12

The Stability of Darcy–Bénard Convection

D. A. S. Rees
University of Bath, Bath, UK

I. INTRODUCTION

Many years have passed since Horton and Rogers (1945) and Lapwood (1948) published their pioneering studies into the onset of convection in porous layers heated from below, the well-known Horton–Rogers–Lapwood or Darcy–Bénard problem. This configuration, a horizontal layer of either finite or infinite extent, has been studied in very great detail and especially so since the weakly nonlinear study of Palm et al. (1972) and the detailed numerical stability analysis of Straus (1974). The main reason for such activity lies in the fact that there are many extensions to the governing Darcy–Boussinesq equations. Commonly cited examples are boundary, inertia, local thermal non-equilibrium, anisotropic, and thermal dispersion effects. The presence of one or more of these or other modifications serves to change the nature of the resulting flow and accounts for the huge research literature. I will refer to such modified problems by the generic term 'Darcy–Bénard', even though further words such as 'Forchheimer' or 'Brinkman' could quite legitimately claim a place in that term.

The Darcy–Bénard problem is striking in its simplicity: a uniform-thickness horizontal porous layer with uniform temperature and impermeable upper and lower surfaces. The macroscopic equations governing the filtration of the fluid in the case where Darcy's law applies, the fluid is Boussinesq, the matrix is rigid and isotropic, and the solid and fluid phases are in local thermal equilibrium, are so straightforward that the linearized stability analysis proceeds analytically within the space of a few lines. Even

[12:09 24/1/00 n:3583 VAFAI.605/3583-012.3d] Ref: 3583 Vafai: Handbook of Porous Media: Chapter 12 Page: 521 521-558

the associated weakly-nonlinear analysis may be undertaken analytically within a few pages. However, this is where the simplicity ends. More complicated scenarios either involve the use of numerical methods or sentence the researcher to heavy-duty algebraic work which, though tractable, will nevertheless take much longer than a directly numerical approach!

The present author is very aware of how much space was set aside in the excellent book by Nield and Bejan (1992) to cover the topic of Darcy–Bénard convection, even though the treatment given even there could not have been comprehensive, especially in terms of presenting detailed results. Indeed, this topic is now sufficiently well advanced and mature to merit a book of its own, much as Koschmeider's (1993) book describes the analogous subject of Bénard convection and takes a more pedagogical stance. With this in mind, it has been necessary to restrict the scope of this chapter so that some material other than references and the author's name and affiliation should fit into the allotted space! Therefore the present review will concentrate exclusively on flows in horizontal layers, rather than on inclined or vertical layers, and it will avoid multi-diffusive flows. The important topic of convection in rotating systems has been dealt with admirably in two very recent reviews by Vadasz (1997, 1998), and therefore little will be mentioned here. The author hopes that offense will not be taken by those whose papers have not been cited.

It seems appropriate to split the review into two main sections, the first of which describes briefly the classical Darcy–Bénard problem where no extensions to Darcy's law or "variations on a theme" are included. This is organized systematically and describes in turn linearized theory, weakly nonlinear theory and strongly nonlinear computations. The second section deals with those aspects which arise from properties of the porous medium (e.g., boundary and inertia effects, anisotropy, and thermal dispersion), arise from fluid properties alone (e.g. temperature-dependent viscosity and chemical reactions), and external influences such as unsteady heating, different types of thermal boundary conditions, and mixed convective effects. These are arranged according to the different type of extension considered. The majority of papers cover various aspects of linearized theory, the onset problem. Decreasing numbers deal in turn with weakly nonlinear theory and strongly nonlinear flows.

II. THE CLASSICAL DARCY–BÉNARD PROBLEM

A. Linearized Theory

In apparently independent investigations, Horton and Rogers (1945) and Lapwood (1948) considered the onset of convection in a horizontal saturated porous layer heated from below. The two bounding surfaces are held at uniform temperatures (i.e., infinitely or perfectly conducting), the flow is assumed to be governed by Darcy's law, and the Boussinesq approximation is taken to apply. If the dimensional horizontal coordinates are $\hat{x}$ and $\hat{z}$ and the dimensional upwards vertical coordinate is $\hat{y}$, then the governing equations of motion are

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} + \frac{\partial \hat{w}}{\partial \hat{z}} = 0 \tag{1}$$

$$(\hat{u}, \hat{v}, \hat{w}) = -\frac{K}{\mu}\nabla \hat{p} + \frac{\rho \tilde{g} \beta K (T - T_c)}{\mu}(0, 1, 0) \tag{2}$$

$$\sigma \frac{\partial T}{\partial \hat{t}} + \hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{v}\frac{\partial T}{\partial \hat{y}} + \hat{w}\frac{\partial T}{\partial \hat{z}} = \kappa \left(\frac{\partial^2 T}{\partial \hat{x}^2} + \frac{\partial^2 T}{\partial \hat{y}^2} + \frac{\partial^2 T}{\partial \hat{z}^2} \right) \tag{3}$$

where all the terms have their usual meanings: K is permeability, ρ is a reference density, β the coefficient of cubical expansion, μ the fluid viscosity, and

$$\sigma = \frac{(1 - \phi)(\rho c)_s + \phi (\rho c)_f}{(\rho c)_f} \tag{4}$$

is the heat capacity ratio of the saturated medium to that of the fluid. Hence c is the heat capacity and ϕ is the porosity of the medium. These equations may be nondimensionalized, using the following substitutions

$$(x, y, z) = (\hat{x}, \hat{y}, \hat{z})/d, \qquad (u, v, w) = \frac{d}{\kappa}(\hat{u}, \hat{v}, \hat{w}) \tag{5}$$

$$p = \frac{K}{\kappa \mu}\hat{p}, \qquad t = \frac{\kappa}{d^2 \sigma}\hat{t}, \qquad \theta = \frac{T - T_c}{T_h - T_c} \tag{6}$$

where T_c and T_h are the upper (cold) and lower (hot) boundary temperatures, respectively, and d is the uniform depth of the layer. Equations (1)–(3) become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{7}$$

$$(u, v, w) = -\nabla p + Ra(0, \theta, 0) \tag{8}$$

$$\sigma \frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \tag{9}$$

where

$$Ra = \frac{\rho \tilde{g} \beta d K (T_h - T_c)}{\mu \kappa} \tag{10}$$

is the Darcy–Rayleigh number; this value plays the same role in porous convection as does the usual Rayleigh number for convecting clear fluids, as it expresses a balance between buoyancy and viscous forces. These equations are to be solved subject to the boundary conditions

$$v = 0, \qquad \theta = 1 - y \qquad \text{at} \qquad y = 0, 1 \tag{11}$$

and it is straightforward to verify that

$$u = v = w = 0, \qquad p_y = Ra(1 - y), \qquad \theta = 1 - y \tag{12}$$

is a solution of equations (7)–(9); this is referred to as the conduction solution. The topic of this chapter is the stability and evolution of disturbances to this and similar basic profiles.

The determination of conditions governing the onset of convection is *usually* undertaken using a linearized theory. Here the solution is perturbed about the basic solution and then linearized. A cellular convective planform is typically assumed for the disturbance profile which involves one or two wavenumbers, and a critical Rayleigh number is derived by solving an ordinary or partial differential eigenvalue problem. In the present case, the onset of two-dimensional disturbances proceeds initially by defining the streamfunction ψ according to $u = \psi_y$, $v = -\psi_x$, $w = 0$. If we also subtract out the basic profile (for which $\psi = 0$) and linearize, there results the system

$$\psi_{xx} + \psi_{yy} = Ra\theta_x, \qquad \theta_t = \theta_{xx} + \theta_{yy} + \psi_x \tag{13}$$

subject to $\psi = 0$ and $\theta = 0$ at $y = 0, 1$. If we assume that convection consists of rolls in the x-direction with wavenumber k, then the substitution

$$\psi = e^{\lambda t} f(y) \sin kx, \qquad \theta = e^{\lambda t} g(y) \cos kx \tag{14}$$

yields

$$f'' - k^2 f + kg = 0, \qquad \lambda g = g'' - k^2 g + kf = 0 \tag{15}$$

subject to $f = g = 0$ at $y = 0, 1$. Here λ is the exponential growth rate. When $\lambda = 0$, disturbances are neutrally stable and neither grow nor decay, and the solution of (15) yields the criterion for the onset of convection. For the present classical problem, the solutions are, simply, that

$$f(y) = A(k^2 + \pi^2) \sin \pi y, \qquad g(y) = Ak \sin \pi y, \qquad Ra = \frac{(k^2 + \pi^2)^2}{k^2} \tag{16}$$

where A is an arbitrary amplitude. A graph of Ra against k is depicted in Figure 1. It is easily shown that the minimum value is $Ra = 4\pi^2$ and this occurs when $k = \pi$, which corresponds to rolls with square cross-section. In Eq. (14) it has been assumed that exchange of stabilities holds (see Drazin and Reid 1981 for details of this concept), which is true for this flow and means that $\lambda = 0$ corresponds to neutral stability. Otherwise, it would be necessary to represent the x-dependencies in (14) in complex exponential form and impose $Re(\lambda) = 0$ for neutral stability; in such cases nonzero values of the imaginary part of λ would correspond to a translational wave-speed of the disturbance. The neutral curve given in Figure 1 should be interpreted as meaning that infinitesimal disturbances will decay whenever both the Rayleigh number and the wavenumber correspond to points below the curve.

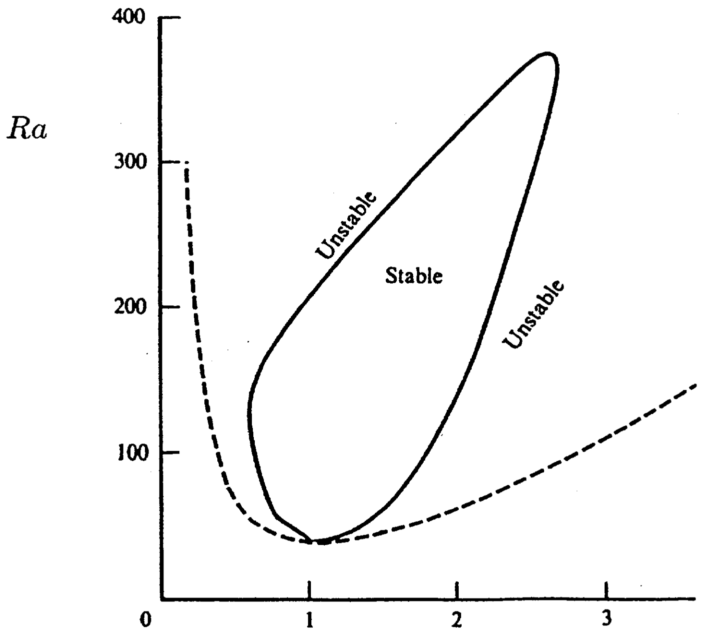

Figure 1. Regions of stable and unstable rolls. The dashed line is the neutral stability curve for the onset of convection and the solid line, from Straus (1974), delineates the region of stable steady rolls. Reproduced by permission of Cambridge University Press.

The above analysis covers only that case with perfectly conducting, impermeable bounding surfaces. Other cases involve different combinations of impermeable (IMP) and constant pressure (FRE), and conducting (CON) and constant heat flux (CHF) for both surfaces (but note that Nield and Bejan (1992) refer to the CHF case as "insulating" and label it INS). Values of the critical Rayleigh number and the associated wavenumber are given in Table 1. It is interesting to note that when a constant heat flux is specified on both boundaries the preferred wavenumber is zero; the only other case where this happens is when both perfectly conducting surfaces are at constant pressure. It is to be presumed that a smooth transition takes place when any thermal boundary condition is varied smoothly from one extreme case to the another. Finally, we note that not all the theoretical cases presented in Table 1 form practical configurations for experimental work.

B. Weakly Nonlinear Analyses

Linearized analyses only yield information on the conditions for which small-amplitude disturbances have a zero growth rate. They do not give information on whether the flow above the critical Rayleigh number is stable, on whether there exists flow at subcritical Rayleigh numbers, or to decide between competing instabilities. These are the roles, in turn, of weakly nonlinear and strongly nonlinear studies. Weakly nonlinear analyses are asymptotic analyses and are therefore valid asymptotically close to the point of onset given by linearized theory. However, much of the qualitative nature of their results carries over to the strongly nonlinear regime.

Table 1. Values of the critical Rayleigh number and wavenumber for various combinations of boundary conditions. After Nield (1968)

Upper surface		Lower surface		Ra_c	k_c
IMP	CON	IMP	CON	$4\pi^2$	π
IMP	CON	IMP	CHF	27.10	2.33
IMP	CHF	IMP	CHF	12	0
IMP	CON	FRE	CON	27.10	2.33
IMP	CHF	FRE	CON	17.65	1.75
IMP	CON	FRE	CHF	π^2	$\pi/2$
IMP	CHF	FRE	CHF	3	0
FRE	CON	FRE	CON	12	0
FRE	CON	FRE	CHF	3	0
FRE	CHF	FRE	CHF	0	0

The seminal paper by Palm et al. (1972) concentrates on the supercritical flow and heat transfer of unit-aspect-ratio convection using a power series expansion in $(1 - Ra_c/Ra)^{1/2}$. Steady convection at Rayleigh numbers up to about 5 times the critical value is described well using this power series, even though it is, strictly speaking, an asymptotic analysis.

Although a rigorous weakly nonlinear analysis for Bénard convection by Newell and Whitehead (1969) was published thirty years ago, the first journal paper to appear from which the analogous Darcy–Bénard stability results may be inferred is Rees and Riley (1989a) (although Joseph (1976) reports the results directly). In these types of analysis, Ra is assumed to be within $O(\varepsilon^2)$ of $Ra_c = 4\pi^2$ and k within $O(\varepsilon)$ of $k_c = \pi$. The temperature field, for example, and the Rayleigh number are expanded in power series in ε as follows

$$\theta = \theta_0 + \varepsilon\theta_1 + \varepsilon^2\theta_2 + \varepsilon^3\theta_3 + \dots, \qquad Ra = R_0 + \varepsilon^2 R_2 + \dots \tag{17}$$

where $R_0 = 4\pi^2$. Here θ_0 is the conduction solution and θ_1 comprises one or more disturbances satisfying the linearized equations of motion. Normally the $O(\varepsilon^2)$ equations may be solved fairly easily, either analytically or numerically. However, the nonlinear interactions between the modes introduced at $O(\varepsilon)$ give rise to generally insoluble equations at $O(\varepsilon^3)$ since the inhomogeneous terms contain components which are proportional to solutions of the corresponding homogeneous equations. A solvability condition may then be derived by choosing an appropriate value of R_2. For example, if we set $\theta_1 = \pi^{-2}A(\tau)\cos\pi x \sin\pi y$ in Eq. (16), where $\tau = \frac{1}{2}\varepsilon^2 t$ is a slow timescale, then the solvability condition is

$$A_\tau = R_2 A - A^3 \tag{18}$$

It is straightforward to show that $A = 0$ is the only steady solution when $R_2 < 0$, which corresponds to $Ra < Ra_c$, and that this solution is unstable when $R_2 > 0$. For $R_2 > 0$ there also exist the solutions $A = \pm R_2^{1/2}$. More complicated stability analyses involve allowing for small wavenumber changes, small changes in orientation of the basic roll, and the inclusion of other rolls. Such stability analyses are known, respectively, as the Eckhaus, zigzag, and cross-roll instabilities. The first two may be accommodated by defining slow x and z variables: $X = \varepsilon x$ and $Z = \varepsilon^{1/2} z$. A three-dimensional analysis yields the amplitude equation

$$A_\tau = R_2 A + \left(2\frac{\partial}{\partial X} - \frac{i}{\pi}\frac{\partial^2}{\partial Z^2}\right)^2 A - A^2\bar{A} \tag{19}$$

Details of the ensuing analysis may be gleaned from Rees and Riley (1989a) and more directly from Rees (1996). In short, if the basic solution takes the form of

$$A = (R_2 - 4K^2)e^{iKX} \tag{20}$$

this corresponds to a roll with a wavenumber $\pi + \varepsilon K$. Solutions with $K < 0$ are unstable to zigzag disturbances, whereas solutions with $4K^2 < R_2 < 12K^2$ are subject to the Eckhaus instability. In the former case two rolls aligned at equal but opposite small angles to the original roll grow at the expense of the basic roll, forming a sinuous roll boundary. One of these two rolls eventually dominates, leaving a stable steady-state configuration. In the latter case a pair of perfectly aligned rolls with wavenumbers either side of the original roll grow, again with one dominating whose wavenumber lies within the stable domain.

Finally, the cross-roll instability may be studied by considering a roll at a finite angle χ to the basic roll; if we let

$$\theta_1 = \pi^{-2} A \cos \pi x \sin \pi y + \pi^{-2} B \cos \pi(x \cos \chi - z \sin \chi) \sin \pi y \tag{21}$$

then A and B may be shown to satisfy the equations

$$A_\tau = R_2 A + 4A_{XX} - A[A\bar{A} + \Omega(\chi) B\bar{B}] \tag{22a}$$

$$B_\tau = R_2 B + 4A_{X_B X_B} - B[B\bar{B} + \Omega(\chi) A\bar{A}] \tag{22b}$$

Here we have suppressed Z-variations in A, and $X_B = \varepsilon(\cos \chi - z \sin \chi)$ is the slow variable perpendicular to the direction of the B-roll. A general analysis of (22) shows that single rolls are the favored mode when $\Omega(\chi) > 1$ for all values of χ, whereas square or rectangular cells are preferred if the minimum value of $\Omega(\chi)$ is less than 1. For the present problem we have

$$\Omega(\chi) = \frac{70 + 28\cos^2 \chi - 2\cos^4 \chi}{49 - 2\cos^2 \chi + \cos^4 \chi} \tag{23}$$

which varies monotonically from 2 at $\chi = 0$ to its minimum, 10/7 at $\chi = \pi/2$ (Rees and Riley 1989b). Therefore rolls are preferred, and it may be shown that the roll given by (20) is stable when $R_2 > 40K^2/3$. This criterion is more restrictive than the Eckhaus criterion, and therefore there exists a band of stable wavenumbers lying between $K = 0$ and $K = +\sqrt{3R_2/40}$, the corresponding domain of existence of the basic roll being $|K| \le \sqrt{R_2/4}$. We note that Riahi (1983) considered the same problem but with finitely conducting boundaries. There is a region in parameter-space wherein square cells are preferred.

As mentioned earlier, a weakly nonlinear analysis is an asymptotic analysis and it is therefore valid only in the $\varepsilon \to 0$ limit. The present analysis corresponds to the bottom of the Straus (1974) stability envelope shown in Figure 1 and which is discussed later. We note that Joseph (1976) presents a more accurate version of the zigzag instability. Similar weakly nonlinear analyses have been undertaken for more complicated versions of the classical Darcy–Bénard problem and a wide variety of phenomena ensue, but we defer discussion of these cases to the next section. Energy stability analyses also exist for the onset of convection, but for the classical Darcy–Bénard problem they yield identical results to those of linearized stability theory.

C. Strongly Nonlinear Convection

One advantage of weakly nonlinear theory is that layers of infinite extent may be considered easily. When the strongly nonlinear regime is encountered, progress is gained through numerical computation. Although spectral methods may, by their nature, be said to apply to an infinite domain, finite difference or finite element techniques must be applied to a finite computational domain. When Darcy flow is considered and sidewalls are taken to be insulated and impermeable, then sidewalls coincide with planes of symmetry in the infinite domain and much may then be said about the infinite case, even though whole classes of possible disturbances are absent due to the presence of the side walls.

At Rayleigh numbers slightly higher than that given by linearized theory, the realized flow is steady and two-dimensional. Straus (1974) undertook a detailed stability analysis of this strongly nonlinear convection and determined the whole regime in *Ra*–*k* space within which convection is stable; this region is displayed in Figure 1. It is important to be able to interpret correctly the information presented in this figure. First, it is essential to note that this represents only the stable regime; it does not give any indication as to the relative probabilities of different stable flows given a random initial condition in a time-dependent simulation. Second, there is no information as to what the final stable planform will be, for this depends on the initial disturbance; on the right-hand part of the stability envelope we know that the most dangerous disturbance is one aligned at right-angles to the basic roll, but the graph does not give the wavenumber of that disturbance. Finally, when *Ra* is larger than the maximum value on that neutral curve, the resulting flow may not be deduced without further knowledge—one could guess that it is three-dimensional, but not whether it is steady.

A long series of papers by Straus, Schubert, Steen and others have sought to extend our knowledge of the flow at increasing Rayleigh numbers; see Holst and Aziz (1972), Horne and O'Sullivan (1974), Horne (1979), Straus and Schubert (1979, 1981), Schubert and Straus (1979, 1982), Kimura et al. (1986, 1987), Aidun (1987), Aidun and Steen (1987), Steen and Aidun (1988), and Caltagirone et al. (1987). In a square box with insulating side walls, the primary transition from a motionless state to steady convection takes place when $Ra = 4\pi^2$, and the second transition to a time-periodic flow occurs at roughly $Ra = 391$. This value was determined very accurately by Riley and Winters (1991), who used a finite element steady flow solver and examined the iteration matrix to find when the linearized disturbance equations admit a Hopf bifurcation. They recorded a value of $Ra = 390.72$ 01 which should now be regarded as being definitive. In the same paper Riley and Winters also computed how this second critical value of *Ra* varies with the box aspect ratio.

At increasing Rayleigh numbers there follows a sequence of transitions between time-periodic flow and aperiodic flow; the precise demarcation of the points of transition are, in the present author's experience, highly dependent on the level of resolution of the numerical scheme. The papers by Kimura et al. (1986, 1987) are devoted to determining this sequence and find that the flow undergoes a short sequence of transitions between periodicity and quasi-periodicity before becoming nonperiodic. A very thorough analysis has been presented by Graham and Steen (1992), who have managed to uncover much of the bifurcation structure of flows at such values of *Ra*. In particular, they show that quasi-periodic motion sets in at $Ra = 505$ where the periodic flow is destabilized by a disturbance at an incommensurate frequency. Other unsteady flows are computed, points of bifurcation examined with a diagram of possible bifurcation relating each solution to one another and inferring the existence of unstable quasi-periodic flows. Furthermore, this bifurcation structure is shown to be able to explain the observations of Kimura et al. (1986) that there are two stable quasi-periodic solutions in the range $500 < Ra < 560$. Other box aspect ratios are considered by Graham and Steen; in particular, the aspect ratio 2.495 yields a double Hopf bifurcation where two periodic solutions bifurcate off the steady solution branch at the same Rayleigh number.

Other details of modal competition between possible steady-state solutions may be found in Riley and Winters (1989), where attention is focused on how the bifurcation structure varies with the box aspect ratio. In Figure 2 we display a sample bifurcation diagram for a box of unit aspect ratio. At increasing values of *Ra* after the primary 1-cell destabilization at $Ra = 4\pi^2$, a 2-cell state and then a 3-cell flow bifurcate off the conduction solution, but are initially unstable. The 2-cell solution eventually gains stability and this point is marked by the bifurcation of a mixed-mode branch of the 2-cell branch. A similar scenario occurs for the 3-cell solution, except that two bifurcations to mixed mode solutions must occur before it is stabilized. We

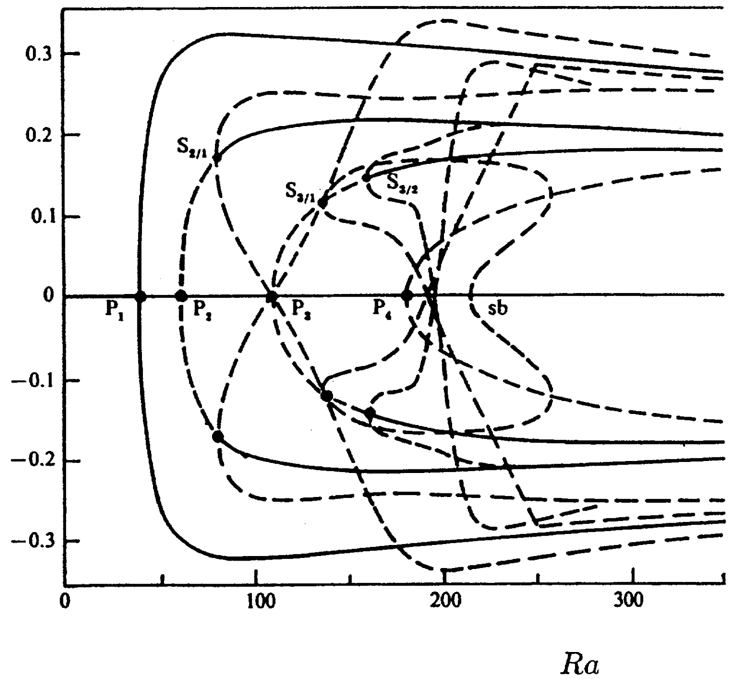

Figure 2. Computed bifurcation structure for two-dimensional flow in a square box. The stable and unstable branches are denoted by full and broken curves respectively. The vertical axis is the temperature at the midpoint of the left-hand sidewall. From Riley and Winters (1989). Reproduced by permission of Cambridge University Press.

note that a similar study of the bifurcation structure in three dimensions, even for a cubic box, has not yet been undertaken, for although three-dimensional flows are unstable in an infinite layer at sufficiently low Rayleigh numbers, the spatial constriction in a cube or a cuboid is sufficient to stabilize such flows; see Steen (1983) and Kimura et al. (1990). A very detailed account of the onset of oscillatory convection is presented in Kimura (1998).

Other early papers are concerned with the effect of a net mass flux through the layer. Sutton (1970) considered vertical flow through permeable boundaries; in this case stabilization (an increased critical Rayleigh number) or destabilization depends on the strength of the throughflow and the aspect ratio of the finite cavity. Prats (1967), on the other hand, studied the effect of a horizontal flow; here the criterion for the onset of convection is unchanged since the Darcy equations retain their precise form when expressed in the frame of reference which is moving with the mean flow. Thus, in an infinite layer, all the above-quoted results of Palm et al. (1972) and of Straus and co-workers are unaffected.

III. VARIATIONS ON THE THEME

A. Inertia Effects

In this subsection we discuss how the presence of fluid inertia (form drag or Forchheimer drag) influences the onset and development of convection. The governing dimensionless equations are given by

$$u_x + v_y + w_z = 0, \qquad \underline{u}(1 + G|\underline{u}|) = -\nabla p + Ra(0, \theta, 0), \\ \theta_t + \underline{u} \cdot \nabla \theta = \nabla^2 \theta \tag{24}$$

where $G = \tilde{K}\rho\kappa/\mu d$ is an inertia parameter and $\tilde{K}$ is a material parameter related to the microscopic lengthscale of the medium (L) and the porosity (ϕ) via Ergun's (1952) relation

$$\tilde{K} = \frac{1.75L}{150(1 - \phi)} \tag{25}$$

for example. Given that the additional term over and above the normal Darcy-flow problem is nonlinear and given that there is no basic flow, it is very straightforward to show that the presence of inertia has no effect on the onset of convection in a horizontal porous layer. This point may also be gleaned from the paper by He and Georgiadis (1990), who also considered the extra effects of internal heating and hydrodynamic dispersion. When there is a basic flow in the porous medium, the presence of inertia serves to modify the stability characteristics from those corresponding to Darcy flow. One example is that of the inclined Darcy–Bénard problem, where unpublished numerical simulations by the present author have shown that convection is delayed by the presence of inertia when compared with that of the corresponding Darcy-flow case given in Rees and Bassom (1998). A more pertinent example is that of the combined effects of a horizontal pressure gradient with inertia, studied by Rees (1998). In this analytical study the author showed that two-dimensional cells (whose axes are perpendicular to the direction of the pressure gradient) have a critical Rayleigh number which rises with both G and the strength Q of the horizontal: horizontal flow

$$Ra_c = \pi^2\left[(1 + G|Q|)^{1/2} + (1 + 2G|Q|)^{1/2}\right]^2 \tag{26a}$$

The critical wavenumber is

$$k_c = \pi \left[\frac{1 + 2G|Q|}{1 + G|Q|} \right]^{1/4} \tag{26b}$$

We note that it is the combined effect of inertia and the horizontal pressure gradient which causes this rise; when inertia is absent, Prats (1967) showed that the dynamics even of strongly nonlinear convection are unchanged when viewed in the frame of reference moving with the mean flow. However, when the rolls are aligned with the mean flow direction the critical Rayleigh is $Ra_c = 4\pi^2(1 + G|Q|)$ with $k_c = \pi$, leading to a clear preference for such rolls in an unbounded layer.

A weakly nonlinear stability analysis was undertaken by He and Georgiadis (1990), who found that the standard pitchfork bifurcation is modified to one with straight lines intercepting the zero amplitude axis; this aspect is a consequence of the Forchheimer term. They also found that the positive amplitude flow arises at a different value of Ra from that of negative amplitude flow. This was attributed to the combined effects of inertia, internal heating, and dispersion. More recently, Georgiadis (personal communication, 1996) has indicated that these combined effects actually cause a splitting of the bifurcation "curve" into two, corresponding to different modes of convection, in much the same way as Rees and Riley (1986) found two different modes in the case of Darcy flow in a layer with symmetrical boundary imperfections. The one arising at the lower value of Ra is generally stable in an infinite layer, while the other is unstable Such results are easily proved using the appropriate amplitude equations.

In the work of He and Georgiadis (1990) the imperfections to the classical problem were sufficiently strong to cause the weakly nonlinear analysis to progress only to second order. A suitable third-order analysis was undertaken by Rees (1996), who assumed that inertia effects were weak. The straight line behavior of the solution curves near onset are reproduced, but at higher Rayleigh numbers the usual square root behavior is re-established. This counter-intuitive result, which says that inertia effects are strongest when the flow is weakest and vice versa, is a direct consequence of the fact that the destabilizing effect of buoyancy is balanced at high Rayleigh numbers by the most nonlinear term in the amplitude equation

$$A_\tau = R_2 A + \alpha A|A| - A^2 \bar{A} \tag{27}$$

where α is a scaled inertia parameter. The author also developed a full weakly nonlinear stability analysis and found that inertia causes some wavenumbers less than the critical value to regain stability, but the cross-roll instability is more effective and reduces the stable wavenumber range.

Strongly convecting flow was considered by Strange and Rees (1996), who undertook a finite difference computation of flow in a square cavity.

Using a 32×32 grid and a streamfunction/temperature formulation they found that unsteady convection is delayed by the presence of inertia, and that, for values of G less than 0.05, the critical value is given by

$$Ra = Ra_{c2} + 2.95 \times 10^4 G \tag{28}$$

where Ra_{c2} is the value corresponding to incipient unsteady flow in the absence of inertia. The same qualitative result was found by Kladias and Prasad (1989, 1990), who also used the Brinkman terms in their fundamental model.

B. Brinkman Effects and Advective Inertia

The appropriate momentum equation is now taken to be

$$\begin{aligned}\phi^{-1}(\underline{u} \cdot \nabla)\underline{u} = \nabla p + (Pr/Ra)^{1/2}\nabla^2\underline{u} - (F\phi Da^{-1})|\underline{u}|\underline{u} \\ - (\phi/Da)(Pr/Ra)^{1/2}\underline{u} + \phi(0, \theta, 0)\end{aligned} \tag{29}$$

(Lage et al. 1992) where the first term is the advective inertia term and the second term on the right-hand side is the Brinkman term. The paper by Lage et al. (1992) discusses at length the role of the Prandtl number on the onset of convection, concluding that the criterion for onset is unaffected by the value of Pr. In this regard these authors show that earlier work by Kladias and Prasad (1989) is incorrect in stating that the criterion is dependent on Pr, and therefore verify much earlier work by Walker and Homsy (1977). In fact, the nondimensionalization used to derive the above equation obscures this result: if we replace all occurrences of $\underline{u}$ by $\underline{u}Da(RaPr)^{-1/2}$, then Eq. (29) becomes

$$\begin{aligned}(Da^2\phi^{-2}Pr^{-1})(\underline{u} \cdot \nabla)\underline{u} = (Ra/\phi)\nabla p + (Da/\phi)\nabla^2\underline{u} \\ - (FDa/Pr)|\underline{u}|\underline{u} - \underline{u} + (0, \theta, 0)\end{aligned} \tag{30}$$

The Prandtl number is now seen to appear only in nonlinear terms and therefore will not affect linearized analyses based on a no-flow solution. However, we see that the criterion for onset will depend only on (Da/ϕ), the coefficient of the Brinkman term, rather than on Da as suggested by Lage et al. (1992). Walker and Homsy (1977) computed the neutral curve shown in Figure 3 which depicts the transition between Darcy flow when Da is very small and clear fluid flow at sufficiently large values (10^{-3} and greater). The reader is referred to the very detailed introductions in Lage et al. (1992) and Georgiadis and Catton (1988) for a description of earlier work on this topic.

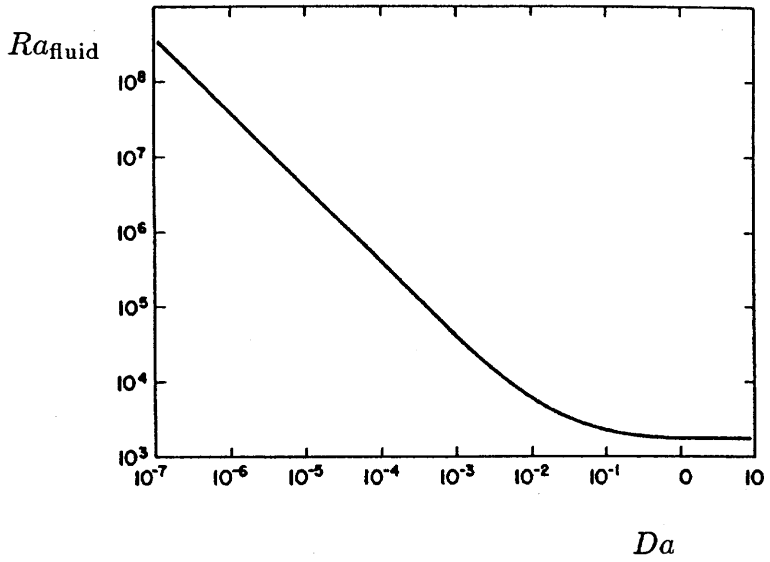

Figure 3. Critical fluid Rayleigh number as a function of the Darcy number. From Walker and Homsy (1977). Reproduced by permission of ASME.

A very recent paper by Néel (1998) considers how a horizontal pressure gradient affects convection in the presence of inertia and Brinkman effects. The results presented therein correspond to a cubic inertia term replacing the usual quadratic term, and therefore they cannot be compared directly with the above papers. In her work, the author does not include advective inertia, but performs a small parameter expansion in terms of an inertia parameter and an equivalent to the Darcy number to show that these effects cause an increase in the critical Rayleigh number. The effects of a horizontal mean flow are also considered.

C. The Effects of Throughflow

Throughflow has been studied in two different types of case: (i) horizontal flow and (ii) vertical flow. We have already discussed briefly the papers by Prats (1967) and Néel (1998) on the effects of horizontal flow, but a further paper by Dufour and Néel (1998) considers Darcy flow in finite horizontal channels. Here various end-wall boundary conditions are imposed and the resulting flow patterns investigated. Weakly nonlinear stability analyses are performed, as are numerical simulations. Given a reasonable entry condition (uniform flow and an imposed temperature profile), an entry-region effect is obtained whereby increasing flow rates yield increasing distances before strong traveling-wave convection is obtained.

A somewhat neglected paper by Sutton (1970) was the first to consider the effect on Ra_c of a vertical throughflow of fluid. On taking the nondimensional upward mean velocity as a small parameter, a perturbation analysis is undertaken which shows that the correction to the critical Rayleigh number is positive, i.e., that the effect of very small throughflow is stabilizing. Some numerical analysis of the perturbation equations was also presented for a small range of values of the throughflow parameter. Sutton showed that, although destabilization occurs for small vertical velocities, stabilization is re-established at higher flow rates. This is consistent with the fact that the overall temperature drop occurs in a decreasingly thick region as the flow rate increases. The small destabilization may be argued as being a consequence of a nonlinear temperature profile. Nield (1987) considered the analogous case of convection between insulating boundaries and showed that throughflow is always stabilizing. In the case of mixed-type boundaries, stabilization or destabilization depends on whether the throughflow is towards or away from the more restrictive boundary.

Riahi (1989) performed a weakly nonlinear analysis using the method of Schlüter et al. (1965). At a sufficiently high Péclet number (equivalent to a scaled throughflow velocity) the resulting finite-amplitude flow takes the form of hexagons (three rolls aligned at 60° to one another) initially with subcritical motions occurring. At higher convection amplitudes, square cells, originally unstable, are stabilized. The existence of multiple stable states implies that the realized flow depends on the initial disturbance.

D. Hydrodynamic Dispersion

Dispersion in the porous medium context is the local mixing which takes place due to the complex paths the fluid takes through the porous medium; the reader is referred to Section 2.2.3 of Nield and Bejan (1992) for a detailed discussion of the modeling. In the papers by Neichloss and Degan (1975) and Kvernvold and Tyvand (1980), dispersion was modeled using an energy equation of the form

$$\theta_t + \underline{u} \cdot \nabla\theta = \nabla \cdot (\mathbf{D} \cdot \nabla\theta) \tag{31}$$

where $\mathbf{D} = (1 + \varepsilon_2 \underline{u} \cdot \underline{u})\mathbf{I} + (\varepsilon_1 - \varepsilon_2)\underline{uu}$ is the dispersion tensor with the values of $\varepsilon_1 = D/15$ and $\varepsilon_2 = D/40$ chosen. Here D is a dispersion factor which takes small values. On the other hand, Georgiadis and Catton (1988) use $\mathbf{D} = 1 + Di|\underline{u}|$ where Di is also a small parameter. It is clear that dispersion is a nonlinear effect and, in the presence of a no-flow basic state, has no influence on the onset problem.

Neichloss and Degan (1975) performed an analysis using the same series expansion technique as Palm et al. (1972) to obtain steady supercritical flows. They show that increasing dispersion decreases the overall Nusselt number, and the influence is significant for coarse materials. Although these authors undertake a two-dimensional stability analysis, they conclude that the flow is stable within the limitations of the theory. A more comprehensive analysis was presented by Kvernvold and Tyvand (1980), who extended the results of Straus (1974) to flows with dispersion. At values of D as small as $1/150$ the stability envelope, similar to that shown in Figure 1, extends to much higher values of Ra and indicates that stable steady convection can persist for a considerably greater range of Rayleigh numbers.

E. Internal Heating

When the porous medium is subject to a uniform distribution of heat sources the nondimensional energy transport equation takes the form

$$\theta_t + \underline{u} \cdot \nabla\theta = \nabla^2\theta + 1 \tag{32}$$

the final term representing the steady uniform generation of heat. Often two Rayleigh numbers are used to categorize the flow: an external Rayleigh number, which is based on any physically imposed temperature difference in the system (i.e., heating from below), and an internal Rayleigh number, Ra_I, which depends on the rate of heat generation. Problems involving internal heat generation are frequently referred to as "penetrative convection," although such a term is used quite legitimately in a wider context such as those flows presented in Section 7.4 of Nield and Bejan (1992).

When the horizontal bounding surfaces are held at identical constant temperatures the critical value of Ra_I is approximately 470. As the temperature of the lower surface is raised, this value decreases until it reaches zero when $Ra = 4\pi^2$. The detailed Ra/Ra_I map is given in Figure 4, where the stable and unstable regions are delineated. A nonlinear stability analysis of the case $Ra = 0$ was carried out by Tveitreid (1977), who showed that down-hexagons (those with fluid flowing downwards in the centre of the hexagonal pattern) are stable up to approximately 8 times the critical value (i.e., 8×470), whereas up-hexagons are always stable. Initially rolls are unstable, but regain their stability at about 3 times the critical value. No information is given about the relative stability of square or rectangular planform flows. Other aspects involving different boundary conditions are discussed briefly in Nield and Bejan (1992).

A substantially different problem ensues when considering a rectangular porous cavity. If all four bounding walls are maintained at the same

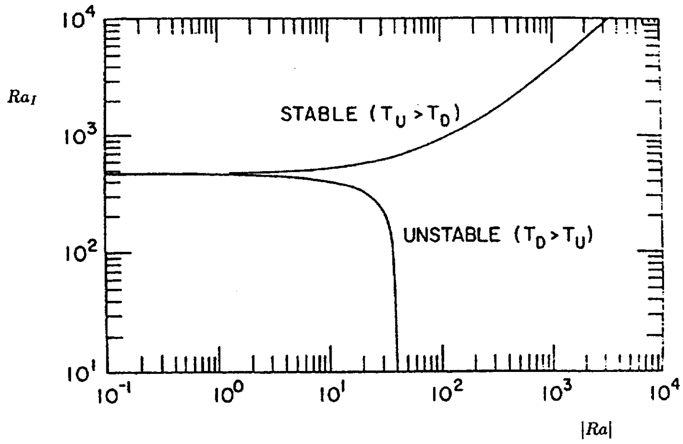

Figure 4. Critical internal Rayleigh number against the external Rayleigh number for stabilizing and destabilizing temperature differences. From Gasser and Kazimi (1976). Reproduced by permission of ASME.

uniform temperature, then there is flow at all nonzero values of Ra_I. Indeed, at high values of Ra_I (with $Ra = 0$) convection at the vertical sidewalls becomes of boundary layer type; see Blythe et al. (1985). However, in such a situation, the upper part of the cavity is unstably stratified and the flow described by Blythe et al. (1985) is unlikely to be realized in practice. A very recent numerical study by Banu et al. (1998) has sought to investigate the onset of unsteady convection in rectangular cavities; the authors have found that incipient unsteady flow occurs at values of Ra_I which are highly dependent on the aspect ratio of the cavity. Since convective instabilities at the time-dependent motion are confined to the top of the cavity, it is possible to show that the critical value is proportional to the inverse third power of the aspect ratio for tall thin cavities. On the other hand, the detailed dynamics becomes very complicated at large aspects ratios where the cavity is shallow. In this case the flow may become chaotic and it loses left/right symmetry; snapshots of a chaotic sequence are shown in Figure 5, where the downward pointing plumes may be seen to be generated whenever there is sufficient room near the top of the cavity, and subsequently travel towards the nearer side wall. A detailed numerical study of possible flow regimes in Ra_I-aspect ratio space is needed.

Nield (1995) considered the case of internal heating which decreases with depth and varies periodically with time, such as would occur in solar energy collectors or solar ponds. The author assumed that the time-depen-

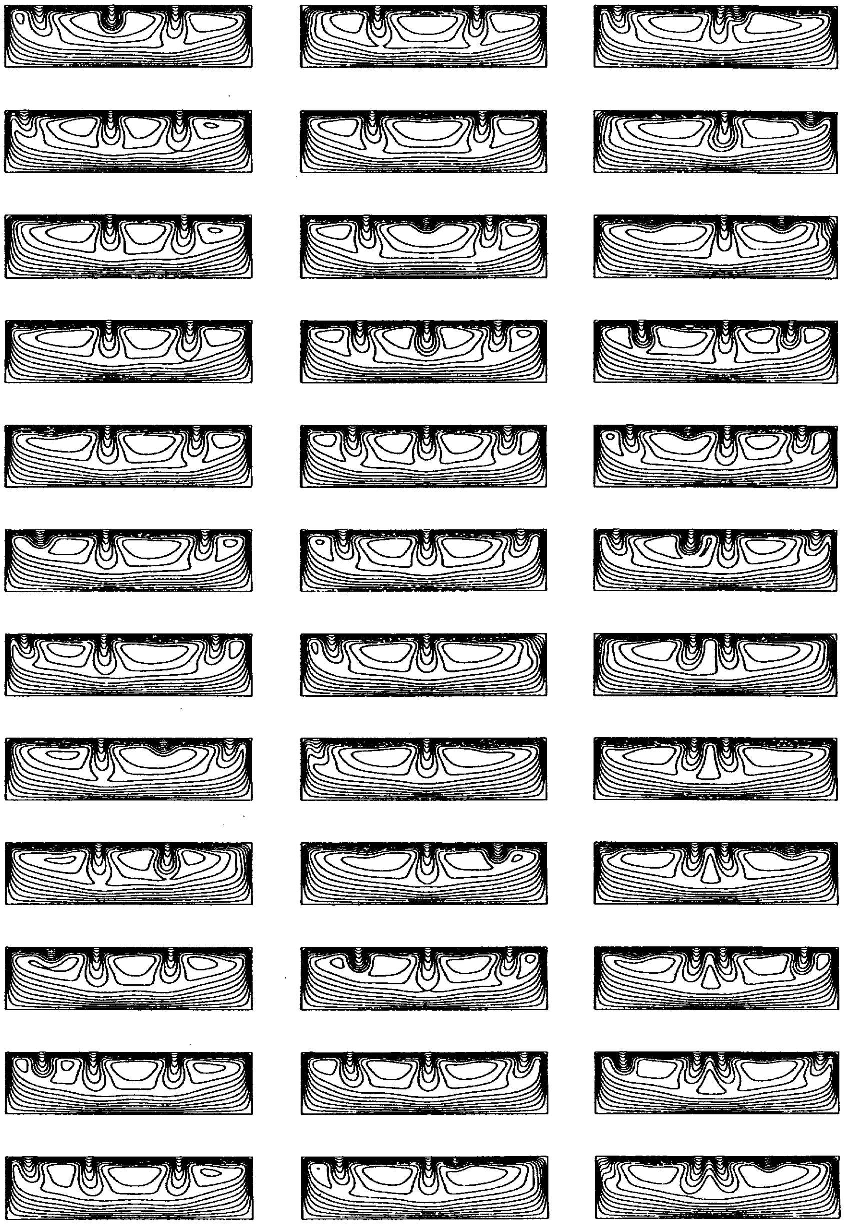

Figure 5. Instantaneous isotherms displaying a chaotic solution for a 4×1 cavity with uniform internal heating at $Ra = 3000$. The individual frames represent snapshots at equal time increments. Time progresses downwards from top left.

dence is quasi-static, in line with the physics of such applications. A linearized analysis using a one-term Galerkin approximation is presented since the flow inhabits a very large parameter space.

Ames and Cobb (1994) consider the onset of convection in a layer with a temperature drop between the infinite horizontal surfaces and where the saturating fluid is water close to its density minimum at approximately 4°C. The results are necessarily detailed, but the authors also use a nonlinear energy theory. In many circumstances the linear and nonlinear onset criteria are different, suggesting that subcritical motions may occur. The extra effect of an anisotropic permeability is examined in Straughan and Walker (1996a); they find that convection arises with a complex growth rate, a Hopf bifurcation to unsteady flow.

The paper by Rionera and Straughan (1990) considers the effect of a y-dependent gravity (to use the present notation) on convection in a layer with a linear density–temperature relationship and nonzero *Ra*. Again, linearized and nonlinear energy analyses were used, and since the critical value of the Rayleigh number using the energy method lies below that obtained using a standard linearized analysis, it is again deduced that subcritical motions arise.

The addition of Brinkman effects was considered by Vasseur and Robillard (1993), who also assumed that the layer (without internal heat generation) is heated using a uniform heat flux at the lower horizontal surface. The other three bounding surfaces are insulated. As there is no heat sink, the mean temperature rises. On subtracting out this rise, the resulting mathematical problem resembles one involving heat generation. As the thermal boundary conditions are different from those assumed by the above-quoted authors, no direct comparison can be made with their conclusions. Analytical solutions are presented for various values of the critical Rayleigh number.

F. Local Thermal Nonequilibrium

The great majority of papers dealing with convective flows in porous media assume that the solid matrix and the saturating fluid are in local thermal equilibrium (LTE), meaning that the difference between the fluid and solid temperatures is negligible throughout the medium. Vafai and Amiri (1998), in their review article on forced convective flows in porous media, state that sufficiently slow dynamic processes allow the use of the LTE assumption. Although we believe that this is true for the Darcy–Bénard problem, recent boundary-layer analyses by Rees and Pop (1999) and Rees (1999) show that LTE is not always recovered in steady-state flows.

If we use θ and Φ to denote the fluid phase and solid phase temperature fields, respectively, then the two energy transport equations may be written in the dimensionless form

$$\theta_t + \underline{u} \cdot \nabla\theta = \nabla^2\theta + H(\Phi - \theta), \qquad \alpha\Phi_t = \nabla^2\Phi + \gamma H(\theta - \Phi) \tag{33}$$

The three parameters, α, γ, and H, are defined according to

$$\alpha = \frac{(\rho c)_s k_f}{(\rho c)_f k_s}, \qquad \gamma = \frac{\varepsilon k_f}{(1-\varepsilon)k_s}, \qquad H = \frac{hd^2}{\varepsilon k_f} \tag{34}$$

respectively, and may be described as a diffusivity ratio, a porosity-scaled conductivity ratio, and a dimensionless inter-phase heat transfer coefficient. Discussions on typical values for h, the dimensional inter-phase heat transfer coefficient, may be found in Kuznetsov (1998) and Vafai and Amiri (1998). For such problems the Darcy–Rayleigh number is based upon the thermal properties of the fluid phase, rather than on the average properties, and is also inversely proportional to the porosity. LTE corresponds to conditions where $|\theta - \Phi|$ is negligible compared with typical values of θ and Φ.

Combarnous (1972) presented a numerical study of steady large-amplitude convection using the above model. At a fixed value of $Ra = 200$ with a unit aspect ratio convection cell, he showed that the Nusselt number is independent of γ when H is sufficiently large (which recovers LTE), and that LTE is established at decreasing values of H as γ increases. Much more recently, Banu and Rees (1999a) have sought to determine conditions for the onset of convection. These authors obtain the same qualitative results as Combarnous in the sense that LTE corresponds either to large values of H or to sufficiently large values of γ. In Figure 6 we show the detailed neutral curves for various values of γ. Comparison with the classical Darcy–Bénard case for which LTE applies is afforded by plotting $\gamma Ra/(1+\gamma)$, which corresponds to the usual Darcy–Bénard Rayleigh number. It is assumed that the layer is infinite in horizontal extent and therefore $\gamma Ra/(1+\gamma)$ has been minimized with respect to the wavenumber. We note that large values of γ or sufficiently large values of H correspond to LTE. When H is very small, the neutral curves correspond to $\gamma Ra/(1+\gamma) = 4\pi^2\gamma/(1+\gamma)$.

?2000

G. Anisotropy, Layering, and Heterogeneity

Storesletten (1998) has very recently presented a thorough and comprehensive review of convection in anisotropic porous media, taking into account anisotropy in both the permeability and thermal diffusivity. Most of that review concentrates on the Darcy–Bénard problem. In the very short time

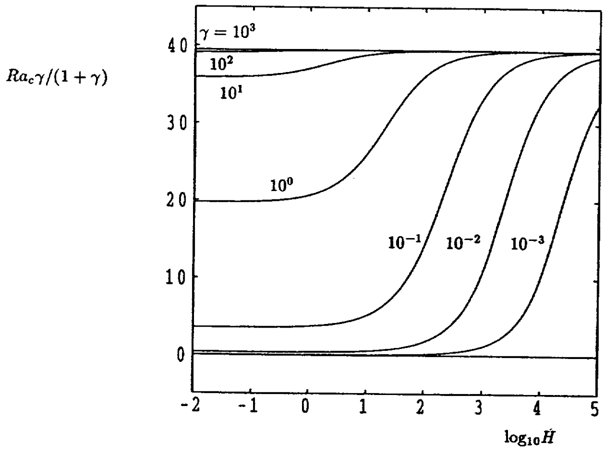

Figure 6. Values of $Ra_c\gamma/(1+\gamma)$ against $\log_{10} H$ for various values of γ. From Banu and Rees (1999a).

since the submission of the manuscript of Storesletten (1998) only two further papers have appeared. Mamou et al. (1998) consider how anisotropy affects the onset of convection in a layer where the upper and lower surfaces are cooled and heated by uniform heat fluxes rather than having constant temperatures imposed. In the isotropic case the critical Rayleigh number for an infinite layer is 12 (Nield 1968). Mamou et al. restrict attention to media where one of the principal axes of the permeability tensor is horizontal. It is shown that the criterion for the onset of convection depends very strongly on the ratio of the principal permeabilities, as does the detailed pattern of convection. The second paper, by Parthiban and Patil (1997), considers the combined effects of anisotropy, internal heating, and an inclined temperature gradient.

Frequent mention is made of the connection between anisotropy and layering in the Darcy–Bénard context. McKibbin (1985) gives a comprehensive account of work in the area of layering, and further comments may be found in McKibbin (1991). Rees and Riley (1990) provide an extension to the earlier weakly nonlinear (two-dimensional) heat transfer analysis of McKibbin and O'Sullivan (1981). In this paper Rees and Riley consider whether or not two-dimensional rolls are stable in layers composed of isotropic sublayers of different permeabilities and thicknesses, and conclude that in some circumstances square cells are to be preferred. The necessary modification to the overall rate of heat transfer is given. Of particular interest is the fact that layering allows the possibility of multimodal neutral curves; these allow for sudden changes in the preferred wavelength and pattern of convection when a parameter (such as a sublayer permeability) changes. Comprehensive details may be found in McKibbin and O'Sullivan (1980). In a three-layer medium Rees and Riley (1990) found a parameter set with the critical Rayleigh number corresponding to three preferred wavenumbers simultaneously, suggesting that increasingly complicated modal interactions may be uncovered as the number of sublayers increases.

The effects of continuous variations in the permeability and thermal diffusivity (as opposed to the discontinuous changes corresponding to layering) were considered in quite general terms in Braester and Vadasz (1993). In general, weak heterogeneities (i.e., small-amplitude variations) modify the manner in which convection ensues. Convection is maintained at all nonzero Rayleigh numbers and a smooth transition to a convecting regime is obtained as the Rayleigh number increases.

H. Inclined Temperature Gradient

Now we consider the case where there is an additional horizontal temperature gradient superimposed on the standard vertical gradient. In nondimensional terms, the horizontal surfaces have the following linear temperature variations

$$y = 0: \quad \theta = 1 - \tilde{\beta}x, \qquad y = 1: \quad \theta = -\tilde{\beta}x \tag{35}$$

where $\tilde{\beta}$ is the ratio of the horizontal to the vertical temperature gradients. Unlike the classical Darcy–Bénard problem in which there is a basic flow whose stability is analyzed, there is always a horizontal flow when $\tilde{\beta} \neq 0$. This is referred to as a Hadley circulation, but there is no net horizontal flow.

Weber (1974) considered small values of $\tilde{\beta}$ and determined criteria for the onset of convection in the form of longitudinal rolls (i.e., with axes in the x-direction): $Ra_c = 4\pi^2(1 + \tilde{\beta}^2)$ at a wavenumber of π. Later, Nield (1991) relaxed the restriction of small values of $\tilde{\beta}$ and used a low-order Galerkin expansion to confirm the qualitative result that a horizontal temperature increases the critical Rayleigh number. He also presented details of stationary and traveling transverse modes. In another paper Nield (1994) discovered that Ra_c for longitudinal rolls does not increase indefinitely as $\tilde{\beta}$ increases, but reaches a maximum before decreasing to zero. The implication is that the vertical temperature difference, which is a characteristic of the Darcy–Bénard problem, is not essential to induce instability in this type of problem. More detailed linear analyses are presented in Lage and Nield (1998), where the authors also provide strongly nonlinear computations (for transverse modes using a variant of the well-known SIMPLE algorithm) and extend the work to doubly diffusive convection. In the strongly nonlinear regime a complicated bifurcation structure involving both stationary and travelling rolls is found. Further references may be found in Lage and Nield (1998). Of some considerable interest is the nonlinear energy stability analysis of Kaloni and Qiao (1997); their results suggest that strongly nonlinear convection may in many circumstances arise at Rayleigh numbers well below that predicted by the standard linearized theory as given by Nield (1991, 1994).

Nield (1990) also considered the additional effect of introducing a horizontal pressure gradient to induce an overall fluid flux in the positive-x direction. As the surface temperature gradient is negative, this additional effect destabilizes the basic flow relative to when there is no mean horizontal flow. Again, it is found that a local vertical temperature difference is not always essential for instability to arise.

The additional effects of anisotropy and an internal heat source were considered by Parthiban and Patil (1997). The authors conclude that longitudinal modes form the preferred pattern of convection at onset, even with the strong tendency towards hexagonal convection brought about by the presence of internal heating, as shown by Tveitreid (1977).

I. Boundary Imperfections

Here we consider how the classical Darcy–Bénard problem is modified by the presence of perturbed boundary conditions such as wavy boundaries and nonuniform bounding temperatures (both steady and unsteady). We note that these two types of boundary imperfection have identical qualitative effects within the weakly nonlinear regime when the imperfection is steady and applies to the upper and lower surfaces, although the quantitative results are different. Here the nonuniformity is described by sinusoidal variations in the x-direction.

Perturbation analyses of temperature variations were first carried out by O'Sullivan and McKibbin (1986), Rees and Riley (1989a,b) and Rees (1990), using weakly nonlinear theory. The general conclusion from these studies is that resulting flow depends very strongly on the wavelength and symmetry of the boundary imperfection. When the imperfection has the critical wavelength (i.e., with a wavenumber of π), and is not symmetric, then the transition to convection is smooth and the concept of a critical Rayleigh number is inapplicable (O'Sullivan and McKibbin 1986; Rees and Riley 1989a). When the wavenumber is greater than 3π longitudinal modes are preferred, causing convection to be three-dimensional (Rees and Riley 1989b). At smaller wavenumbers it is possible for rectangular planforms to arise abruptly, and these are composed of a pair of rolls aligned at equal but opposite angles from the direction of the x-axis. At slightly higher Rayleigh numbers these cells destabilize and one of the pair dominates the other. This is a mutual resonance effect mediated by the boundary imperfection. At wavenumbers close to the critical value rolls with spatially deformed axes or spatially varying wavenumbers may arise (Rees and Riley 1989a), and when the wavenumber is very small the pattern of convection at onset may then be quasi-periodic (Rees 1990).

Further and more detailed weakly nonlinear analyses have been undertaken by Riahi (1993, 1995, 1996, 1998) and a centre manifold analysis by Néel (1992). The first three of these papers and that of Néel form an extension of the work of O'Sullivan and McKibbin (1986) and Rees and Riley (1989a,b), who consider the boundary imperfection to have only one sinusoidal component, to the multicomponent case. Given the complexity of the possible patterns when given one component (Rees and Riley 1989b), the number of different possibilities multiplies enormously in the more general cases. Generally, convection is three-dimensional and nonperiodic, although many special cases exist for which this is not true. Examples are cited where the flow may undertake a transition between different patterns as the imperfection amplitude is altered. Riahi (1998) considers the same type of fundamental problem, but assumes that the boundaries are poorly conducting, an extension of earlier work contained in Riahi (1983). Other aspects are studied, using reduction methods, by Néel (1990a,b).

Limited results are available for strongly nonlinear convection in the presence of boundary imperfections. As part of their study, Rees and Riley (1986) undertook a two-dimensional numerical simulation of convection in a symmetric layer with wavy boundaries. The onset of convection is abrupt and is delayed by the presence of the nonuniformity. However, the onset of time-periodic flow takes place at much smaller Rayleigh numbers than those corresponding to the uniform layer. The mechanism generating unsteady flow is no longer a thermal boundary layer instability, but appears to be a cyclical interchange between two distinct modes which support each other via the imperfection, and its onset is not a Hopf bifurcation. At relatively high amplitudes of the wavy surface, the basic flow may bifurcate directly to unsteady flow. Rathish Kumar et al. (1998) have also studied the effects of large-amplitude undulations of the lower surface of a porous cavity. They investigate the effects of varying the wave phase, the wave amplitude, and the number of waves, using a finite element technique.

In the above papers the boundary imperfections were held stationary in space and in most cases the preferred mode of convection at onset is steady. Indeed, rolls show no tendency to exhibit a horizontal mean motion in the absence of moving boundary conditions or a horizontal pressure gradient, although the presence of an inclined temperature gradient may sometimes cause such motion, as mentioned earlier. Just as stationary small-amplitude imperfections cause weak fluid motion whose presence affects the onset of convective instability, so a moving imperfection will cause the basic fluid motion to be unsteady, which may influence the location of the convective cells.

The first paper in this subtopic deals with moving thermal boundary conditions of the form

$$y = 0: \quad \theta = 0, \qquad y = 1: \quad \theta = a\cos[k(x + Ut)] \tag{36}$$

In their paper, Ganapathy and Purushothaman (1992) assume that the wavenumber of the imperfection, k, is large and also include Brinkman and advective inertia effects. This is not a stability problem as the mean temperature gradient across the layer is zero. They perform a long-wave approximate analysis to determine the subsequent motion. Mamou et al. (1996) use the boundary conditions

$$y = 0: \quad \theta = 1 + a\sin[k(x - Ut)], \qquad y = 1: \quad \theta = 0 \tag{37}$$

in a numerical simulation of strong two-dimensional convection. These authors assume that $k = \pi$, the critical value for the classical Darcy–Bénard problem, that the flow is spatially periodic, and that both a and U take finite (rather than infinitesimal) amplitudes. Additionally, they compute the flow in a frame of reference which is moving with the imperfection, and reduce the problem to one in which the imperfection is stationary but which has an overall mean horizontal flow. The main result of their computational work is to demonstrate that there are two main flow regimes: (i) where the convective motion follows the thermal wave, and (ii) where the convective pattern drifts at a slower speed than the thermal wave. In particular they deduce that there is a critical value of Ra at which one behavior gives way to the other. Much of this qualitative behavior has been explained by Banu and Rees (1999b), who perform a weakly nonlinear stability analysis based on the boundary conditions

$$\begin{aligned} y = 0: &\quad \theta = 1 + a\cos[\pi(x - Ut)], \\ y = 1: &\quad \theta = a\cos[\pi(x - Ut)] \end{aligned} \tag{38}$$

The amplitude of convection is taken to be $O(\varepsilon)$ and the transitional case, mimicking the computations of Mamou et al. (1996), requires $a = O(\varepsilon^3)$ and $U = O(\varepsilon^2)$. For a given wave speed, convection follows the boundary wave until a certain value of R_2 (see Section II) is exceeded. The resulting nonlinear motion is governed by a straightforward ordinary differential system and has to be computed numerically. As R_2 increases the flow may be either periodic (taking all possible multiples of the forcing period) or quasi-periodic, but the overall mean horizontal motion of the convective pattern decreases. An asymptotic analysis, using the technique of multiple scales analysis for large values of R_2, shows that the horizontal drift is proportional to R_2^{-1} in this limit; a typical flow for such a regime is given in Figure 7.

An entirely different type of imperfection has been investigated by Impey et al. (1990) and Vadasz and Braester (1992). These authors consider the effect of imperfectly insulated sidewalls on convection in a horizontal

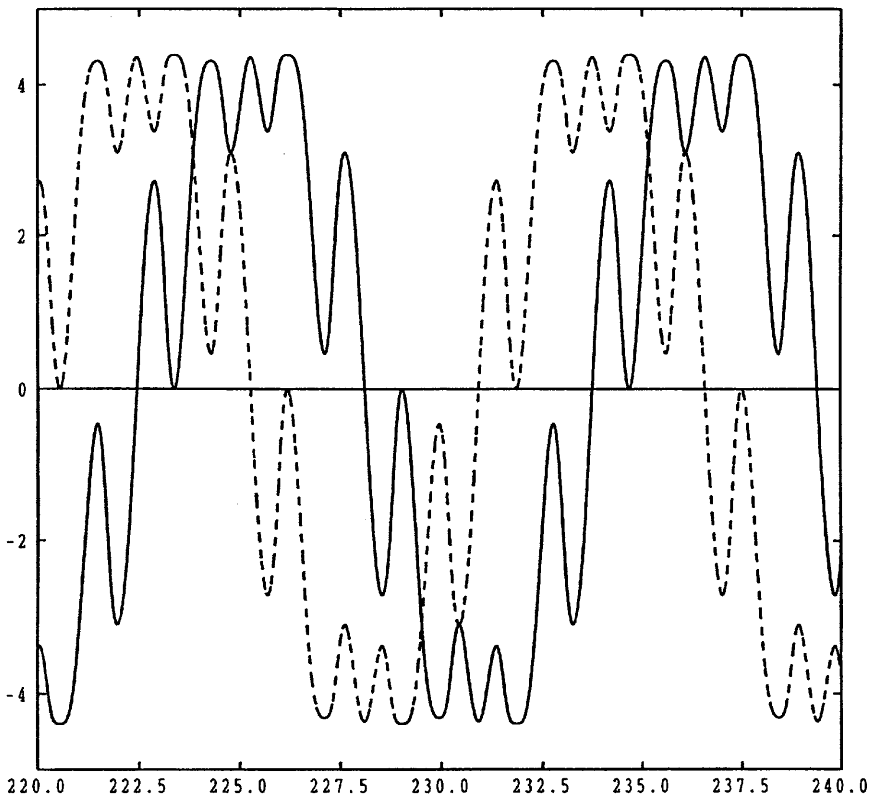

Figure 7. Evolution of the flow for $R_2 = 19.05707$ with $\alpha = 0.5$. The real part of the complex amplitude is given by the solid curve, and the imaginary part corresponds to the dashed curve. This flow has 9 times the forcing period. From Banu and Rees (1999b).

1999

cavity with upper and lower surfaces at a uniform temperature. Vadasz and Braester (1992) use weakly nonlinear theory to show that weak imperfections modify the usual pitchfork bifurcation to a so-called imperfect bifurcation. Thus weak convection persists at relatively low Rayleigh numbers and grows sharply in strength near the critical Rayleigh number corresponding to the classical Darcy–Bénard problem. A second stable flow also exists within the weakly nonlinear regime if the Rayleigh number is sufficiently large. Impey et al. (1990) provide a comprehensive account of the effects of different shapes and symmetries of the sidewall imperfections, and extend their results well outside the weakly nonlinear regime by using numerical continuation techniques drawn from bifurcation theory. Great emphasis is laid upon how different solution branches gain or lose stability as the Rayleigh number increases.

Vadasz et al. (1993), while also summarizing the results of Vadasz and Braester (1992), also consider strong convection in a cavity with perfectly conducting sidewalls. Conditions are determined under which a conducting no-flow state may be obtained. For general sidewall temperature profiles, flow exists at all nonzero Rayleigh numbers. In shallow cavities convection is confined to the regions near the sidewalls at subcritical Rayleigh numbers, but the patterns migrate towards the middle and fill the cavity as the Rayleigh number increases. Cases with heating from above as well as heating from below are considered.

J. Large Wavenumber Effects

In two separate recent studies the flow within a tall cavity heated from below has been considered, but different sidewall boundary conditions are imposed and this leads to very great qualitative differences between the two cases. Lewis et al. (1997) allow the sidewalls to be insulated, and perform a large-wavenumber asymptotic analysis of nonlinear convection. Three separate nonlinear regimes appear as the Rayleigh number increases, but convection remains unicellular. On the other hand, Rees and Lage (1997) use perfectly conducting boundary conditions by setting a linearly decreasing temperature profile up the sidewalls. For all cell aspect ratios the onset problem is degenerate in the sense that any combination of an odd and an even mode is destabilized simultaneously at the critical Rayleigh number. This degeneracy persists even into the nonlinear regime. A large-wavenumber analysis of the tall cavity problem shows that strong boundary layers form on the top and the bottom of the cavity and a third may exist in the interior.

K. Recent Experimental Advances

A primary difficulty in experimental work involving porous media is the determination of the detailed macroscopic flowfield. Conventional visualization techniques suffer due to the random nature of most media and the difference in refractive indices of the fluid and the matrix of those media which are transparent. Invasive techniques yield only a small amount of information, and others provide information only at boundaries. Howle et al. (1993) overcame these difficulties using suitably regular media (stacked square cross-section bars or punched disks) and a shadowgraph technique. Despite the relatively coarse nature of the media, a remarkably sharp transition to convection was observed for the barred medium. A more comprehensive account is given in Howle et al. (1997).

The magnetic resonance imaging technique was used by Shattuck et al. (1995, 1997) to provide accurate imaging and heat transport measurements for both ordered and disordered media consisting of spheres. It was found that pattern selection in disordered media is influenced strongly by the presence of packing defects. On the other hand, well-defined convective patterns appear when the spheres are uniformly packed. The authors provide a very comprehensive account of the effects of different container geometries and conclude that theoretical modeling of macroscopic laws may well need more detailed information on the pore structure.

L. Miscellaneous Topics

Finally, we present brief descriptions of various studies which cannot be categorized easily into the above subsections.

Parthiban and Patil (1996) consider the onset of convection of a rarefied gas for which the Knudsen number is close to unity. Compressibility effects are included, as are thermal dissipation effects (in the usual fluid-dynamical sense of heat being generated by sufficiently high fluid velocities). The authors study both the Darcy and Darcy–Brinkman models. It is found that rarefaction serves to increase both the critical Rayleigh number and the wavenumber. A simpler model was used by Stauffer et al. (1997) in an independent study of strong convection using numerical methods. We also note that the time derivative of the density in the continuity equations used in these papers is multiplied by 1 in Parthiban and Patil (1996) and by the porosity in Stauffer et al. (1997), and that the Rayleigh numbers are defined differently. Therefore it is difficult to compare these studies.

Malashetty et al. (1994) performed a linear stability analysis of a layer saturated with a chemically reacting fluid. The basic state consists of a nonlinear temperature profile whose degree of nonlinearity depends on the size of the Frank–Kamenetskii parameter. A low-order Galerkin expansion of the linearized stability equations was used to yield criteria for the onset of convection. It is found that this type of chemical reaction enhances convection and destabilizes the layer in comparison with the classical Darcy–Bénard problem.

The presence of a vertical baffle occupying part of the vertical extent of a porous cavity was considered by Chen and Wang (1993). The primary effect of the presence of the baffle is to inhibit the onset of convection by restricting the space within which convection can occur, although a suitably placed baffle will not influence onset criteria.

Sometimes it is advantageous to delay the onset of convection in order to reduce the rate of heat transfer across a porous layer. With this in mind, Tang and Bau (1993) have presented and analyzed a particular feedback control stabilization strategy. The controller modifies the boundary temperature using (i) a measured deviation of the temperature from the conductive state, and (ii) its time derivative. It is found that the critical Rayleigh number may be increased by a factor of 4. The conduction state is maintained through small modulations in the boundary temperatures.

The possibility of combining surface tension effects in a highly porous medium with standard instability mechanisms in order to explain the frequent occurrence of hexagonal planforms was postulated by Hennenberg et al. (1997). The Darcy flow model with a linear equation of state is insufficient to cause stable hexagonal patterns, and the addition of Brinkman effects does not change this qualitative result as the symmetry of the layer has not been changed. Hennenberg et al. refer to their paper as tentative, and detailed comments are given in Nield (1998) about the modeling of the free surface and about other mechanisms which could explain the presence of hexagonal cells.

Skeldon et al. (1997) present a method for investigating the bifurcation structure associated with competition between hexagonal patterns and rolls. Although the paper uses directional solidification as the application, the primary aim is to devise suitable finite element grids which will capture correctly all the symmetries involved in hexagonal/roll interactions. The computation of points of bifurcation is of great importance in understanding the roles played by the many competing solutions of a convection problem. In this regard much information may be gained from analyzing the eigenvalues of the Newton–Raphson iteration matrix when computing steady-state solutions. At relatively low Rayleigh numbers this is quite inexpensive, but efficiency and accuracy are of increasing importance at increasing Rayleigh numbers. Both may be achieved by means of two different methods described in detail in Straughan and Walker (1996b), who also give applications in porous medium convection. The first is a modified version of the compound matrix method, and the second is the Chebyshev tau method.

Porous rocks may exhibit large-scale imperfections such as layering or faulting. Although much work has been presented in the literature on the onset problem for layered media, little exists for faulted media, such as is presented in Joly et al. (1996). In that paper the authors devise a method for analyzing stability which involves the finite element technique. The method is able to test whether complicated two-dimensional flows are unstable to three-dimensional disturbances. It is illustrated by an application to a layered medium, the middle sublayer of which has vertical faulting modeled by a relatively high permeable porous medium.

NOMENCLATURE

Roman Letters

a	amplitude of thermal wave
A, B	amplitudes of weakly nonlinear convection
c	heat capacity
d	macroscopic length scale
D, Di	dispersion factors
Da	Darcy number
$\mathbf{D}$	dispersion tensor
f	reduced streamfunction
F	inertial coefficient
g	temperature
G	inertia parameter
$\tilde{g}$	gravity
h	inter-phase coefficient of heat transfer
H	nondimensional inter-phase coefficient of heat transfer
k	wavenumber of convection
K	permeability, small perturbation to wavenumber
$\tilde{K}$	dimensional inertia parameter
L	microscopic length scale
p	pressure
Pr	Prandtl number
Q	fluid flux speed
Ra	Darcy–Rayleigh number
t	time
T	temperature
u, v, w	fluid flux velocities in the x, y, and z directions
U	nondimensional velocity of thermal wave
x, y, z	Cartesian coordinates
X, X_B, Z	slow spatial variables

Greek Letters

α	scaled inertia parameter, diffusivity ratio
β	coefficient of cubical expansion
$\tilde{\beta}$	ratio of temperature gradients
γ	porosity-scaled conductivity ratio
ε	small expansion parameter
θ	scaled temperature
κ	thermal diffusivity
λ	exponential growth rate
μ	viscosity
ρ	fluid density
σ	heat capacity ratio
τ	slow time scale
ϕ	porosity
Φ	temperature of the solid phase
χ	angle between rolls
ψ	streamfunction
Ω	inter-roll coupling coefficient

Superscripts and Subscripts

$\hat{}$	dimensional
c	cold, critical
f	fluid
h	hot
s	solid
0,1,2,3	terms in weakly nonlinear theory

REFERENCES

Aidun CK. Stability of convection rolls in porous media. ASME J Heat Transfer 94:331–336, 1987.

Aidun CK, Steen, PH. Transition of oscillatory convective heat transfer in a fluid-saturated porous medium. AIAA J Thermophys Heat Transfer 1:268–273, 1987.

Ames KA, Cobb SS. Penetrative convection in a porous medium with internal heat sources. Int J Eng Sci 32:92–105, 1994.

Banu N, Rees DAS. Onset of Darcy–Bénard convection using a thermal nonequilibrium mode. In preparation, 1999a.

Banu N, Rees DAS. The effect of a travelling thermal wave on weakly nonlinear convection in a porous layer heated from below. To appear in: Proceedings of Workshop on Fluid Mechanics, Sylhet, Bangladesh (September 1998), 1999b.

Banu N, Rees DAS, Pop I. Steady and unsteady free convection in porous cavities with internal heat generation. Proceedings of 11th International Heat Transfer Conference, Kyongju-ju, Korea, 1998, Vol 4, pp 375–380.

Blythe PA, Daniels PG, Simpkins PG. Convection in a fluid-saturated porous medium due to internal heat generation. Int Commun Heat Mass Transfer 12:493–504, 1985.

Braester C, Vadasz P. The effect of a weak heterogeneity of a porous medium on natural convection. J Fluid Mech 254:345–362, 1993.

Caltagirone JP, Fabrie P, Combarnous M. De la convection naturelle oscillante en milieu poreux au chaos temporel? C R Acad Sci Paris II 305:549–553, 1987.

Chen F, Wang CY. Convective instability in saturated porous enclosures with a vertical insulating baffle. Int J Heat Mass Transfer 36:1897–1904, 1993.

Combarnous M. Description du transfert de chaleur par convection naturelle dans une couche poreuse horizontale à l'aide d'un coefficient de transfert solide–fluide. C R Acad Sci Paris II A275:1375–1378, 1972.

Drazin PG, Reid WH. Hydrodynamic Stability. Cambridge: Cambridge University Press, 1981.

Dufour F, Néel M-C. Numerical study of instability in a horizontal porous channel with bottom heating and forced horizontal flow. Phys Fluids 10:2198–2207, 1998.

Ergun S. Fluid flow through packed columns. Chem Eng Prog 48:89–94, 1952.

Ganapathy R, Purushothaman R. Free convection in a saturated porous medium due to a travelling thermal wave. Z Anger Math Mech 72:142–145, 1992.

Gasser RD, Kazimi MS. Onset of convection in a porous medium with internal heat generation. ASME J Heat Transfer 98:49–54, 1976.

Georgiadis JG, Catton I. Dispersion in cellular thermal convection in porous media. Int J Heat Mass Transfer 31:1081–1091, 1988.

Graham MD, Steen PH. Strongly interacting traveling waves and quasiperiodic dynamics in porous medium convection. Physica D 54:331–350, 1992.

He XS, Georgiadis JG. Natural convection in porous media: effect of weak dispersion on bifurcation. J Fluid Mech 216:285–298, 1990.

Hennenberg M, Ziad Saghir M, Rednikov A, Legros JC. Porous media and the Bénard–Marangoni problem. Transp Porous Media 27:327–355, 1997.

Holst PH, Aziz K. Transient three-dimensional natural convection in confined porous media. Int J Heat Mass Transfer 15:73–90, 1972.

Horne RN. Three-dimensional natural convection in a confined porous medium heated from below. J Fluid Mech 92:751–766, 1979.

Horne RN, O'Sullivan MJ. Oscillatory convection in a porous medium heated from below. J Fluid Mech 66:339–352, 1974.

Horton CW, Rogers FT. Convection currents in a porous medium. J Appl Phys 16:367–370, 1945.

Howle L, Behringer RP, Georgiadis J. Visualization of convective fluid flow in a porous medium. Nature 362:230–232, 1993.

Howle L, Behringer RP, Georgiadis JG. Convection and flow in porous media, Part 2, Visualization by shadowgraph. J Fluid Mech 332:247–262, 1997.

Impey MD, Riley DS, Winters KH. The effect of sidewall imperfections on pattern formation in Lapwood convection. Nonlinearity 3:197–230, 1990.

Joly N, Bernard D, Ménégazzi, ST2D3D: A finite-element program to compute stability criteria for natural convection in complex porous structures. Numer Heat Transfer B29:91–112, 1996.

Joseph DD. Stability of fluid motions II. Berlin: Springer, 1976.

Kaloni PN, Qiao Z. Non-linear stability of convection in a porous medium with inclined temperature gradient. Int J Heat Mass Transfer 40:1611–1615, 1997.

Kimura S. Onset of oscillatory convection in a porous medium. In: Ingham DB, Pop I, eds. Transport Phenomena in Porous Media. Oxford: Pergamon, 1998, pp 77–102.

Kimura S, Schubert G, Straus JM. Route to chaos in porous-medium thermal convection. J Fluid Mech 166:305–324, 1986.

Kimura S, Schubert G, Straus JM. Instabilities of steady, periodic and quasi-periodic modes of convection in porous media. ASME J Heat Transfer 109:350–355, 1987.

Kimura S, Schubert G, Straus JM. Time dependent convection in fluid-saturated porous cube heated from below. J Fluid Mech 207:153–189, 1990.

Kladias N, Prasad V. Natural convection in horizontal porous layers: effects of Darcy and Prandtl numbers. ASME J Heat Transfer 112:675–684, 1989.

Kladias N, Prasad V. Flow transitions in buoyancy-induced non-Darcy convection in a porous medium heated from below. ASME J Heat Transfer 112:675–684, 1990.

Koschmeider EL. Bénard cells and Taylor vortices. Cambridge: Cambridge Monographs on Mechanics and Applied Mathematics, 1993.

Kuznetsov AV. Thermal nonequilibrium forced convection in porous media. In: Ingham DB, Pop I, eds. Transport Phenomena in Porous Media. Oxford: Pergamon, 1998, pp 103–130.

Kvernvold O, Tyvand PA. Dispersion effects on thermal convection in porous media. J Fluid Mech 99:673–686, 1980.

Lage JL, Nield DA. Convection induced by inclined gradients in a shallow porous medium layer. J Porous Media. 1:57–69, 1998.

Lage JL, Bejan A, Georgiadis JG. The Prandtl number effect near the onset of Bénard convection in a porous medium. Int J Heat Fluid Flow 13:408–411, 1992.

Lapwood ER. Convection of a fluid in a porous medium. Proc Camb Phil Soc 44:508–521, 1948.

Lewis S, Rees DAS, Bassom AP. High wavenumber convection in tall porous containers heated from below. Q J Mech Appl Math 50:545–563, 1997.

McKibbin R. Thermal convection in layered and anisotropic porous media: a review. In: Wooding RA, White I, eds. Convective Flow in Porous Media. Wellington, New Zealand: DSIR, 1985, pp 113–127.

McKibbin R. Convection and heat transfer in layered and anisotropic porous media. Proceedings of ICHMT International Seminar on Heat and Mass Transfer in Porous Media, Dubrovnic, Yugoslavia, 1991, pp 327–336.

McKibbin R, O'Sullivan MJ. Onset of convection in a layered porous medium heated from below. J Fluid Mech 96:375–393, 1980.

McKibbin R, O'Sullivan MJ. Heat transfer in a layered porous medium heated from below. J Fluid Mech 111:141–173, 1981.

Malashetty MS, Cheng P, Chao BH. Convective instability in a horizontal porous layer saturated with a chemically reacting fluid. Int J Heat Mass Transfer 37:2901–2908, 1994.

Mamou M, Robillard L, Bilgen E, Vasseur P. Effects of a moving thermal wave on Bénard convection in a horizontal saturated porous layer. Int J Heat Mass Transfer 39:347–354, 1996.

Mamou M, Mahidjiba A, Vasseur P, Robillard L. Onset of convection in an anisotropic porous medium heated from below by a uniform heat flux. Int Commun Heat Mass Transfer 25:799–808, 1998.

Néel M-C. Convection in a horizontal porous layer of infinite extent. Eur J Mech B 9:155–176, 1990a.

Néel M-C. Convection naturelle dans une couche poreuse horizontale d'extension infinie: chauffage inhomogène. C R Acad Sci Paris II 309:1863–1868, 1990b.

Néel M-C. Inhomogeneous boundary conditions and the choice of the convective patterns in a porous layer. Int J Eng Sci 30:507–521, 1992.

Néel M-C. Convection forcée en milieux poreux: écarts à la loi de Darcy. C R Acad Sci Paris II 326:615-620, 1998.

Neichloss H, Dagan G. Convective currents in a porous layer heated from below: the influence of hydrodynamic dispersion. Phys Fluids 18:757–761, 1975.

Newell AC, Whitehead JA. Finite bandwidth, fine amplitude convection. J Fluid Mech 38:279–303, 1969.

Nield DA. Onset of thermohaline convection in a porous medium. Water Resour Res 11:553–560, 1968.

Nield DA. Convective instability in porous media with throughflow. AIChemEng J 33:1222–1224, 1987.

Nield DA. Convection in a porous medium with inclined temperature gradient and horizontal mass flow. Heat Transfer 1990, Hemisphere, 1990, Vol 5, pp 153–158.

Nield DA. Convection in a porous medium with inclined temperature gradient. Int J Heat Mass Transfer 34:87–92, 1991.

Nield DA. Convection in a porous medium with inclined temperature gradient: additional results. Int J Heat Mass Transfer 37:3021–3025, 1994.

Nield DA. Onset of convection in a porous medium with nonuniform time-dependent volumetric heating. Int J Heat Fluid Flow 16:217–222, 1995.

Nield DA. Modelling the effect of surface tension on the onset of natural convection in a saturated porous medium. Transp Porous Media 31:365–368, 1998.

Nield DA, Bejan A. Convection in Porous Media. New York: Springer-Verlag, 1992.

O'Sullivan MJ, McKibbin R. Heat transfer in an unevenly heated porous layer. Transp Porous Media 1:293–312, 1986.

Palm E, Weber JE, Kvernvold O. On steady convection in a porous medium. J Fluid Mech 54:153–161, 1972.

Parthiban C, Patil PR. Convection in a porous medium with velocity slip and temperature jump boundary conditions. Heat Mass Transfer 32:27–31, 1996.

Parthiban C, Patil PR. Thermal instability in an anisotropic porous medium with internal heat source and inclined temperature gradient. Int Commun Heat Mass Transfer 24:1049–1058, 1997.

Prats M. The effect of horizontal fluid motion on thermally induced convection currents in porous mediums. J Geophys Res 71:4835–4838, 1967.

Rathish Kumar BV, Murthy PVSN, Singh P. Free convection heat transfer from an isothermal wavy surface in a porous enclosure. Int J Numer Meth Fluids 28:633–661, 1998.

Rees DAS. The effect of long-wave thermal modulations on the onset of convection in an infinite porous layer heated from below. Q J Mech Appl Math 43:189–214, 1990.

Rees DAS. The effect of inertia on the stability of convection in a porous layer heated from below. J Theor Appl Fluid Mech 1:154–171, 1996.

Rees DAS. The effect of inertia on the onset of mixed convection in a porous layer heated from below. Int Commun Heat Mass Transfer 24:277–283, 1998.

Rees DAS. Vertical thermal boundary layer flow in a porous medium using a thermal nonequilibrium model: elliptic effects. Proceedings of Workshop on Fluid Mechanics, Sylhet, Bangladesh (September 1998), 1999.

Rees DAS, Bassom AP. The onset of convection in inclined porous layers. Proceedings of International Conference on Heat Transfer, Kyong-ju, Korea, 1998, Vol 4, pp 497–502.

Rees DAS, Lage JL. The effect of thermal stratification on natural convection in a vertical porous insulation layer. Int J Heat Mass Transfer 40:111–121, 1997.

Rees DAS, Pop I. Vertical thermal boundary layer flow in a porous medium using a thermal nonequilibrium model. J Porous Media 3: 31–44, 2000.

Rees DAS, Riley DS. Convection in a porous layer with spatially periodic boundary conditions: resonant wavelength excitation. J Fluid Mech 166:503–530, 1986.

Rees DAS, Riley DS. The effects of boundary imperfections on convection in a saturated porous layer: near-resonant wavelength excitation. J Fluid Mech 199:133–154, 1989a.

Rees DAS, Riley DS. The effects of boundary imperfections on convection in a saturated porous layer: non-resonant wavelength excitation. Proc R Soc Lond A421:303–339, 1989b.

Rees DAS, Riley DS. The three-dimensional stability of finite-amplitude convection in a layered porous medium heated from below. J Fluid Mech 211:437–461, 1990.

Riahi DN. Nonlinear convection in a porous layer with finite conducting boundaries. J Fluid Mech 129:153–171, 1983.

Riahi DN. Nonlinear convection in a porous layer with permeable boundaries. Int J Non-linear Mech 24:459–463, 1989.
Riahi DN. Preferred pattern of convection in a porous layer with a spatially non-uniform boundary temperature. J Fluid Mech 246:529–543, 1993.
Riahi DN. Finite amplitude thermal convection with spatially modulated boundary temperatures. Proc R Soc Lond 449:459–478, 1995.
Riahi DN. Modal package convection in a porous layer with boundary imperfections. J Fluid Mech 318:107–128, 1996.
Riahi DN. Finite bandwidth, long wavelength convection with boundary imperfections: near-resonant wavelength excitation. Int J Math Sci 21:171–182, 1998.
Riley DS, Winters KH. Modal exchange mechanisms in Lapwood convection. J Fluid Mech 215:309–329, 1989.
Riley DS, Winters KH. Time-periodic convection in porous media: the evolution of Hopf bifurcations with aspect ratio. J Fluid Mech 223:457–474, 1991.
Rionera S, Straughan B. Convection in a porous medium with internal heat source and variable gravity effects. Int J Eng Sci 28:497–503, 1990.
Schlüter A, Lortz D, Busse FH. On the stability of finite-amplitude convection. J Fluid Mech 23:129–144, 1965.
Schubert G, Straus JM. Three-dimensional and multicellular steady and unsteady convection in fluid-saturated porous media at high Rayleigh numbers. J Fluid Mech 94:25–38, 1979.
Schubert G, Straus JM. Transitions in time-dependent thermal convection in fluid-saturated porous media. J Fluid Mech 121:301–303, 1982.
Shattuck MD, Behringer RP, Johnson GA, Georgiadis JG. Onset and stability of convection in porous media: visualization by magnetic resonance imaging. Phys Rev Lett 75:1934–1937, 1995.
Shattuck MD, Behringer RP, Johnson GA, Georgiadis JG. Convection and flow in porous media, Part 1, Visualization by magnetic resonance imaging. J Fluid Mech 332:215–246, 1997.
Skeldon AC, Cliffe KA, Riley DS. Grid design for the computation of a hexagon–roll interaction using a finite element method. J Comput Phys 133:18–26, 1997.
Stauffer PH, Auer LH, Rosenberg ND. Compressible gas in porous media: a finite amplitude analysis of natural convection. Int J Heat Mass Transfer 40:1585–1589, 1997.
Steen PH. Pattern selection for finite-amplitude convection states in boxes of porous media. J Fluid Mech 136:219–241, 1983.
Steen PH, Aidun CK. Time-periodic convection in porous media: transition mechanism. J Fluid Mech 196:263–290, 1988.
Storesletten L. Effects of anisotropy on convective flow through porous media. In; Ingham DB, Pop I, eds. Transport Phenomena in Porous Media. Oxford: Pergamon, 1998, pp 261–283.
Strange R, Rees DAS. The effect of fluid inertia on the stability of free convection in a saturated porous medium heated from below. Proceedings of International Conference on Porous Media and their Applications in Science, Engineering and Technology, Kona, Hawaii, 1996, pp 71–84.

Straughan B, Walker DW. Anisotropic porous penetrative convection. Proc R Soc Lond 452:97–115, 1996a.

Straughan B, Walker DW. Two very accurate and efficient methods for computing eigenvalues and eigenfunctions in porous convection problems. J Comp Phys 127:128–141, 1996b.

Straus JM. Large amplitude convection in porous media. J Fluid Mech 64:51–63, 1974.

Straus JM, Schubert G. Three-dimensional convection in a cubic box of fluid-saturated porous material. J Fluid Mech 91:155–165, 1979.

Straus JM, Schubert G. Modes of finite-amplitude three-dimensional convection in rectangular boxes of fluid-saturated porous material. J Fluid Mech 103:23–32, 1981.

Sutton FM. Onset of convection in a porous channel with throughflow. Phys Fluids 13:1931–1934, 1970.

Tang J, Bau HH. Feedback control stabilization of the no-motion state of a fluid confined in a horizontal porous layer heated from below. J Fluid Mech 257:485–505, 1993.

Tveitreid M. Thermal convection in a horizontal porous layer with internal heat sources. Int J Heat Mass Transfer 20:1045–1050, 1977.

Vadasz P. Flow in rotating porous media. Fluid transport in porous media. In: Duplessis P, ed. Advances in Fluid Mechanics. Southampton: Computational Mechanics Publications, 1997.

Vadasz P. Free convection in rotating porous media. In: Ingham DB, Pop I, eds. Transport Phenomena in Porous Media. Oxford: Pergamon, 1998, pp 285–312.

Vadasz P, Braester C. The effect of imperfectly insulated sidewalls on natural-convection in porous-media. Acta Mech 91:215–233, 1992.

Vadasz P, Braester C, Bear J. The effect of perfectly conducting side walls on natural convection in porous media. Int J Heat Mass Transfer 36:1159–1170, 1993.

Vafai K, Amiri A. Non-Darcian effects in confined forced convective flows. In: Ingham DB, Pop I, eds. Transport Phenomena in Porous Media. Oxford: Pergamon, 1998, pp 313–329.

Vasseur P, Robillard L. The Brinkman model for natural convection in a porous layer: effects of nonuniform thermal gradient. Int J Heat Mass Transfer 36:4199–4206, 1993.

Walker K, Homsy GM. A note on convective instabilities in Boussinesq fluids and porous media. ASME J Heat Transfer 99:338–339, 1977.

Weber JE. Convection in a porous medium with horizontal and vertical temperature gradients. Int J Heat Mass Transfer 17:241–248, 1974.