than both of the stick forces. This information is used to ‘harmonise’ the controls. The controls are said to be harmonised if the aileron, elevator and rudder force ratios are 1:2:4 to achieve equal effect, defined as response in angular rate (i.e. the rudder pedal force required for a 10°/s yaw is double the stick force for a 10°/s pitch, which in turn is double the force for a 10°/s roll).

6.2 Analysis to calculate stick forces

The stick force to trim, generally given the symbol $\overline{P_e}$, is equal to the hinge moment multiplied by the gearing ratio (m_e, between the angular movement of the elevator and the distance the stick moves). Hence:

$$\overline{P_e} = m_e \frac{\rho V^2}{2} S_\eta c_\eta \overline{C_H}$$

We therefore need to find the hinge moment required to trim, $\overline{C_H}$. We have already found a way to calculate the tab angle required to trim for zero stick force (§3.2). Calculating the stick force required at different tab angles is very similar.

We start, as for the zero stick force (stick free) case, with the definition of the hinge moment, C_H:

$$C_H = b_0 + b_1\alpha_{\mathrm{T}} + b_2\eta + b_3\beta.$$

For the stick free case, C_H=0, which is not now true. However, we can still rearrange to find the elevator angle as a function of the hinge moments:

$$\eta = \frac{C_H - b_0 - b_1\alpha_{\mathrm{T}} - b_3\beta}{b_2}.$$

We also know, from the equation sheet, that:

$$C_{L_T} = a_1\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)\frac{C_L}{a} + a_1(\eta_{\mathrm{T}} - \epsilon_0) + a_2\eta + a_3\beta.$$

If we substitute η into this equation we get, after substituting for α_{T} and using the definitions of $\overline{a_1}$ and $\overline{a_3}$:

$$C_{L_T} = \overline{a_1}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)\frac{C_L}{a} + \overline{a_1}(\eta_{\mathrm{T}} - \epsilon_0) + \overline{a_3}\beta + \frac{a_2}{b_2}(C_H - b_0).$$

This is the more general case of an equation that we have already derived for C_H=0 (i.e. the stick free case). We know an equation that links the lift at the tailplane to the pitching moments:

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V}C_{L_T}.$$

Substituting for C_{L_T} in this equation results in:

$$C_M = C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_{\mathrm{T}} - \epsilon_0) + \overline{a_3}\beta + \frac{a_2}{b_2}(C_H - b_0)\right].$$

This can easily be re-arranged to find the tab angle to trim with zero hinge moments, $\overline{\beta}$:

$$\overline{V}\,\overline{a_3}\overline{\beta} = C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_{\mathrm{T}} - \epsilon_0) - \frac{a_2 b_0}{b_2}\right] \tag{17}$$

Alternatively, we can rearrange to get the hinge moments required to trim:

$$\overline{V}\frac{a_2\overline{C_H}}{b_2} = C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_{\mathrm{T}} - \epsilon_0) + \overline{a_3}\beta - \frac{a_2 b_0}{b_2}\right] \tag{18}$$

We could use this expression to calculate the elevator hinge moments for an arbitrary trimmed flight condition, but there is actually a shortcut to doing this. If we subtract equation 18 from equation 17 most of the terms cancel and we get:

$$\overline{V}\left(\overline{a_3}\overline{\beta} - \frac{a_2}{b_2}\overline{C_H}\right) = \overline{V}\,\overline{a_3}\beta$$

which can be rearranged to get:

$$\overline{C_H} = \frac{b_2}{a_2}\overline{a_3}(\overline{\beta} - \beta)$$

and hence:

$$\overline{P_e} = m_e \frac{\rho V^2}{2} S_\eta c_\eta \frac{b_2}{a_2}\overline{a_3}(\overline{\beta} - \beta).$$

Therefore, to calculate the stick force required to trim simply calculate the tab angle to trim 'stick-free' and use this equation.

6.3 More flight testing

It is theoretically possible to use the stick forces to calculate the stick free neutral point. We will only go through this briefly here, since in practice this method is problematic due to friction between the stick and the elevator.

We have already shown that:

$$\overline{C_H} = \frac{b_2}{a_2}\overline{a_3}(\overline{\beta} - \beta).$$

Differentiating with respect to lift coefficient, if the tab angle is held constant, results in:

$$\frac{\partial \overline{C_H}}{\partial C_L} = \frac{b_2}{a_2}\overline{a_3}\frac{\partial \overline{\beta}}{\partial C_L}$$

but from §4.4:

$$\frac{\mathrm{d}\overline{\beta}}{\mathrm{d}C_L} = -\frac{K_n'}{\overline{V}}\frac{1}{\overline{a_3}}.$$

Hence,

$$\frac{\mathrm{d}\overline{C_H}}{\mathrm{d}C_L} = -\frac{b_2 K_n'}{\overline{V} a_2}$$

Therefore, by measuring the hinge moments (stick forces) at different flight conditions and repeating for different c.g. positions (in a similar way to that used for tab angle and elevator angle measurements of the neutral points) we can measure the location of the stick-free neutral point.

6.4 Modifi cation of stick forces

Three methods are available to decrease the pilot's effort, bearing in mind that the pilot must retain some 'feel' of the control forces:

Gearing between stick and control surface. This has limited application since the gearing is restricted by stick travel limits.

Power assistance, either by 'load sharing' or by using full power assistance with artificial feel.

Aerodynamic assistance can be used, by using 'horn balances', by moving the hinge line of the elevator or by using balance/anti balance tabs (see figure 26).

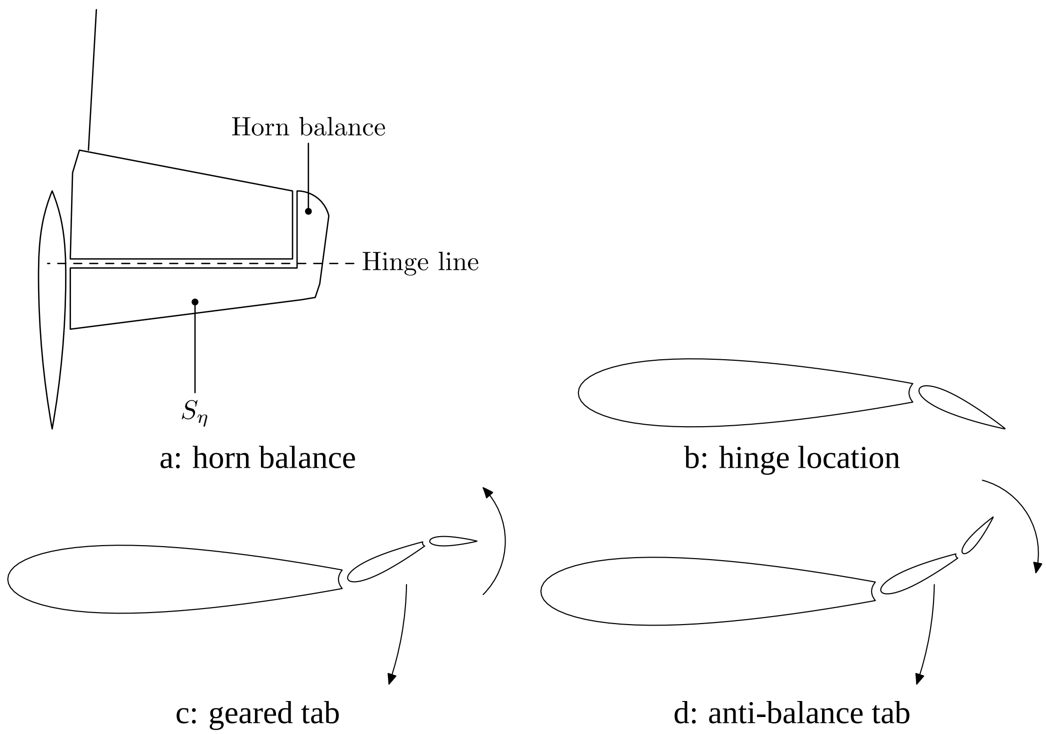


Figure 26: Aerodynamic assistance

What are we trying to do when we use aerodynamic balancing? Effectively, we are trying to reduce the control force (hinge moment) required for a given elevator deflection (i.e. we are trying to reduce $\mathrm{d}P_e/\mathrm{d}\eta$):

$$\overline{P_e} = m_e \frac{\rho V^2}{2} S_\eta c_\eta C_H,$$

but,

$$C_H = b_0 + b_1\alpha_{\mathrm{T}} + b_2\eta + b_3\beta,$$

so that

$$\frac{\mathrm{d}\overline{P_e}}{\mathrm{d}\eta} = m_e \frac{\rho V^2}{2} S_\eta c_\eta b_2.$$

Therefore, to achieve aerodynamic balance we are trying to reduce b_2. However, $\mathrm{d}P_e/\mathrm{d}\eta$ and hence b_2, must be negative for the correct feel of the controls. Therefore, reduction of b_2 enables us to reduce the control forces. This is useful at high speeds. However, at low speeds this solution may give inadequate 'feel' for the pilot. Therefore, aerodynamic assistance can also be limited.

7 Manoeuvre stability—stick fixed

7.1 Introduction

We have now completed our analysis of 'straight and level' static stability. The next step is to examine longitudinally symmetric manoeuvres (i.e. manoeuvres that affect the left and right hand side of the aircraft equally). The most straightforward example of this is a steady 'pullout' at constant velocity. Somewhat surprisingly, the elevator angle required for pitch trim in a steady banking turn can also be calculated in the same way. This is because the radius of a typical banked turn is very large. Hence, the asymmetry in the flow is small once the turn has been initiated.

7.2 Analysis of a steady pullout

Consider two conditions, shown in figure 27:

1. The aircraft is in steady, level flight at speed V.
2. The aircraft is in a steady pullout at speed V.

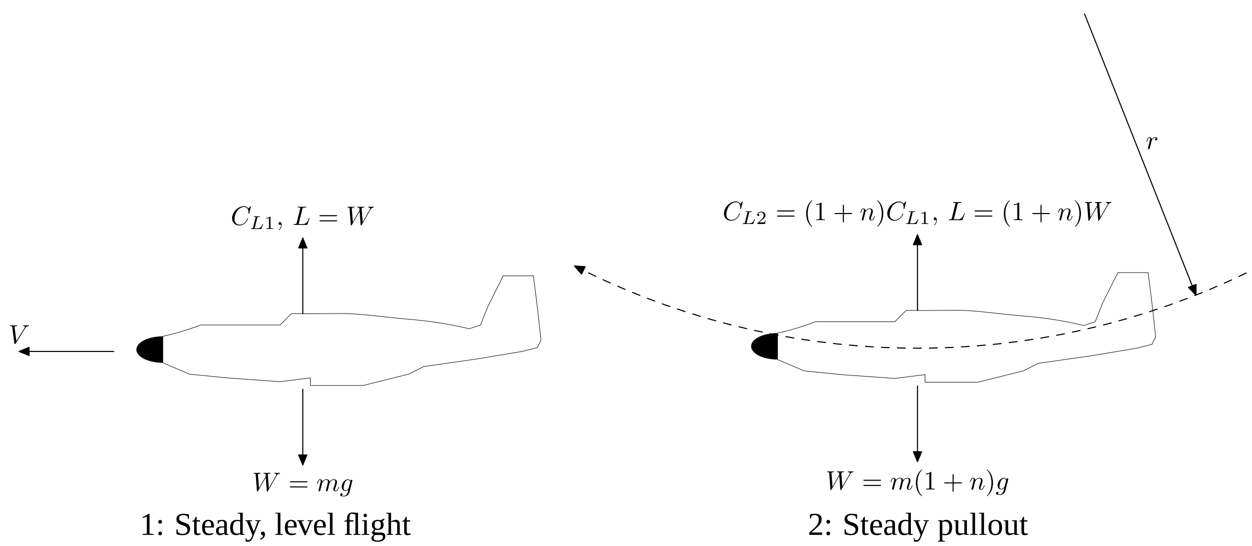


Figure 27: Manoeuvre conditions

In the steady pullout the aircraft has a radial (centripetal) acceleration $V^2/r = ng$. The difference in lift between (1) and (2) is $nmg = nW$. Hence:

$$\Delta C_L = nC_L = \frac{nW}{\rho V^2 S/2}$$

where C_L is the lift coefficient in the straight and level case.

The other key difference between the two cases is that in the steady pullout the aircraft has an angular (pitch) velocity as well as a linear velocity. This angular velocity can easily be found by considering the amount of time that the aircraft would take to complete a full circle at constant speed. If the aircraft completed a full circle it would pitch though 2π radians and would therefore cover a distance of $2\pi r$, where r is the radius of the circle. The time taken, t, at speed V would be:

$$t = \frac{2\pi r}{V}$$

but by considering the centripetal acceleration we also know that:

$$r = \frac{V^2}{ng}.$$

Hence,

$$t = \frac{2\pi V}{ng}.$$

The aircraft has pitched through a total angle of 2π radians in this time. Therefore the pitch rate, q, is:

$$q = \frac{2\pi}{2\pi V/ng} = \frac{ng}{V}.$$

But why is this pitch rate, q, important? The pitch rate will cause the tail of the aircraft to move down relative to the incoming air velocity. This causes the incidence at the tailplane to increase by an amount:

$$\Delta\alpha_{\mathrm{T}} = \frac{ql_T}{V}$$

where l_T is the tail arm *measured from the centre of gravity*. We already have an expression for q, so we get:

$$\Delta\alpha_{\mathrm{T}} = \frac{ngl_T}{V^2}.$$

Unfortunately, this expression has a V^2 term, and hence will vary with the flight conditions. We can get rid of this awkward term by applying the definition of the lift coefficient, C_L, for the straight and level case:

$$C_L = \frac{W}{\rho V^2 S/2}.$$

Hence

$$V^2 = \frac{W}{\rho S C_L/2}.$$

Substituting this back into the expression for $\Delta\alpha_{\mathrm{T}}$ results in:

$$\Delta\alpha_{\mathrm{T}} = \frac{\rho g S l_T}{W}\frac{nC_L}{2}$$

or

$$\Delta\alpha_{\mathrm{T}} = \frac{nC_L}{2\mu_1}.$$

where $\mu_1 = W/\rho g S l_T$ and is known as the *longitudinal relative density*.

The change in incidence of the tailplane causes the lift coefficient of the tailplane to alter by an amount $a_1\Delta\alpha_{\mathrm{T}}$.Therefore, remembering that the lift coefficient of the aircraft in the steady pullout is $(1+n)C_L$:

$$C_{L_T} = \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)(1+n)C_L + a_1(\eta_{\mathrm{T}} - \epsilon_0) + a_2\eta + a_3\beta + a_1\Delta\alpha_{\mathrm{T}}.$$

Hence,

$$C_{L_T} = \frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)(1+n)C_L + a_1(\eta_{\mathrm{T}} - \epsilon_0) + a_2\eta + a_3\beta + a_1\frac{nC_L}{2\mu_1}.$$

The basic pitching moment equation is still valid, since it makes no assumptions about the source of the lift and moments—it is simply the result of non-dimensionalising a free body diagram. Therefore, this revised expression for C_{L_T} can be substituted. Again, remembering that the lift coefficient in the steady pullout is $(1+n)C_L$:

$$\begin{aligned} C_M &= C_{M_0} - (h_0 - h)(1+n)C_L \\ &\quad - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)(1+n)C_L + a_1(\eta_{\mathrm{T}} - \epsilon_0) + a_2\eta + a_3\beta + a_1\frac{nC_L}{2\mu_1}\right]. \end{aligned}$$

For the straight and level flight of the aircraft, in trim, we have previously derived the equation:

$$C_M = 0 = C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + a_1(\eta_{\mathrm{T}} - \epsilon_0) + a_2\overline{\eta} + a_3\beta\right]. \quad (19)$$

In trim, the pitching moments acting on the manoeuvring aircraft will be zero if the aircraft is undertaking a steady manoeuvre. If we now look at the expression for trim in a steady pullout, and look at the change in elevator angle required for trim, such that the elevator angle is now $\overline{\eta} + \Delta\overline{\eta}$ we get:

$$\begin{aligned} 0 &= C_{M_0} - (h_0 - h)(1+n)C_L - \\ &\overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)(1+n)C_L + a_1(\eta_{\mathrm{T}} - \epsilon_0) + a_2(\eta + \overline{\eta}) + a_3\beta + a_1\frac{nC_L}{2\mu_1}\right]. \end{aligned} \quad (20)$$

Equations 19 and 20 are very similar. We can therefore perform a similar mathematical trick to the one we used to get the stick forces to trim: we subtract one equation from the other. If we subtract equation 19 from equation 20 we get:

$$0 = -(h_0 - h)nC_L - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)nC_L + a_1\frac{nC_L}{2\mu_1} + a_2\Delta\overline{\eta}\right]. \qquad (21)$$

This can be rearranged to get the elevator deflection$/g$ required for a steady pullout:

$$\frac{\Delta\overline{\eta}}{n} = -\frac{C_L}{\overline{V}a_2}\left[(h_0 - h) + \overline{V}\left\{\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{a_1}{2\mu_1}\right\}\right].$$

This must always be negative, otherwise the pilot would pitch nose-down on pulling back on the stick.

7.3 Stick fixed manoeuvre point

When $\Delta\overline{\eta}/n = 0$ the c.g. is at the *stick fixed manoeuvre point*. Hence, at $h = h_m$:

$$0 = -\frac{C_L}{\overline{V}a_2}\left[(h_0 - h_m) + \overline{V}\left\{\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{a_1}{2\mu_1}\right\}\right],$$

$$h_m = h_0 + \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{a_1}{2\mu_1}\right].$$

This should be compared with the neutral point location stick fixed, h_n, which we have previously shown to be:

$$h_n = h_0 + \overline{V}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right).$$

Therefore, the stick fixed manoeuvre point is a distance $a_1\overline{c}/2\mu_1$ aft of the stick fixed neutral point. It is worth noting that the location of the stick fixed manoeuvre point varies with altitude, since μ_1 is a function of the air density as well as of geometry.

7.4 Stick fixed manoeuvre stability

The *stick fixed manoeuvre margin*, H_m, is defined by:

$$H_m = h_m - h.$$

We showed in §7.2 that:

$$\frac{\Delta\overline{\eta}}{n} = -\frac{C_L}{\overline{V}a_2}\left[(h_0 - h) + \overline{V}\left\{\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{a_1}{2\mu_1}\right\}\right].$$

Hence,

$$h = h_0 + \frac{\overline{V} a_2}{C_L}\frac{\Delta\overline{\eta}}{n} + \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{a_1}{2\mu_1}\right]$$

Therefore,

$$H_m = -\frac{\overline{V} a_2}{C_L}\frac{\Delta\overline{\eta}}{n}.$$

This result is important because it demonstrates that there is a relationship between the stick fixed manoeuvre margin and the elevator angle to trim. There is, seemingly, a discrepancy between the fact that h_m moves with changing altitude and the above expression. How can this discrepancy be explained?

8 Manoeuvre stability—stick free and tailless aircraft

8.1 Introduction

'Stick fixed' analysis has enabled us to calculate the elevator angles required to trim the aircraft in a steady pullout/bank, but tells us nothing about the stick forces required for the manoeuvres (just as stick fixed analysis told us nothing of the stick forces for straight and level flight). 'Stick free' manoeuvre stability analysis will allow us to calculate these stick forces.

8.2 Analysis

In §6.2, we derived an expression that allowed us to calculate the hinge moments for trim in straight and level flight:

$$C_M = 0 = C_{M_0} - (h_0 - h)C_L - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)C_L + \overline{a_1}(\eta_{\mathrm{T}} - \epsilon_0) + \overline{a_3}\beta + \frac{a_2}{b_2}(\overline{C_H} - b_0)\right].$$

The same process can be undertaken to find the hinge moments for trim in a steady pullout, $\overline{C_H} + \Delta\overline{C_H}$. For the pullout, assuming that the tab is not used:

$$C_M = 0 = C_{M_0} - (h_0 - h)(1+n)C_L - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)(1+n)C_L + \overline{a_1}(\eta_{\mathrm{T}} - \epsilon_0) + \overline{a_3}\beta + \overline{a_1}\frac{nC_L}{2\mu_1} + \frac{a_2}{b_2}(\overline{C_H} + \overline{C_H} - b_0)\right].$$

where C_L is, again, the lift coefficient in straight and level flight.

Equating these two expressions and cancelling the identical terms results in:

$$0 = -(h_0 - h)nC_L - \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)nC_L + \overline{a_1}\frac{nC_L}{2\mu_1} + \frac{a_2\Delta\overline{C_H}}{b_2}\right].$$

Hence,

$$\frac{\overline{V}a_2}{b_2 C_L}\frac{\Delta\overline{C_H}}{n} = -(h_0 - h) - \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{\overline{a_1}}{2\mu_1}\right].$$

The *stick free manoeuvre point*, h'_m, is defined as the c.g. position that gives $\Delta\overline{C_H}/n = 0$.

Therefore,

$$h'_m = h_0 + \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{\overline{a_1}}{2\mu_1}\right].$$

The *stick free manoeuvre margin*, H'_m, is defined as:

$$H'_m = h'_m - h$$

which can easily be shown to be:

$$H'_m = -\frac{\overline{V}a_2}{b_2 C_L}\frac{\Delta\overline{C_H}}{n}.$$

The stick force per g is calculated from the hinge moment per g, in exactly the same way as for straight and level flight:

$$\frac{\Delta P_e}{n} = m_e \frac{\rho V^2}{2} S_\eta c_\eta \frac{\Delta\overline{C_H}}{n}.$$

For handling safety the stick force required to pull high g should be appreciable to avoid accidentally exceeding the structural limitations of the aircraft. A typical value for a non-aerobatic aircraft is usually of the order of 20 N/g.

8.3 Tailless aircraft

The analysis for tailless aircraft is very similar to that for conventional aircraft. The flight conditions, as for conventional aircraft, are shown in figure 28.

For a tailless aircraft in steady trimmed flight we have already derived the equation (§5.2):

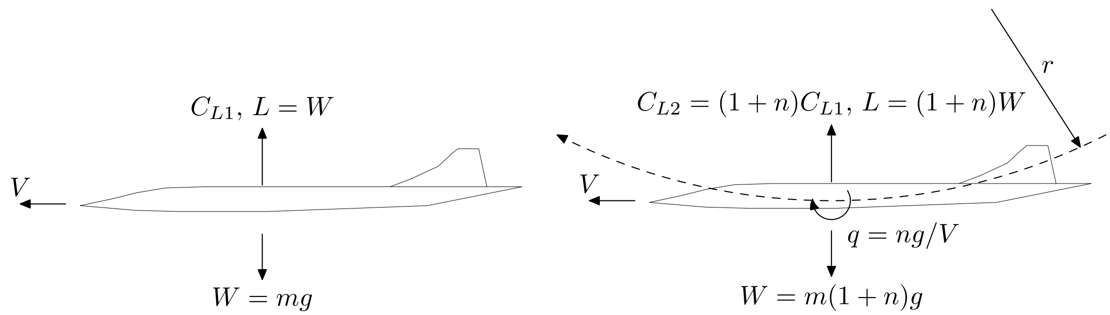
$$C_M = 0 = C_{M_0} + \frac{\partial C_{M_0}}{\partial \eta}\overline{\eta} - (h_0 - h)C_L.$$

When the aircraft is in a steady pullout with radial acceleration ng and with pitch rate q we can write:

$$C_M = 0 = C_{M_0} + \frac{\partial C_{M_0}}{\partial \eta}(\overline{\eta} + \Delta\overline{\eta}) - (h_0 - h)(1+n)C_L + \frac{\partial C_M}{\partial q}q.$$

Again, we can use the trick of subtracting one equation from the other to get:

$$\Delta\overline{\eta} = \frac{1}{\partial C_{M_0}/\partial \eta}\left[(h_0 - h)nC_L - \frac{\partial C_M}{\partial q}q\right].$$



1: Steady, level flight 2: Steady Pullout

Figure 28: Manoeuvre conditions for a tailless aircraft

$\partial C_M/\partial q$ is what is known as an aerodynamic derivative. There are a large number of aerodynamic derivatives that can be defined for any aircraft, and they enable us to calculate the aerodynamic behaviour of the aircraft. We will encounter more aerodynamic derivatives when we examine the dynamic stability of aircraft. There is a standard non-dimensionalised form for each of these parameters. The non-dimensional form of $\partial C_M/\partial q$ is given the symbol m_q, and is defined as:

$$m_q = \frac{1}{\rho V S c_0^2}\frac{\partial M}{\partial q}.$$

Hence,

$$\frac{\partial C_M}{\partial q} = \frac{1}{\rho V^2 S c_0/2}\frac{\partial M}{\partial q} = \frac{2c_0}{V} m_q$$

and we know that

$$q = \frac{ng}{V}.$$

Therefore,

$$\Delta\overline{\eta} = \frac{1}{\partial C_{M_0}/\partial\eta}\left[(h_0 - h)nC_L - \frac{2c_0}{V} m_q \frac{ng}{V}\right].$$

As for the conventional aircraft, we have an expression that includes the flight velocity. Again, we can remove this by using the definition of the lift coefficient and rearranging such that:

$$V^2 = \frac{W}{\rho S C_L/2}.$$

Making this substitution results in:

$$\Delta\overline{\eta} = \frac{1}{\partial C_{M_0}/\partial\eta}\left[(h_0 - h)nC_L - m_q n C_L \frac{\rho g c_0 S}{W}\right].$$

The longitudinal relative density, μ_1, for a tailless aircraft is defined as:

$$\mu_1 = \frac{W}{\rho g S c_0}$$

Using this definition and rearranging results in:

$$\frac{\Delta\overline{\eta}}{n} = \frac{1}{\partial C_{M_0}/\partial\eta}\left[(h_0 - h) - \frac{m_q}{\mu_1}\right] C_L.$$

This expression can be used to calculate the elevator deflections required to undertake manoeuvres.

8.4 Tailless aircraft manoeuvre point

As for the conventional aircraft, the manoeuvre point is defined by the c.g. location that results in $\Delta\overline{\eta}/n = 0$. Therefore,

$$h_m = h_0 - \frac{m_q}{\mu_1}.$$

The resulting aerodynamic forces due to a positive pitch rate are shown in figure 29.

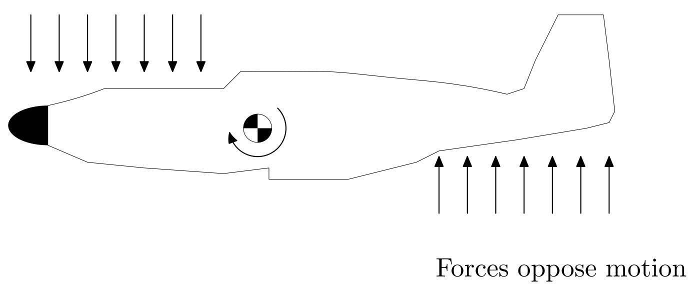


Figure 29: Aerodynamic forces during pitching motion

These forces all oppose the motion of the aircraft. Hence m_q is always negative. This means that the manoeuvre point for a tailless aircraft is always aft of the neutral point for the aircraft (which is at $h = h_0$). The damping in pitch has therefore increased the stability of the aircraft.

8.5 Tailless aircraft manoeuvre margins

The manoeuvre margin for a tailless aircraft, H_m, is defined identically to that for a conventional aircraft:

$$H_m = h_m - h.$$

Hence

$$H_m = (h_0 - h) - \frac{m_q}{\mu_1} = K_n - \frac{m_q}{\mu_1}.$$

The elevon angle per g can therefore be written as:

$$\frac{\Delta\eta}{n} = \frac{H_m C_L}{\partial C_{M_0}/\partial\eta}.$$

The elevon angle per g is therefore directly proportional to the manoeuvre margin.

8.6 Relationships between static and manoeuvre margins

Conventional aircraft

We have shown that the static margins, stick fixed and stick free, for conventional aircraft are:

$$K_n = (h_0 - h) + \overline{V}\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right)$$
$$K_n' = (h_0 - h) + \overline{V}\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right).$$

Also, the manoeuvre margins for conventional aircraft are:

$$H_m = (h_0 - h) + \overline{V}\left[\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{a_1}{2\mu_1}\right]$$
$$H_m' = (h_0 - h) + \overline{V}\left[\frac{\overline{a_1}}{a}\left(1 - \frac{\mathrm{d}\epsilon}{\mathrm{d}\alpha}\right) + \frac{\overline{a_1}}{2\mu_1}\right].$$

Therefore,

$$H_m = K_n + \frac{\overline{V}a_1}{2\mu_1}$$
$$H_m' = K_n' + \frac{\overline{V}\,\overline{a_1}}{2\mu_1}.$$

The manoeuvre points of conventional aircraft are aft of the respective neutral points. This is due to the stabilising influence of additional lift at the tailplane due to the pitch rate. Note that since μ_1 is a function of the air density the manoeuvre margin decreases with increasing altitude.

Tailless aircraft

We have shown that the static margin for tailless aircraft is:

$$K_n = h_0 - h$$

and that the manoeuvre margin is:

$$H_m = (h_0 - h) - \frac{m_q}{\mu_1}.$$

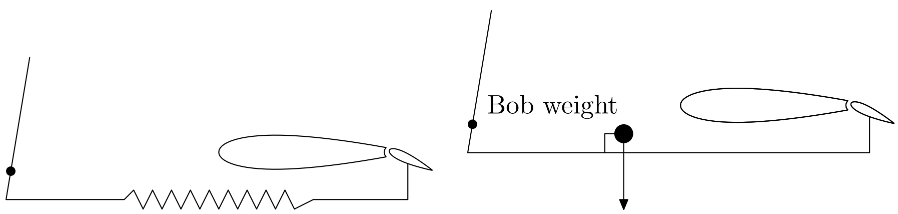
Therefore,

$$H_m = K_n - \frac{m_q}{\mu_1}.$$

As for conventional aircraft, a tailless aircraft is more stable when manoeuvring due to the stabilising effect of the pitch damping term m_q (remember, m_q is negative). Again, the manoeuvre margin is reduced at high altitudes due to the presence of a density term in μ_1.

8.7 Modification of stick free neutral and manoeuvre points

Two common ways of modifying the stick free neutral and manoeuvre points are shown in figure 30.



a: Spring *b*: Bob weight

Figure 30: Modification of neutral and manoeuvre points

A spring or bob-weight is defined as positive if it exerts a moment that would cause a positive deflection of the elevator.

These two additions have *no* effect on the stick-fixed neutral or manoeuvre points since the calculation of these locations does not require the consideration of hinge moments. However, it can be shown that a positive spring moves the stick free neutral point aft but has no effect on the stick free manoeuvre point. In contrast, a positive bob-weight results moves both the stick free neutral point and the stick free manoeuvre point of an aircraft aft. These effects are shown in figure 31.

By combining positive and negative springs and bob-weights it is possible to move the two stick free points independently of each other. Since the stick forces are directly proportional to the stick free static margin and the stick free manoeuvre margin this enables the stick forces to be modified by a simple mechanical addition to the system.

For example, an aircraft might have suitable levels of stick free static stability but insufficient manoeuvre margins. This results in a stable aircraft with good feel for the pilot and suitable stick loads for trim, but the low stick force per g resulting from the low manoeuvre margin might cause a risk of inadvertently overstressing the aircraft. The addition of a negative spring together with a positive bob-weight would solve this problem since the stick free

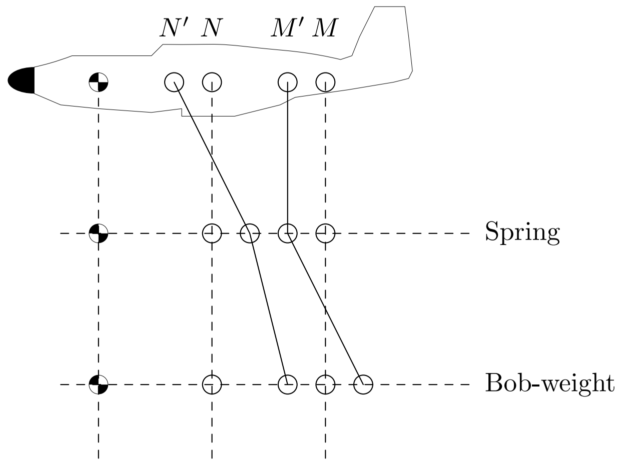


Figure 31: Effects of positive springs and bob-weights

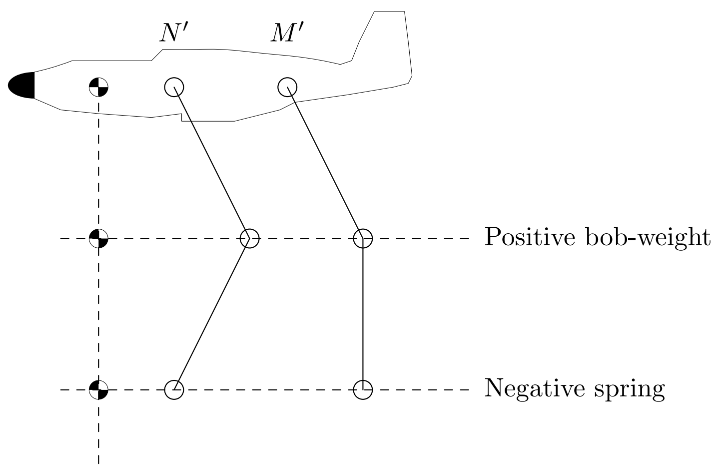


Figure 32: Modification of stick force per g

static margin would be unchanged but the stick free manoeuvre margin would increase. This is shown in figure 32.

9 Compressibility effects on static stability

9.1 Introduction

Everything that we have examined so far has assumed:

- linear aerodynamics;
- incompressible flow;
- rigid aircraft.

In reality, of course, none of these assumptions will be valid at all flight conditions. At higher angles of attack the aerodynamics become non-linear (e.g. C_L not proportional to α) and as the aircraft flies faster other effects become important. In this section we will briefly outline the major changes that occur at high speeds, and the effect that this has on the control of the aircraft.

9.2 High speed effects

Changes from the low speed case arise primarily from Mach number (compressibility) and distortion (aeroelastic) effects. The dominant effects of increasing Mach number come from:

1. change of lift curve slope with Mach number;

2. movement of aerodynamic centre rearwards, moving from quarter-chord at low speed to mid-chord at supersonic Mach numbers.

These effects are shown in figure 33.

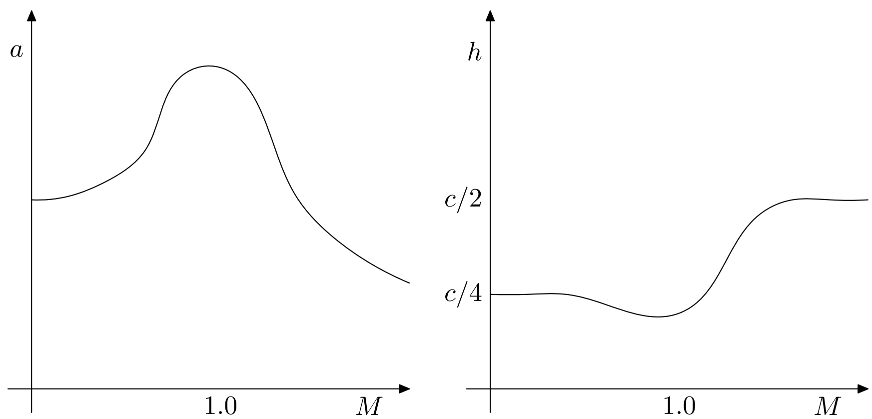


Figure 33: Compressibility effects on lift curve slope and aerodynamic centre

Combined with these are the effects of Mach number on zero-lift pitching moment and zero-lift angle.

The effects of aeroelasticity are to reduce lift curve slopes with increasing $\rho V^2/2$, dynamic pressure. The loads acting on an aircraft are proportional to the dynamic pressure, if the lift and drag coefficients are constant. The deflections are, similarly, proportional to the forces. Hence, all aeroelastic effects are dependent on the dynamic pressure. This results in changes in the aeroelastic response of the aircraft at different altitudes, since the ambient air density varies with altitude. If we superimpose altitude effects onto the variation of lift curve slope with Mach number for a typical (aft) swept wing aircraft, we get a result as in figure 35.

There are similar effects on the effectiveness of the tailplane and the elevator, as shown in figure 36.

The downwash at the tail typically varies as shown in figure 37.

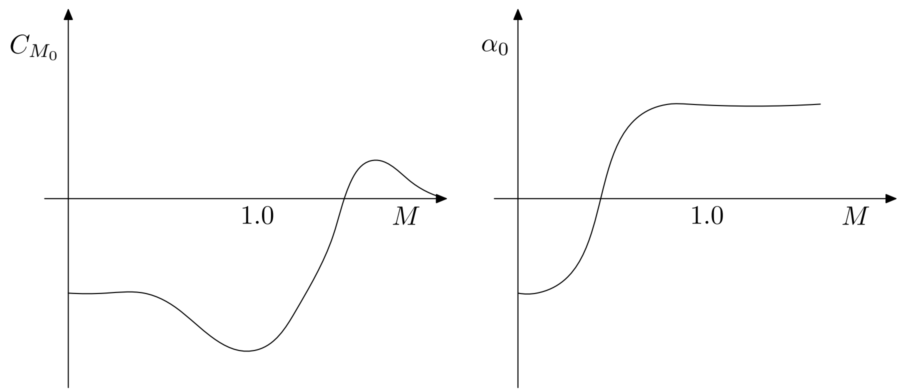


Figure 34: Compressibility effects on zero lift pitching moment and zero lift angle

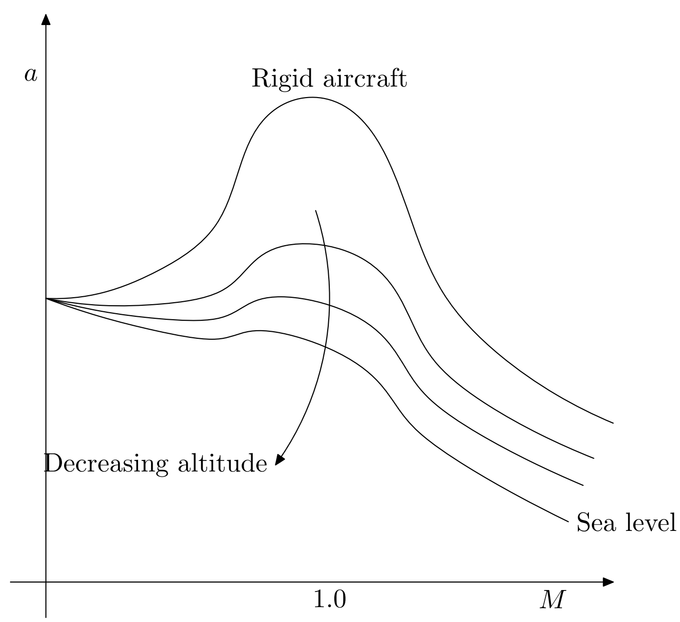


Figure 35: Aeroelastic effects on lift curve slope

It decreases to zero at high supersonic speeds, since any ‘downwash’ is confined to the volume of air influenced by the wing.

The usual result of the combined effects is that stability reduces as the Mach number nears unity and then increases, sometimes rapidly, to a higher value at supersonic speeds. The variation of C_M for a typical aircraft is shown in figure 38.

To counteract the nose down pitching moment that often occurs on swept wing aircraft (subsonic jet transports—Boeing 707, 747, etc.) an up-elevator or stabilisation input is provided by a Mach number sensing system. This is known as ‘Mach trim’. If the nose down moment were allowed to take effect the stick force gradient would be reversed, and there is also a danger that the maximum allowable speed of the aircraft due to structural limits would be exceeded. The stick forces for such an aircraft are shown in figure 39.

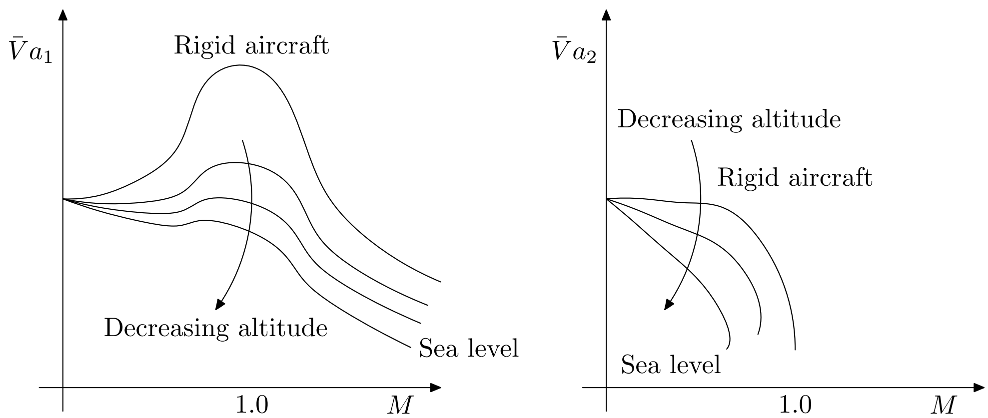


Figure 36: Aeroelastic effects on tailplane and elevator

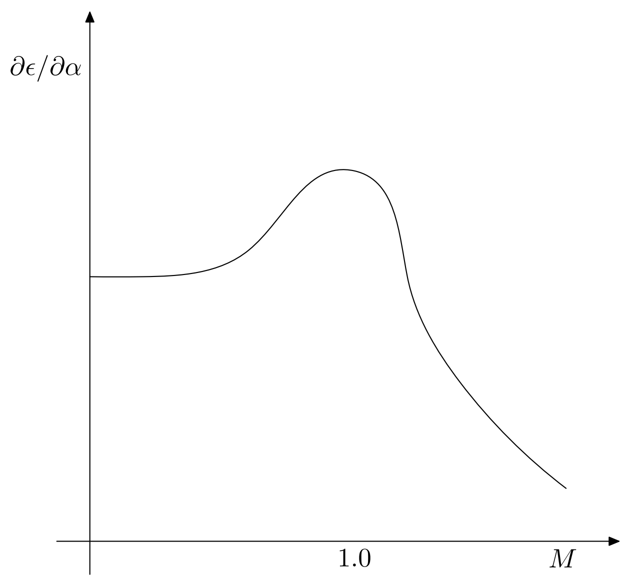


Figure 37: Variation of downwash with Mach number

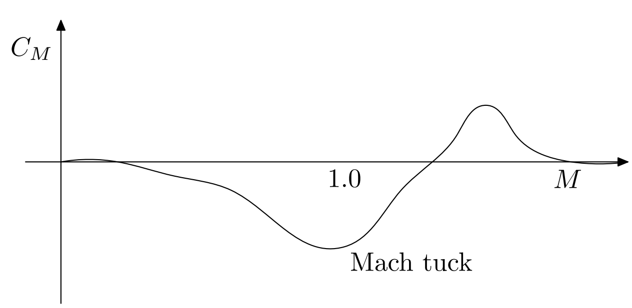


Figure 38: Variation of pitching moment with Mach number

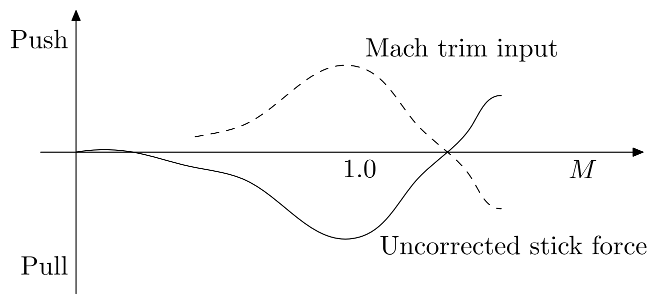


Figure 39: Variation of stick forces with Mach number