UNIVERSITY COLLEGE LONDON

A STUDY OF DYNAMIC STABILITY USING THE ROLLING BALL ANALOGY

G.W.HUNT
A.N.BRANDES

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DEPARTMENT OF CIVIL AND MUNICIPAL ENGINEERING

GOWER STREET,

LONDON, W.C.1.

CONTENTS.

SUMMARY

The dynamic stability of a generalised structure is studied using a mechanical "rolling ball" analogy, particular stress being placed on the relationships between astatic and dynamic collapse loads, represented in the analogy by the Critical Contour and Stability Boundary respectively. The effects of changes in damping are also considered. No direct relationship is established, but there is considered to be a possibility of prediction of the dynamic collapse load and mode of collapse of a structure using a behaviour pattern of the type attained in these tests.

INTRODUCTION

This paper is concerned with the effects of dynamic loading on the stability of certain types of structure and sets out to investigate this experimentally using a mechanical analogy. Dynamic stability is of interest because it results from instantaneous loading, which may give collapse of the structure at a lower load than if it had been continuously loaded. The analogy to be used is a "rolling ball" type in two degrees of freedom, which is a development of the single degree of freedom case in which the ball rolls along a rail.

Previous work in this subject has been carried out by various researchers, notably Thompson at University College, London and Budianski and Hutchinson at Harvard. Their work led to the idea of a surface of the kind used in this experiment, but it was not until 1966 that one was built at University College by Barke and Maclaren under the guidance of Dr. J. Thompson. The purpose of this subsequent experiment is to improve the surface and further the work carried out in the previous year.

THEORETICAL OUTLINE.

The purpose of this experiment was to construct a mechanical analogy to the dynamic collapse of a structure, and to attempt to predict the behaviour of such a structure using this analogy. The validity of a mechanical analogy may be explained in the following manner.

Consider a structure which is capable of a "snap" type of collapse, such as the low arch given in Fig. 1. Under increasing load, P, the structure would deflect continuously until a stage is reached at which the structure snaps through to a further equilibrium state shown as a dotted line on the diagram. This would have a static equilibrium path of the type shown in Fig. 2, plotting generalised load P to corresponding generalised deflection E. The snapping point is indicated on the diagram and occurs at the "static buckling load" Ps.

Fig. 1.

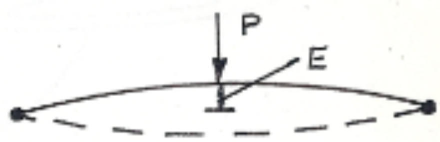
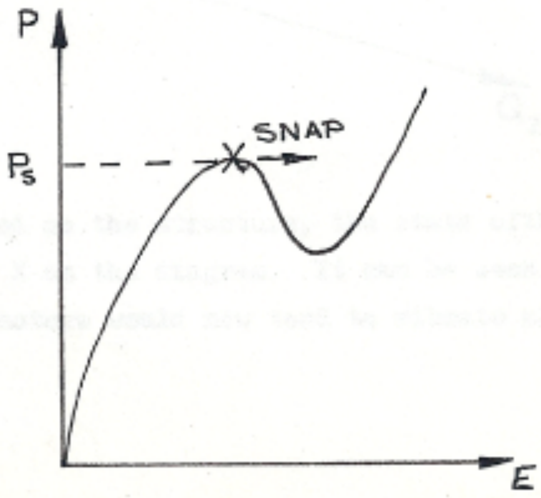
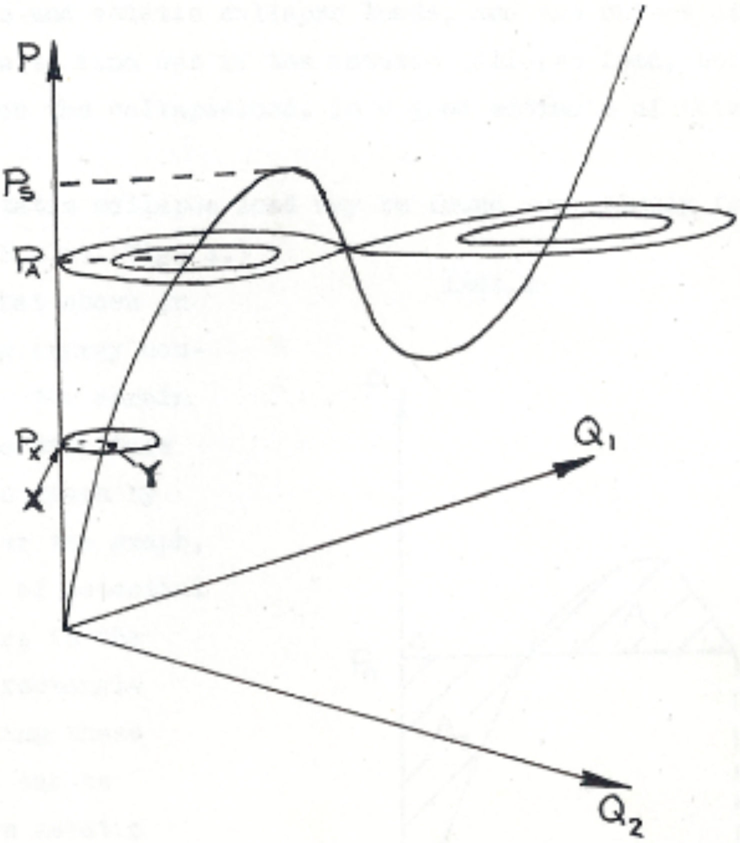


Fig. 2.



If we now give the problem two degrees of freedom and plot the load P against generalised co-ordinates Q_1, and Q_2 we obtain a graph of the type shown in Fig. 3., the static equilibrium path still containing the inherent static buckling load Ps. We must now consider the effect of dynamic loading, in which the entire load is placed on the structure before it had time to suffer any deformation.

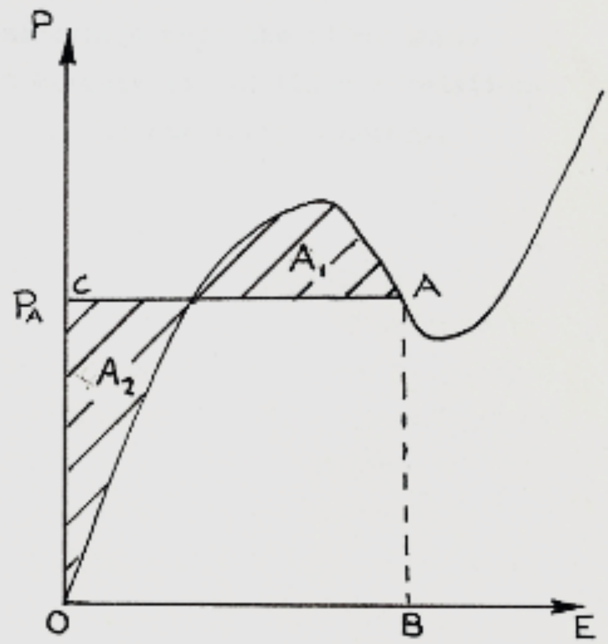
Fig. 3.



If a load Px is placed on the structure, the state ofthe system is represented by point X on the diagram. It can be seen from the diagram that the structure would now tend to vibrate about point Y, the point on the static equilibrium path corresponding to the load Px. The energy of the system may be represented by a contour line around point Y, and thus a family of energy contoursmay be built up associated with this point. Thus it can be seen that there is a load P_A at which the energy contour touching the P axis also touches the unstable portion of the static equilibrium path. This means the structure is potentially capable of collapsing and the load associated with this point is known as the astatic buckling load P_A. It can be seen that the actual collapse load is bounded by the static and astatic collapse loads, and the object of this experiment is to find out if the astatic collapse load, being a lower bound on the collapseload, is a good estimate of this load.

The astatic collapse load may be found very simply from a static P, E plot (Fig. 4.) similar to that shown in Fig. 2, using energy considerations. The strain energy of the structure at point A is given by the area under the graph, and the loss of potential of the load P_A is the area of the rectangle ABOC. Equating these two areas it can be seen that the astatic collapse load occurs when areas A_1 and A_2 are equal. This provides a quick and simple way of finding the astatic collapse load from a plot

Fig. 4.



of the static equilibrium path.

The basis of the mechanical "rolling ball" analogy lies in the existence of the energy level contours. These may be regarded as analogous to the potential energy contours of a smooth surface, and the vibration of the structure about a point on the static equilibrium path by the releasing of a ball from a point on this surface and the rolling of this ball about a minimum point of potential energy. Thus we arrive at the idea of a surface containing two points of minimum potential energy, i.e. two bowls, separated by a saddle point, associated with the unstable portion of the static equilibruim path. The contour line on the surface passing through the saddle point is termed the Critical Contour, and is associated with the astatic buckling load.

The actual buckling load is associated with a line on the surface termed the Stability Boundary, above which the ball will always pass over the ridge line and escape into the other bowl. Thus the purpose of the experiment becomes to establish a relationship between the Critical Contour and the Stability Boundary.

PREPARATION OF THE SURFACE.

Preliminary Considerations.

In order to investigate the behaviour of a particle upon a surface it is essential for the particle to move in a path dictated as nearly as possible by inertia and gravity forces alone. To achieve this the ideal solution would be to use an infinitely small mass sliding with no friction on a smooth, frictionless surface, or to use an infinitely small ball rolling with no sliding on a smooth surface. If an approximation to the latter case is used, the motion must be one of perfect rolling, otherwise energy will be lost in non-translational rotation, or sliding together with a rolling motion. Allied with this is the consideration that the surface must be free from any bumps or roughness which might cause the particle to leave the surface and land again with resulting loss of energy.

The Surface.

In the previous work on the subject, a surface was made, mounted in an open wooden box, 5 ft. x 4 ft. x 1 ft. deep, consisting of chicken wire covered with bandages soaked in plaster. This base was covered with another layer of plaster to provide strength and smoothness. The surface consists of two hollows connected by a saddle point at a level roughly half the distance between the minimum point of the two hollows and the highest point of the wall.

The final working surface was of sanded down plaster covered with a layer of emulsion paint. This surface, although possessing a general curvature in the required directions, had superimposed upon it, bumps varying in wavelength up to 2 ft. or so, and a roughness of a very short wavelength imposed on these bumps. In the previous experiments, a $1\frac{1}{2}$ in. ball bearing had been used, but owing to the short wavelength bumps, this ball lost a great deal of energy in very rapid bumping. In addition to this, it could be seen that the ball bearing was slipping relative to the surface, especially at points where the acceleration was high.

Improvements Carried Out Upon The Surface.

To improve upon the surface before the start of the experimental work, it was decided that the long and short wavelength bumps should be removed as far as possible. Since the existing surface could not be sanded down to a depth sufficient to remove these bumps, the surface was covered with a layer of "Polyfilla" which was smoothed by hand to produce a surface which filled in the long wavelength bumps leaving only short wavelength bumps of its own. "Polyfilla" was used since it stays workable for long enough to be able to adjustthe overall shape and is easier to sand down than the original plaster.

After approximately 8 successive applicationsof "Polyfilla" the surface was as good as could be obtained without a fundamental change in approach.

The "Polyfilla" surface was, however, quite unsuitable as it stood, being much too soft and easily scratched. To prepare a hard permanent surface upon which to perform the experiments, the surface was painted with two coats of white emulsion paint, followed by 3 layers of Poly-Eurethane paint, one of undercoat and two of top-coat, which sets to a hard glossy surface.

Marking Of The Surface.

To establish some sort of framework of reference for the various experiments, the surface had to have contours marked upon it. Two inch contours were used and were found using a level and a small staff with a locating pin embedded in the bottom of it. Filling of the surface with water was considered as a method of marking the contours, but was discarded because of the possible damage to the surface (its load bearing strength was not great) and the possibility of deformations giving false contours when the surface was filled with water. The contours obtained by surveying were accurate to ± 0.1 in.. As a further reference, the ridge line was also marked on the surface. The ridge line is the line passing through the saddle point of the surface which follows the direction of the greatest slope.

EXPERIMENTAL WORK

Choice of Ball.

Once the surface had been finished the problem of which ball to use was looked into once more. The hard shiny surface proved to be even more unsuitable for the ball bearing than the previous surface. The rolling was much smoother than before, especially at small amplitudes, but the slipping at high acceleration was very much more pronounced. Solid Carbon Di-oxide was tried, but this again performed adequately at small amplitudes and low velocities only. On larger oscillations the CO_2 did not sublime rapidly enough to carry the weight of the particle continuously. Next, two rubber balls were tried, a solid subber "Superball" with an extremely high coefficient of restitution and a larger hollow rubber ball. The latter, although possessing smoother rolling characteristics, lost more energy than the "Superball" which was finally used despite its tendency to start bouncing on hitting small imperfections. The bouncing was attributed to the high coefficient of restitution which does not allow small vertical oscillations to decay quickly as in the case of the larger hollow ball.

The Stability Boundary and Zones of Indeterminacy.

A record of the Stability Boundary on one of the hollows had been found from the previous work on the surface, and a first step in the experimental work was to plot the new Stability Boundary for the whole surface. The Stability Boundary and modes of escape were found by trial and error, releasing the ball from various points until one was found at which the ball just failed to escape into the other hollow or just succeeded. This process was repeated until the locus of points representing a boundary had been obtained. Boundaries for each different mode of escape were plotted in the same manner.

With release from certain areas of the surface the motion of the ball was found to be irregular, the ball not necessarily following the same mode of escape with each release. It was decided that these areas should be defined and they were painted grey on the surface and termed Zones of Indeterminacy.

Fixed Paths.

In each hollow of the surface there existed a series of points from which the ball when released performed a complete oscillation about the minimum point before coming to rest momentarily at its point of release, assuming the absence of damping. The loci of these points were termed Fixed Paths, and they were plotted for each surface using a trial and error technique similar to that described above.

Damping.

In order to discover the effect of damping upon the Stability Boundary, a very soft rubber squash ball was used to replot the boundary on one side of the surface. This type of ball has a very low coefficient of restitution and permits very smooth rolling with high damping because any inconsistencies in the surface are absorbed by deformations of the ball.

Filming.

Before an analysis of the motion ofthe balls and their different dampings could be undertaken, it was essential to film the motion of the ball. In the previous work, the filming was done from a tripod to one side of the surface. This gave an oblique view of the surface and consequently distorted a plan of the paths of the balls. In order to get a true picture from above the surface, a DEXION frame was designed and constructed to support the camera approximately 15 feet above the surface, and once the frame had been stiffened to prevent inherent vibrations in the movie camera the mounting provedto be perfectly adequate. (Photo 2.) The various different modes of release and the damping of the different balls were filmed and from the film, the motion of the balls on particular releases was recorded using an analytical projector which allowed the film to be shown frame by frame so that the path of the ball could be plotted on a screen in front of the projector.

A 5 minute demonstration film illustrating the scope of the model analogy was made and may be found with the department of Civil Engineering.

PRESENTATION OF RESULTS AND EXPLANATION OF DIAGRAMS.

The Stability Boundary and Fixed Paths.

The Stability Boundary and Fixed Paths which were found can be seen on diagram (a) showing the boundariesfor the whole surface plotted with the "Superball". Each separate area behind the Stability Boundary corresponds to a different mode of escape from one side of the surface. Diagrams b, c, d, e and f show some of the more significant releases and the two bifurcating paths which were investigated. Diagram "g" shows the zones of Indeterminacy hatched which are also visible in photograph 1. The new Stability Boundary found for a highly damped squash ball is shown for one side of the surface only on diagram "h".

The results obtained from an analysis of the film gave a plan view of the distance travelled horizontally by each ball on each oscillation. From this the graphs 1 and 2 were plotted showing the height above the minimum point of the surface against number of oscillations. This was in fact a time base since the period of oscillation of all the balls were practically constant. The graphs are drawn for the "Superball", the $1\frac{1}{2}$ inch ball bearing and the squash ball for small oscillations; and for the "Superball" and $1\frac{1}{2}$ inch ball bearing for large oscillations. The height of the ball was found by interpolation from the known height of the contours. Damping factors "ν", of 4.25×10^{-3} and 62.3×10^{-3} were found for the "Superball" and the squash ball respectively by consideration of the horizontal logarithmic decrement assuming a simple damped vibration theory.

DISCUSSION OF RESULTS.

Quality of the Finished Surface.

While the finished surface was a considerable improvement upon the original model, it still proved very difficult to produce a completely smooth rolling motion of the ball on the surface. This was due partly to the difficulty of producing a perfectly smooth surface and partly to the disadvantages of the ball chosen to perform the experiments. The roughness of the surface was almost completely eliminated but it proved extremely difficult to iron out the long wave length bumps since the "Polyfilla" covering tended to assume exactly the shape of the surface beneath it, and sanding merely smoothed the roughness while retaining the long wave inconsistencies. The effect of the bouncing of the "Superball" is difficult to assess, on simple releases it was usually possible to get a very smooth rolling motion, but on the longer releases the bouncing motion mentioned earlier was usually present, dying away once the energy and velocity of the ball had lessened. The bouncing appeared to start spontaneously but seemed to be less pronounced when the surface had been thoroughly wiped clear of any dust and dirt, the orientation of the ball at release also affected the tendency towards bouncing, and it is probable thatthe ball was not perfectly spherical.

Plotting of the Boundaries.

At the conclusion of the previous work it was suggested that the method of releasing the ball should be improved upon. An electro-magnetic device was suggested but this was of course impossible due to the choice of a different ball. Besides this, it was found that the method of release, providing it is not too clumsy, is relatively unimportant in affecting the overall pattern of the Stability Boundary. The main difficulty experienced in the plotting lay in determining the exact point of release of the ball. As a preliminary measure the point of contact of the ball with the surface was marked since it was easier to locate than any other point. It is easily seen that this is not the most meaningful point which might be recorded, because for the ball to escape its lowest point it must attain the level of the Critical Contour. Thus for such a release the point of contact would in fact lie above the Critical Contour. Once the boundaries had been established by plotting the points of contact, they were adjusted by marking the level of the lowest point of the ball in significant places along the boundaries and drawing in the "true" boundaries.

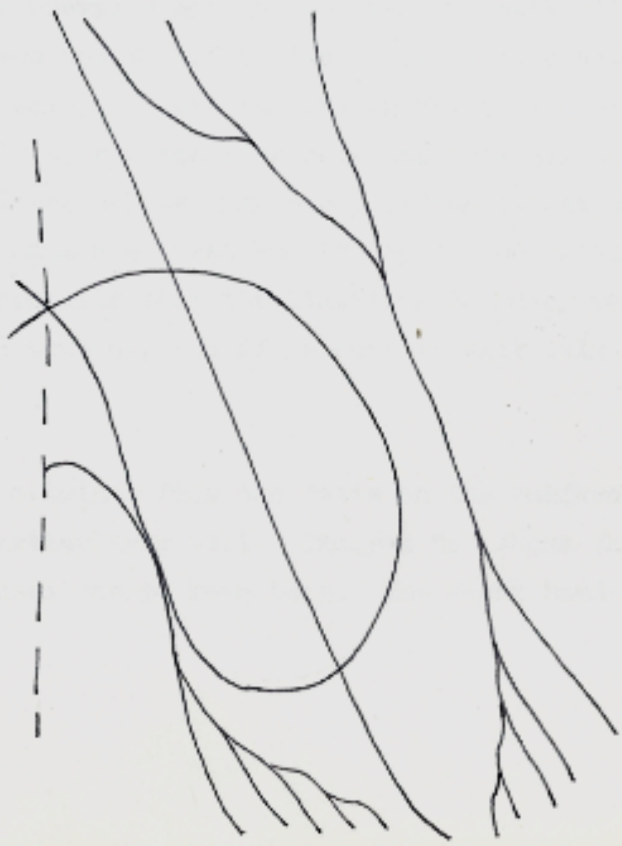
The Boundaries.

It is immediately apparent from the diagram of the boundaries, that there is a marked difference between the behaviour of the ball in the two hollows of the surface. While only five different releases were found for the Right Hand side of the surface, nearer twenty were found to be clearly defined upon the left hand side, apart from those which were thought not consistent enough to be marked down as releases and contained in the Zones of Indeterminacy. This is due to the steeper sides and more elongated shape of the Right Hand side which tend to make the ball follow the line of the "valley" and to pick up an oscillatory motion near to the fixed path. The Left Hand side has gentler slopes with no well-defined "valley" shape, and it can be seen how the Stability Boundary comes very close to the Fixed Path. If the walls of the Right Hand side could be made higher, it is very probable that similar higher releases would be found and the Fixed Path and the Stability Boundary tend to meet.

Several of the releases which were found from the Left Hand side of the surface are curious and it is hard to decide whether they are the result of an imperfect system or an inherent feature of the surface. On diagram "d" can be seen the different releases from Zones A. B. C. where similar releases from A and C are interrupted by an area of different release from B, even though the minimum energy required for escape increases very smoothly along the Stability Boundary. Area B was one of the few entirely enclosed areas found on the surface.

It was suggested in the previous work that higher releases on the Right Hand side of the surface might be represented by successive scollops coming out of each preceding release area as shown in Fig. 5.

Fig. 5.



This idea which looks likely from the Right Hand side of the surface is not borne out by the Left Hand side where the overall pattern becomes very complicated. It does not seem likely that a simpler pattern would be found with a system entirely free from damping.

A certain relationship between the Critical Contour and the Stability Boundary may be established ~~ascertain~~ in the following hypothesis. Consider a surface of the type used in the experiment but free from effectsof damping, and releases from a small area containing the saddle point. In this area the motion of the ball may be linearised and as the size of this area tends to zero, a particular spatial path of the ball is approached which relates to release from the saddle point. A ball released along this path must continue in motion for an infinite length of time, and may at some time come instantaneously to rest. In this case the ball will stop somewhere on the Critical Contour, and the further path of the ball will be the reverse of the previous motion, meaning the ball will reach the saddle point. There are therefore two cases to be considered, one in which the ball comes to rest and one in which it continues in motion forever. If we now consider releases from all points on the Critical Contour there can be only one position corresponding to this point of instantaneous rest and it may not be present at all. Therefore we may conclude that the Stability Boundary may or may not touch the Critical Contour, and if it does it will only touch in one place.

The results obtained from the tests on the surface demonstrate this phenomenon particularly well. Diagram "a" shows the surface in detail and both cases can be seen here. The Right Hand side shows the Stability Boundary and Critical Contour touching in one place whereas they definitely do not touch on the Left Hand side at any point, and presumably would never do so even with negligible damping, since the ball did not cross the ridge path at the saddle point for any release position.

Damping.

The curvesof damping show clearly how the steel ball bearing loses energy very rapidly at the start of its large oscillations, but settles down to a steady decay once the slipping has stopped.

The effect of damping upon the Stability Boundary can be seen on diagram "h" shown only for the Right Hand side of the surface. To achieve a maximum discrepancy between the lines, the boundaries here have been plotted using the least and most damped balls, the "Superball" and the squash ball. Since the boundary investigated is one associated with a very simple release the difference is of course small. Many of the higher releases woulddisappear completely if the squash ball were used for the complete surface. This suggests that a structure, which would presumably be highly damped, when subjected to normal loading, would probably have only simple modes of buckling open to it.

The damping factor "υ" found for the squash ball was approximately 62.3×10^{-3}, and about 16 times the value calculated for the "Superball". Although the order of damping was very different in these cases, there was not a correspondingly large retreat of the Stability Boundary, and it may be assumed that the effect of structural damping is not of primary importance when simple modes of buckling are considered.

Validity of the Analogy.

For any one particular release the rolling ball analogy is direct, but since the experimental surface is fixed, while the energy contours of the structure change with increasing load, the analogy is not direct for variations in load. However, since the energy contours vary in a continuous manner as the load is increased, the analogy may be considered meaningful, especially in areas in which the Stability Boundary and Critical Contour lie close together. It should also be noted that releases from all points on the surface were considered, whereas for a particular structure, since the P axis is a line in space, there will be only one release for each value of load, and thus for each contour on the surface. It can therefore be seen that all values of loading on a particular structure are represented by a line on the surface passing from the minimum point and increasing continuously in height. The surface therefore does not relate to only one structure and may be associated with a number of particular structures.

CONCLUSIONS

It would seem from the tests carried out on the surface that, although the Critical Contour is a lower bound on the Stability Boundary it cannot be regarded as a good estimation of the Stability Boundary for all positions of release. It is possible, however, to draw some conclusions from the experiment.

If the static equilibrium path of loading shown in Fig. 3 of the Theoretical Outline approximates to a straight line in plan view then the point of release, the minimum point and the saddle point would be approximately a straight line in plan. Therefore for a simple series of contours of the type considered the releases behind the Stability Boundary are most likely to be in the zone corresponding to first release, Zone I or Zone II shown on diagram "a". It would therefore be possible to predict the chance of a first mode of collapse from a static equilibrium plot in two degrees of freedom. However, it can be seen from the surface that the areas in which the Stability Boundary and Critical Contour are closest are not predicted by drawing a straight line in plan from the saddle point through the minimum, as perhaps might have been presupposed.

The area of the surface around the fixed paths was very definitely that associated with the widest separation of the Critical Contour and Stability Boundary. This, then, is presumably the most safe area for dynamic loading, but prediction of behaviour would be very difficult in this region. Fixed paths were found in both bowls, but an intensive theoretical study of rolling balls on general surfaces would have to be carried out before the presence of fixed paths could be assumed or predicted for any surface.

Finally, a mention should be made on the scale aspect of the model. The distance between the Critical Contour and Stability Boundary is only meaningful if it is non-dimensionalised with a typical length in the surface. The height of the Critical Contour above the minimum point was first considered for this length but this was rejected because it did not seem valid to assume that changes in this height would result in marked differences in behaviour of the boundaries. The least distance between the minimum point and the Critical Contour was also considered and although this would be a better length to take, it is not entirely acceptable, and this problem was not satisfactorilly solved.

ACKNOWLEDGEMENTS

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G. W. Hunt

Nicholas Brandes

University College, London.

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Diagram (a)

The Surface

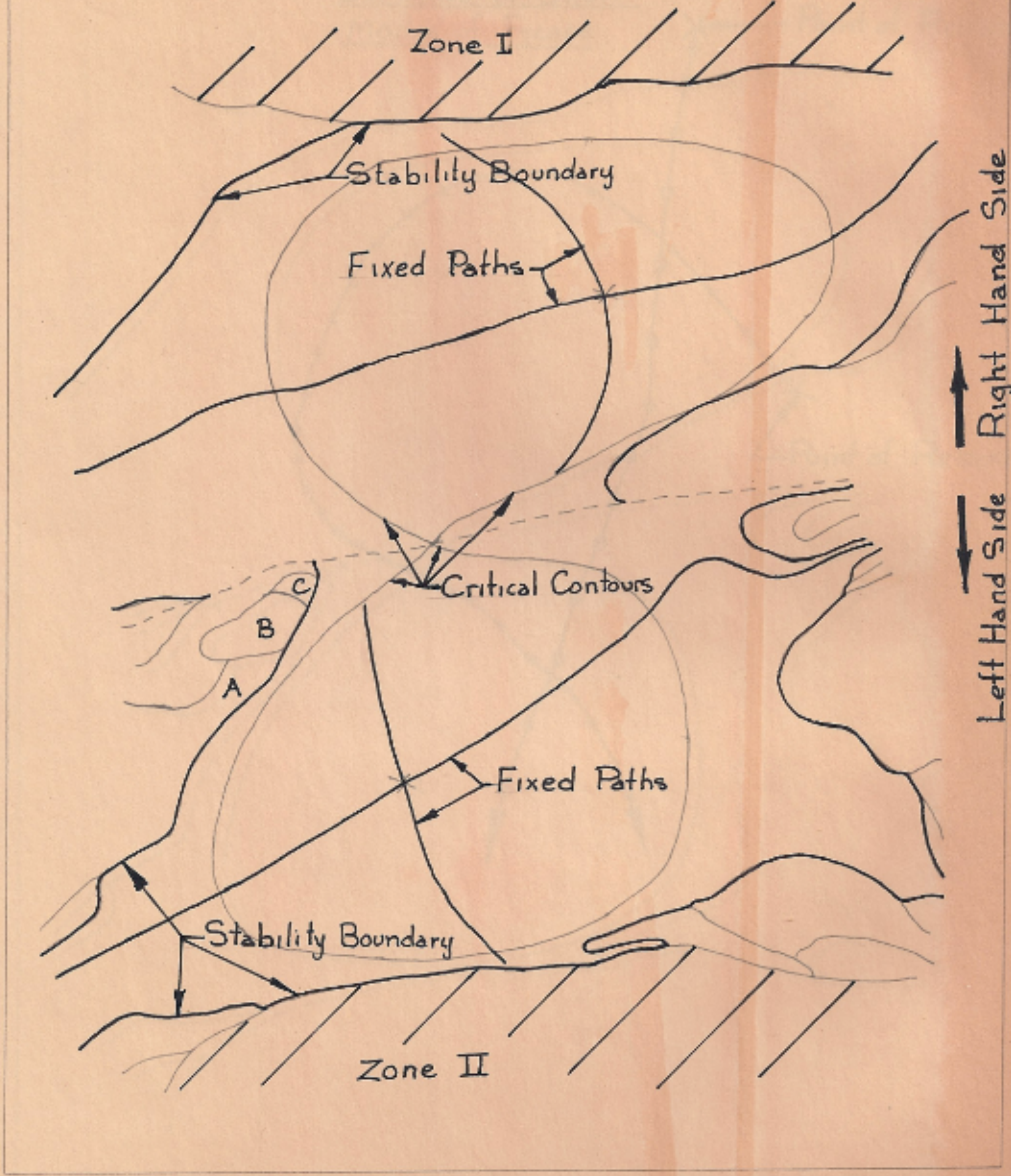


Diagram (b)

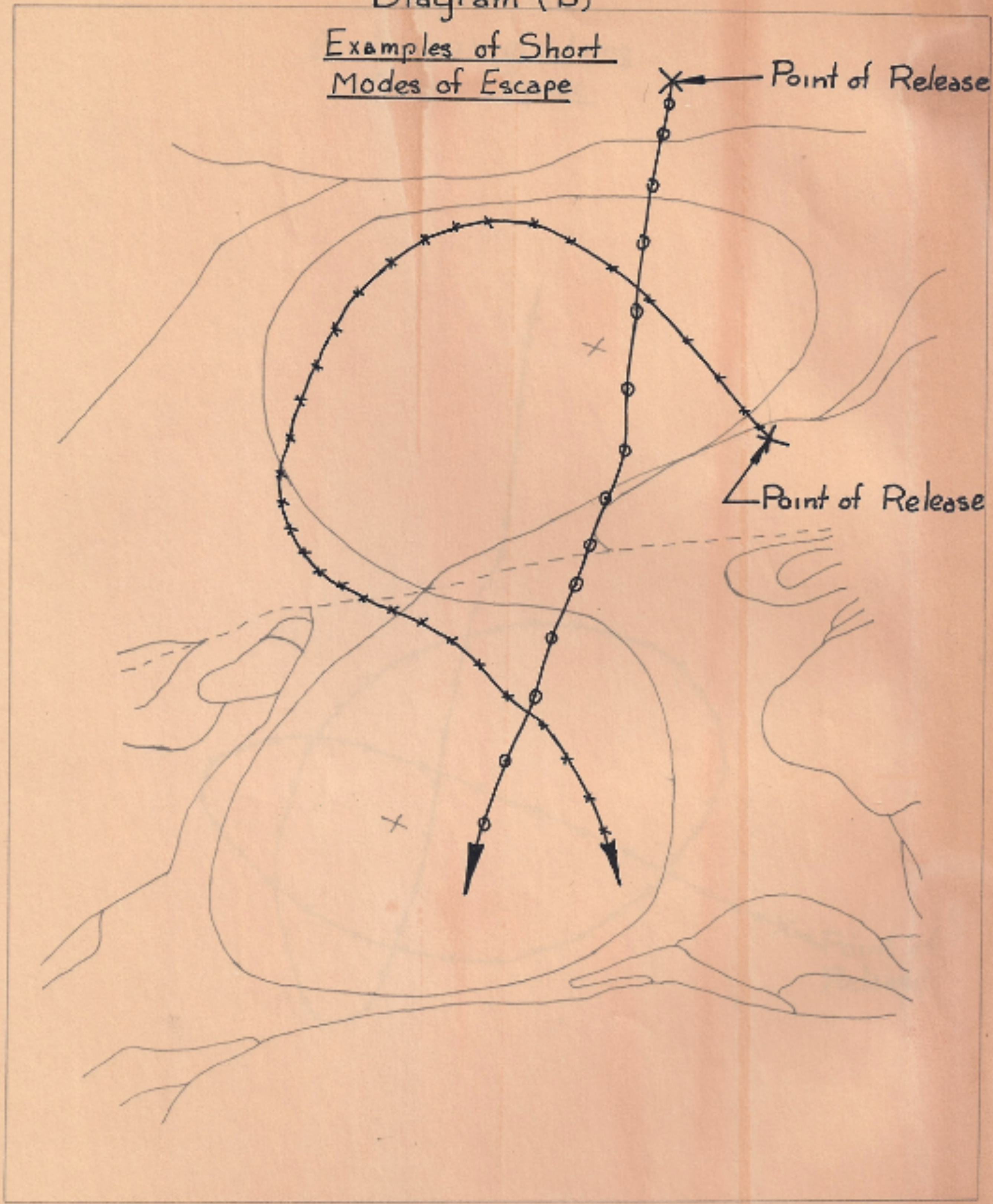

Examples of Short
Modes of Escape
Point of Release
Point of Release

Diagram (c)

An Example of a Long Mode of Escape

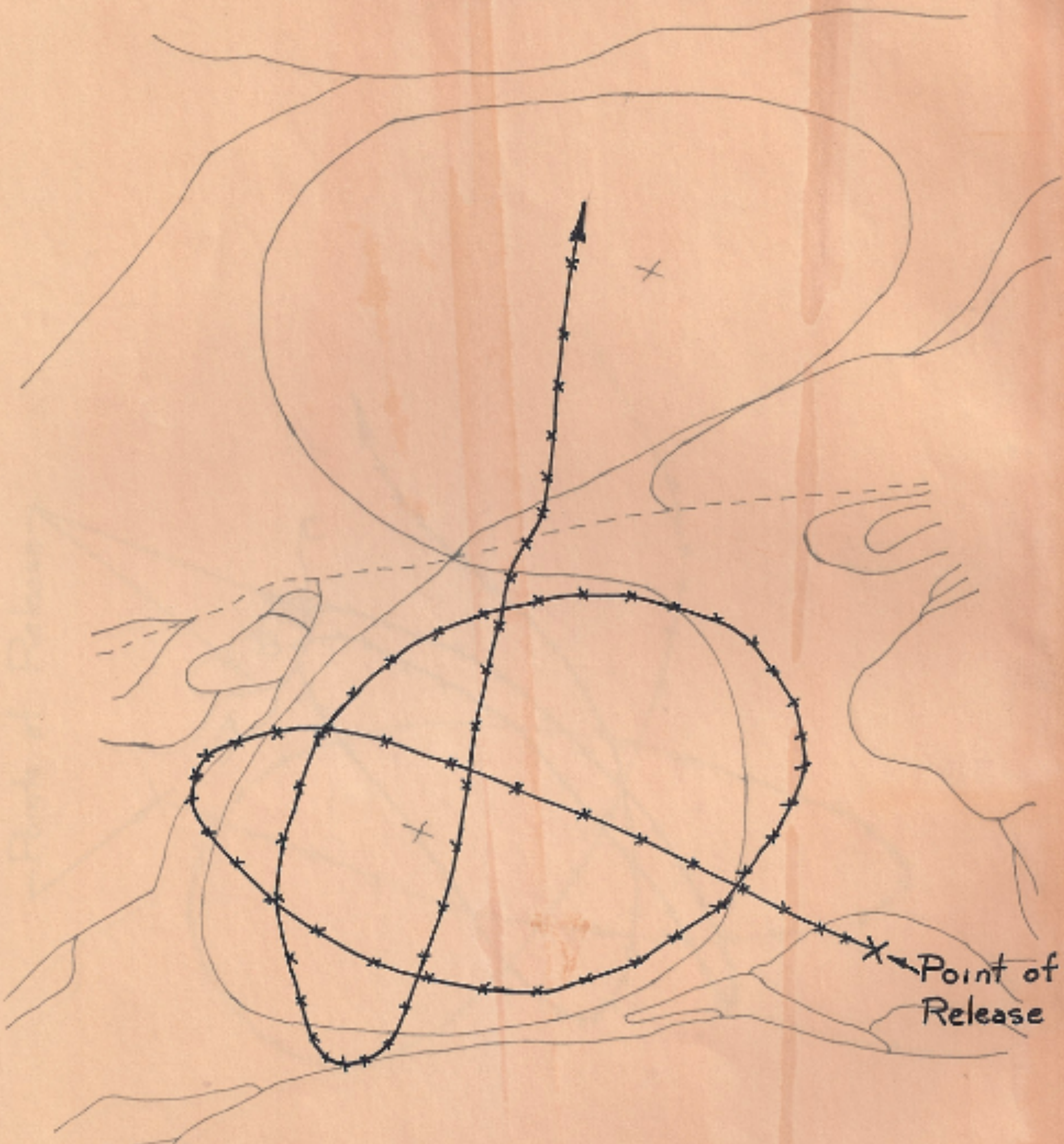


Diagram (d)

Release from Zones A,B,C

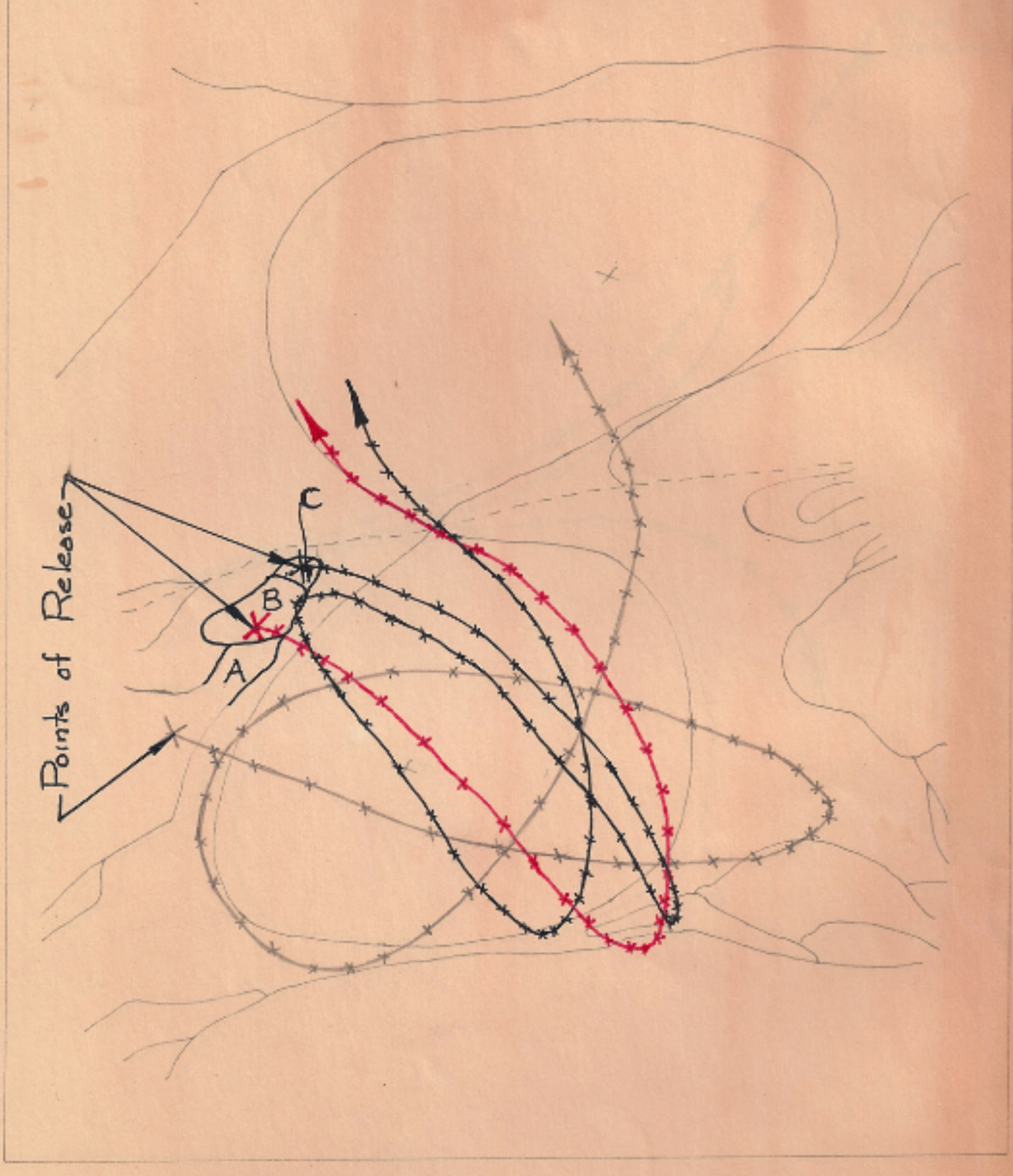


Diagram (e)

Bifurcating Path I

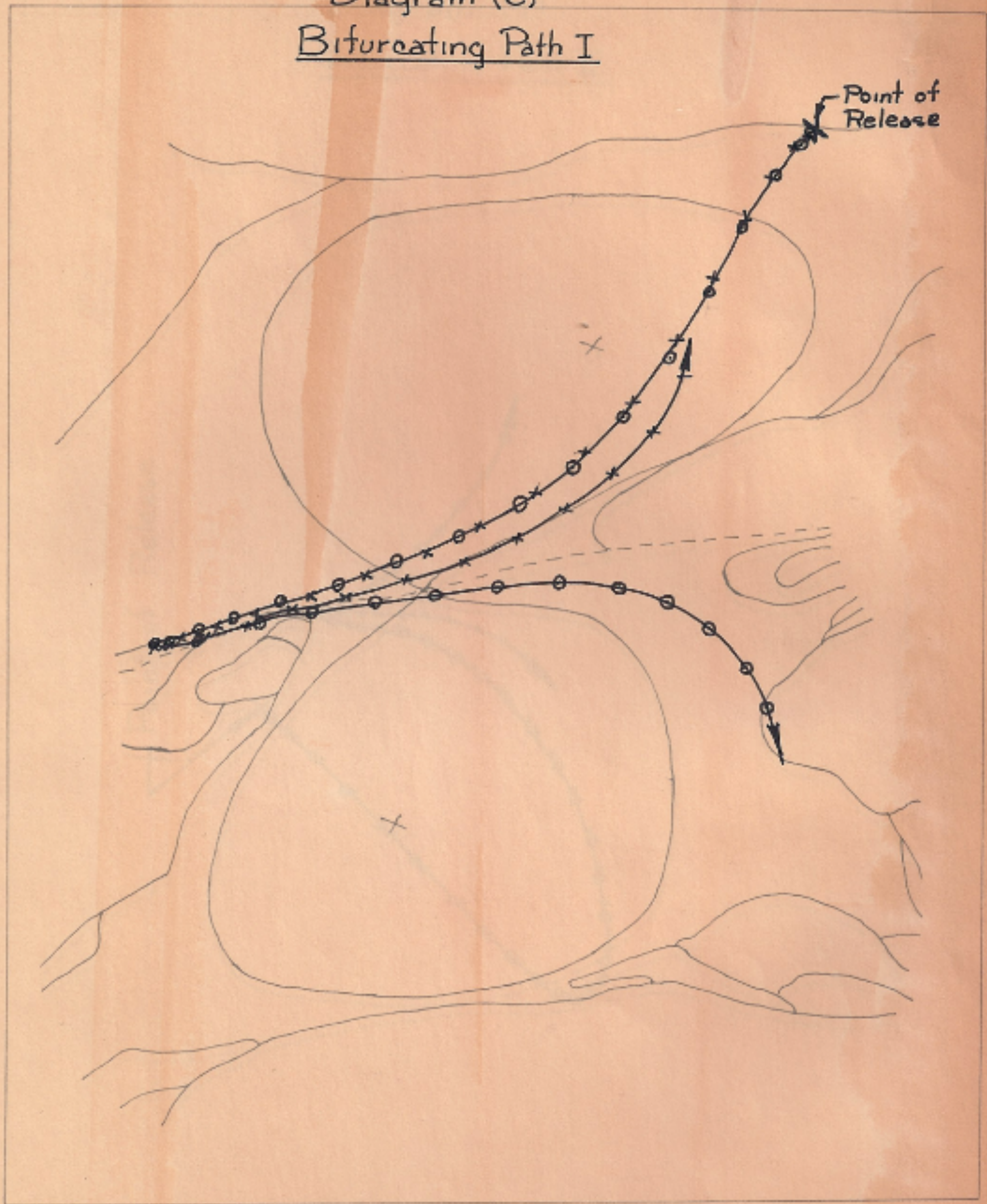


Diagram (f)

Bifurcating Path II

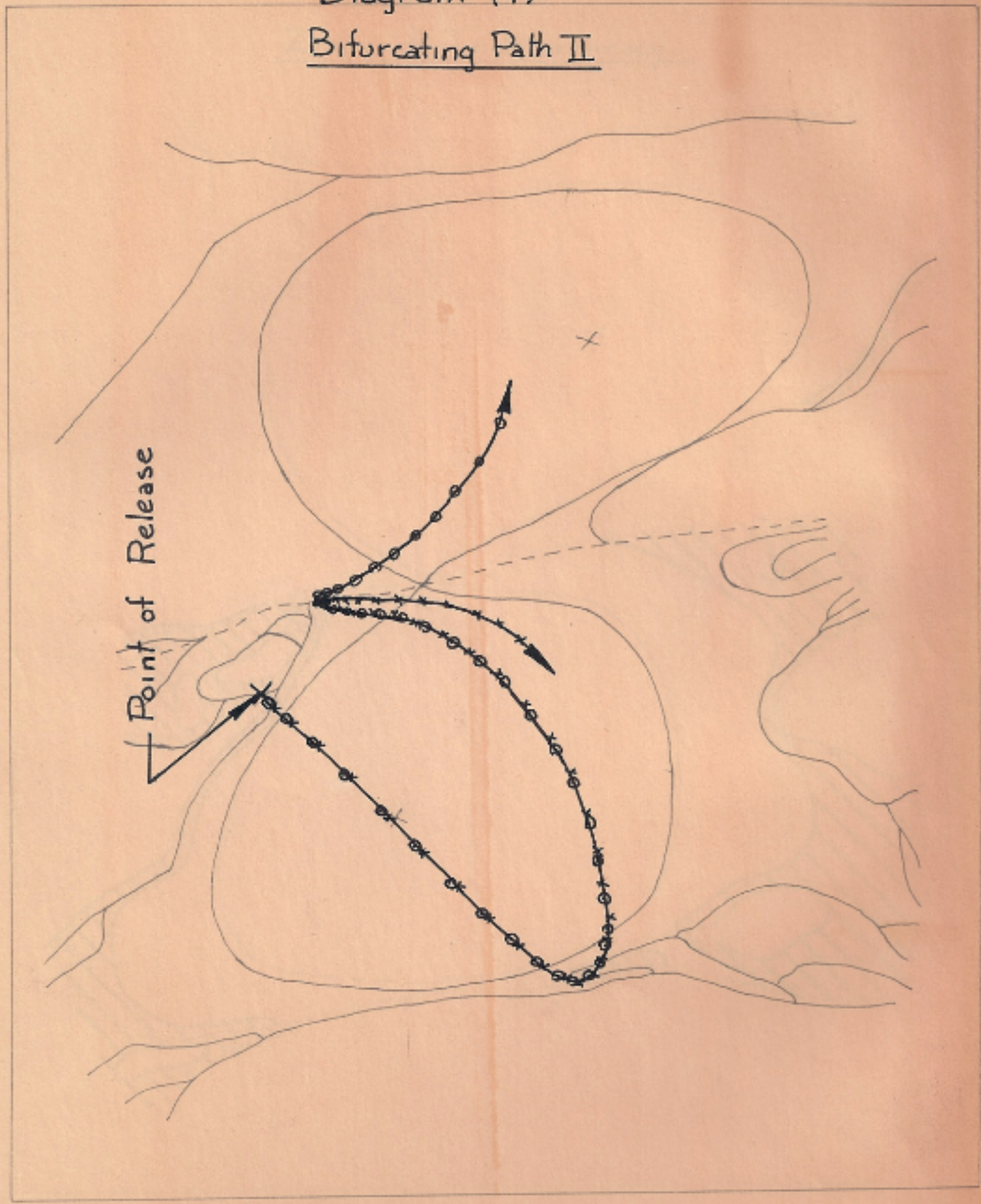


Diagram (g)

Zones of Indeterminacy

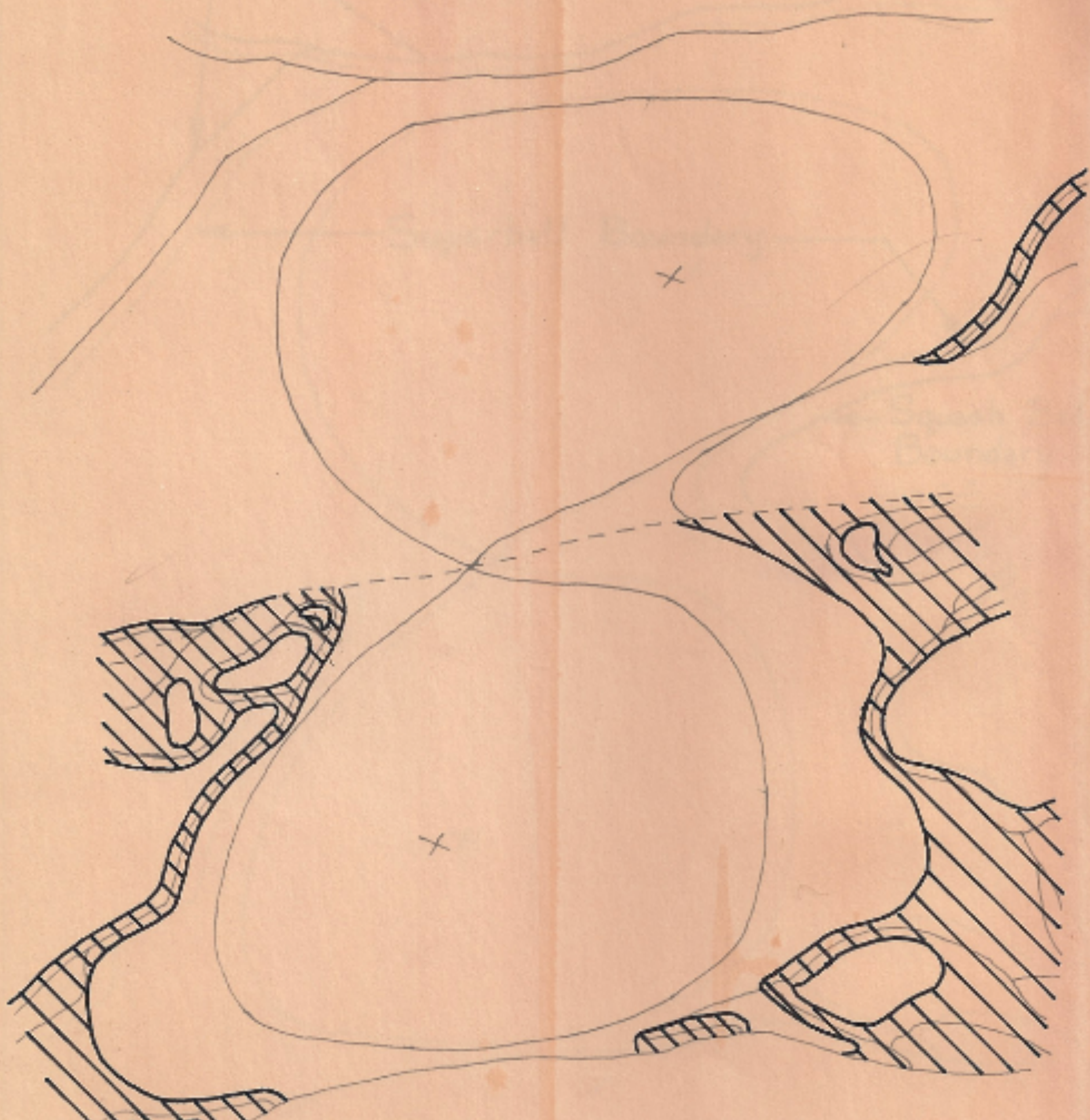
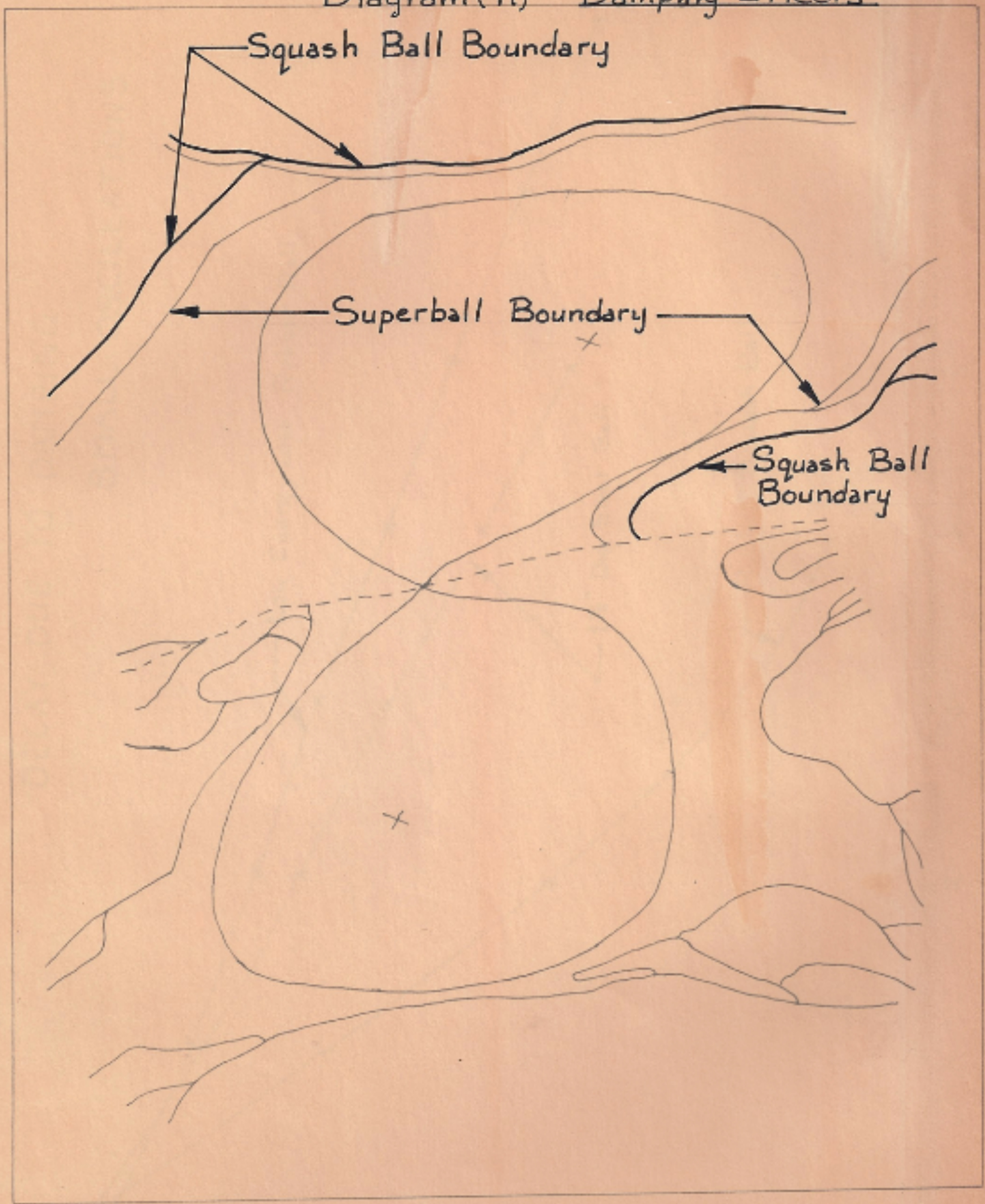
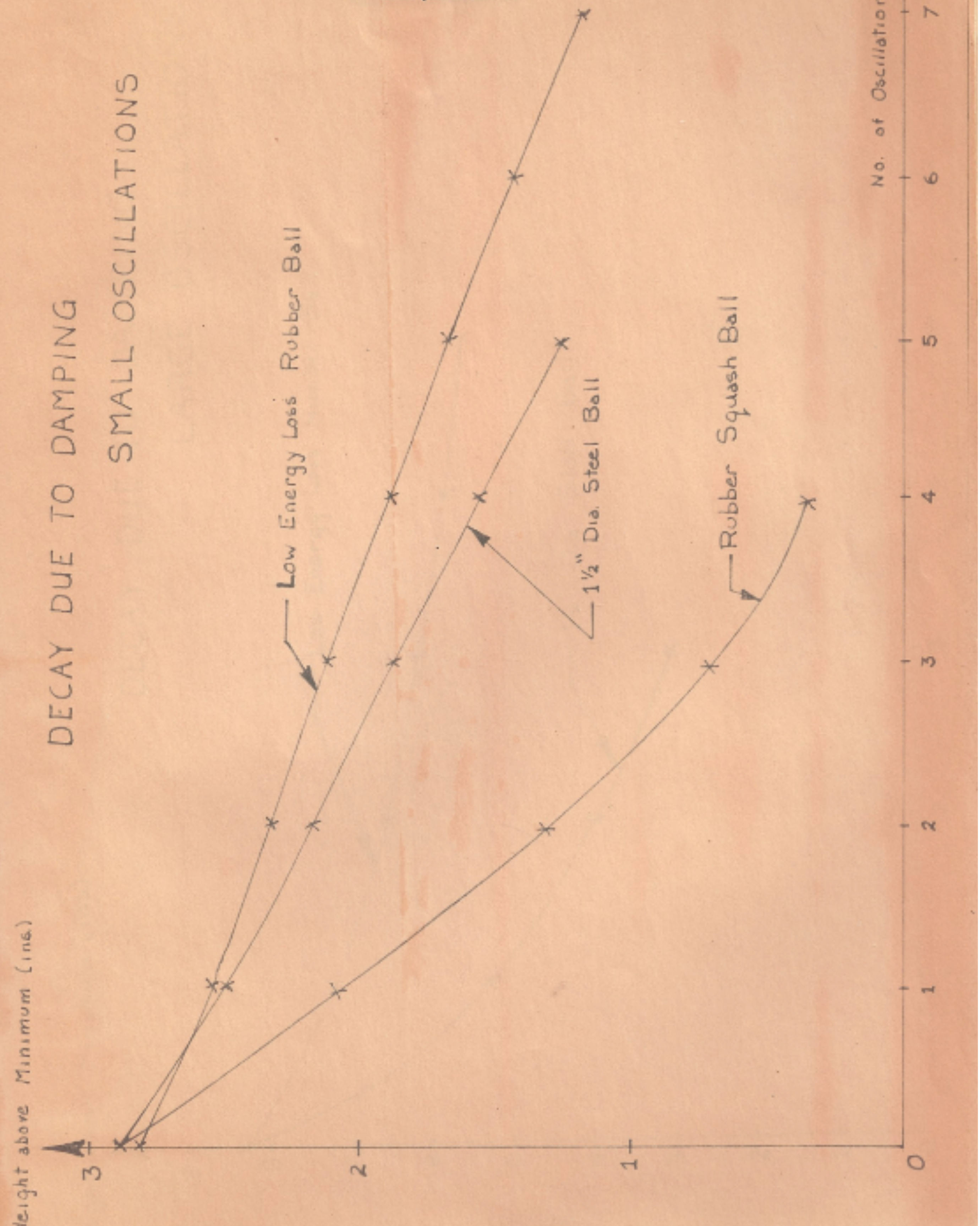


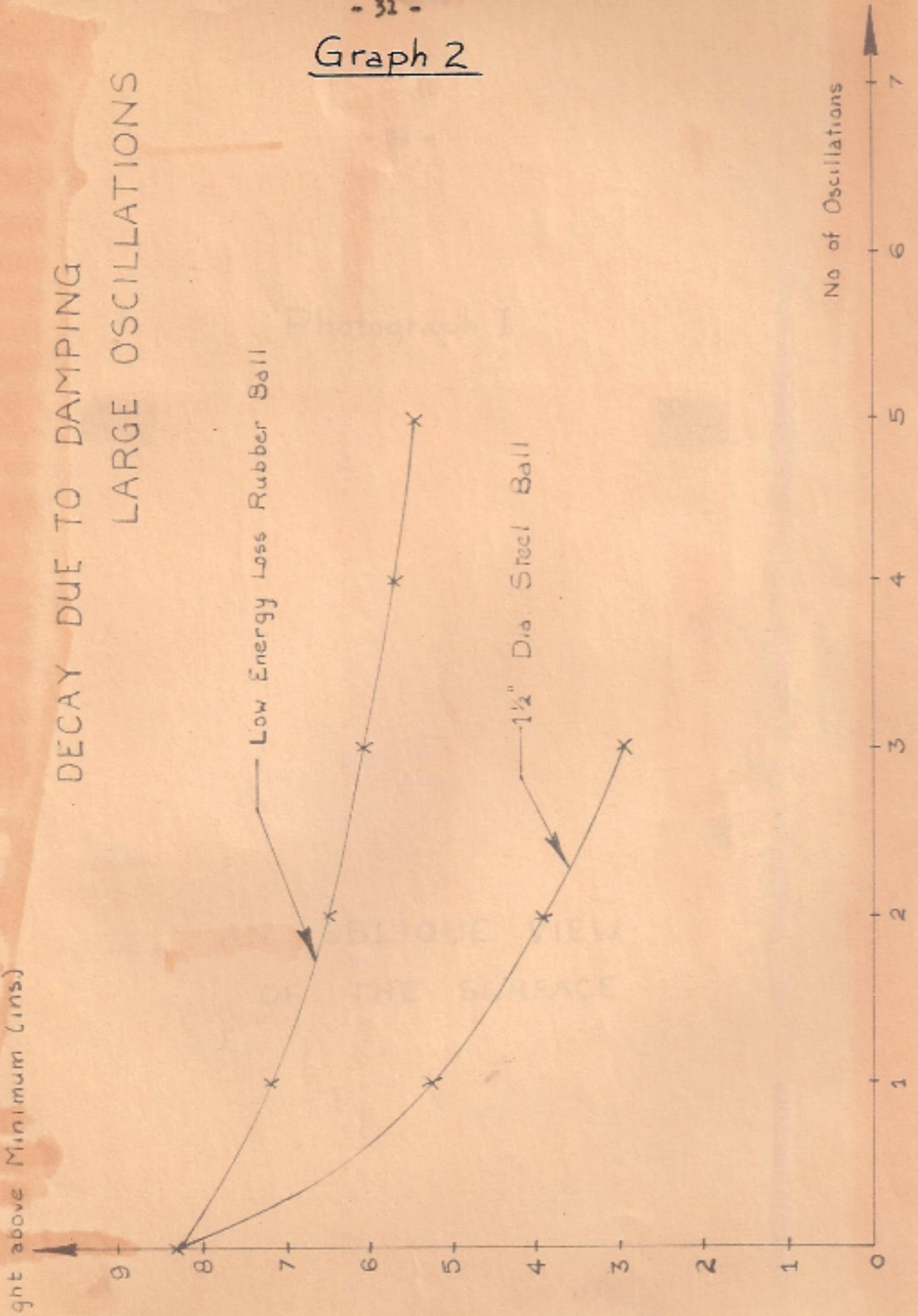
Diagram (h) Damping Effects



Graph 1


DECAY DUE TO DAMPING
SMALL OSCILLATIONS
Low Energy Loss Rubber Ball
1½" Dia. Steel Ball
Rubber Squash Ball
Height above Minimum (ins.)
No. of Oscillations
0
1
2
3
4
5
6
7

Graph 2


DECAY DUE TO DAMPING
LARGE OSCILLATIONS
Low Energy Loss Rubber Ball
1½" Dia Steel Ball
Height above Minimum (ins)
No of Oscillations
0
1
2
3
4
5
6
7
8
9

Photograph I

AN OBLIQUE VIEW
OF THE SURFACE

Photograph 2

DEXION FRAME FOR
CAMERA MOUNTING