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Technical Note

The effect of *g*-jitter on free convection near a stagnation point in a porous medium

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1. Introduction

Convective motion in a porous medium has attracted considerable attention from many authors because of its applications in geophysics, oil recovery technique, thermal insulation, engineering, and heat storage systems. The archival publications on this topic are excellently reviewed in the recent books by Nield and Bejan [1], and Ingham and Pop [2].

A well-understood problem in buoyancy-driven convection in porous media is the flow near the stagnation point of a heated cylindrical surface which is embedded in a fluid-saturated porous medium in the presence of a constant downward gravitational field. This problem has been the subject of several numerical and analytical treatments by Pop and Merkin [3], Merkin and Mahmood [4], and Merkin and Pop [5].

In the present paper, we wish to study the effect of periodical gravity modulation on the free convection near the forward stagnation point of a cylindrical surface which is embedded in a porous medium, i.e. if $\mathbf{g}$ denotes this gravity vector modulation, we assume that this is of the type

$$\mathbf{g}(\bar{t}) = g_0\left[1 + a\sin(\pi\bar{\omega}\bar{t})\right]\mathbf{k} \tag{1}$$

where g_0 is the time-averaged value of $\mathbf{g}$ acting along the direction of the unit vector $\mathbf{k}$ pointing vertically downward, $\bar{t}$ is the time, $\bar{\omega}$ is the frequency of the single-harmonic component of oscillation and a is a scaling parameter which gives the magnitude of the forcing term $\sin(\pi\bar{\omega}\bar{t})$ relative to the magnitude of the mean gravity g_0. If $a \ll 1$ then the forcing may be seen as a perturbation of the mean gravity. Since the governing equations of this problem are non-linear, this kind of forcing leads to the phenomenon of streaming, where a time-periodic forcing with zero mean produces a periodic response consisting of a steady-state solution with a non-zero mean and time-dependent fluctuations involving higher harmonics. This streaming field is significant because it can contribute to various transport phenomena, such as heat transfer or the distribution of chemical species in a time-averaged sense.

Previous works by Amin [6], Biringen and Danabasoglu [7], Biringen and Pelter [8], Alexander et al. [9], Farooq and Homsy [10,11], Li [12], and Pan and Li [13] have estimated and calculated the effect of time varying *g*-jitter induced free convection in a Boussinesq viscous (non-porous) and incompressible fluid. Amin [6] has investigated the heat transfer from a sphere immersed in an infinite fluid medium in a zero-gravity environment under the influence of *g*-jitter. Her conclusion is that heat transfer is negligible for high-frequency *g*-jitter but under special circumstances, when the Prandtl number is high enough, low-frequency *g*-jitter may play an important role. Biringen and Danabasoglu [7] have solved the full non-linear, time-dependent Boussinesq equations for *g*-jitter in rectangular cavities. The part of their results is that obtained for non-zero

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Nomenclature

a	amplitude of the modulation
f	reduced stream function
$\mathbf{g}$	acceleration due to gravity vector
g_0	mean acceleration due to gravity
$\mathbf{k}$	unit vector
K	permeability
ℓ	characteristic length
Ra	Rayleigh number
$S(x)$	shape function
t	non-dimensional time
T	fluid temperature
T_w	wall temperature
T_∞	ambient fluid temperature
ΔT	temperature scale
u, v	non-dimensional velocity components along x- and y-axes
U_c	characteristic velocity
x, y	non-dimensional Cartesian coordinates
α	thermal diffusion
β	thermal expansion coefficient
τ	reduced non-dimensional time
θ	non-dimensional temperature
ν	kinematic viscosity
σ	ratio of composite material heat capacity to convective fluid heat capacity
ψ	non-dimensional stream function
ω	non-dimensional frequency of gravitational field oscillations

Superscripts

$'$	differentiation with respect to y
-	dimensional variables

terrestrial gravity and modulation that is perpendicular to the applied temperature gradient. Although in their work the modulation is much larger than the time-averaged gravity, yet their results show the response to consist of a harmonic time-dependent component superposed over steady streaming. The results of Farooq and Homsy [10,11] are complementary to those reported by Biringen and Danabasoglu [7], since by a weakly non-linear calculation, Farooq and Homsy [10,11] were able to explore parametric dependencies that explain physical mechanisms and scalings. Alexander et al. [9] have carried a numerical investigation of the effect of g-jitter on dopant concentration in a modeled crystal growth reactor. They concluded that low-frequency g-jitter can have a significant effect on dopant concentration. Li [12], and Pan and Li [13] reported analytical results of the g-jitter induced flows in microgravity under the influence of a transverse magnetic field for a simple system consisting of two vertical plates held at different temperatures. Results showed that the g-jitter frequency, applied magnetic field and temperature gradients all contribute to affect the convective flow. It was found that the amplitude of the velocity decreases at a rate inversely proportional to the g-jitter frequency and with increase in the applied magnetic field. The induced flow oscillates at the same frequency as the affecting g-jitter, but out of a phase angle. The phase angle is a function of geometry, applied magnetic field, temperature gradient and frequency. While a magnetic field can be applied to suppress oscillatory flows associated with g-jitter, it is more effective in damping frequency flows, but only has a moderate damping effect on the flow induced by high frequency g-jitter.

Undoubtedly, these studies have shed light on the basic nature of g-jitter effects and have provided a thrust to devise useful mechanisms by which the g-jitter induced convective flows may be further produced and used. One of the ideas is the use of a porous matrix as an effective heat transfer augmentation technique for g-jitter induced flows in microgravity. To the authors' best knowledge, only little work has appeared so far on the effect of g-jitter induced flows in a porous medium, even though it is of interest in many areas, including material processing. Malashetty and Padmavathi [14] have studied the stability of a Boussinesq fluid-saturated horizontal porous layer, heated from below, for the case of a time-dependent buoyancy force generated by gravity modulation using the Brinkman–Forchheimer flow model. A method based on small amplitude of the modulation has been used to compute the critical values of the Rayleigh number and wave number. The shift in the critical Rayleigh number has been calculated as a function of frequency and modulation, Prandtl number, porous parameter, and the ratio of the effective viscosity to the viscosity of the fluid. It has been found that the low-frequency g-jitter can have a significant effect on the stability of the system and that the effect of gravity modulation can be used to stabilize the system in case of a porous medium, for a large Prandtl number. In a very recent paper, Rees and Pop [15] have shown how the boundary-layer flow induced by a constant temperature vertical surface embedded in a fluid-saturated porous medium is modified by time-periodic variations in the gravitational acceleration. A small amplitude expansion up to the fourth-order has been used to determine the detailed effect of the g-jitter. It has been shown that the mathematical problem reduces to the classical Cheng–Minkowycz [16] similarity problem with no free parameters, and to a set of non-similar boundary-layer equations which are solved numerically using the Keller-box method. The numerical solutions are complemented by an asymptotic analysis which shows that the g-jitter effect is eventually confined to a thin layer embedded within the main boundary-layer, and becomes weak at increasing distances from the leading edge.

As in the previous paper by Rees and Pop [14], the focus of the present study is to examine the response of a non-linear system, such as one consisting of a flow near the front stagnation point of a cylindrical surface with a constant temperature, to a time-periodic perturbation of the gravitational field as given by Eq. (1).

2. Basic equation

If we assume that the Boussinesq approximation holds, then equations which govern the high Rayleigh number, unsteady free convection (boundary-layer flow) near a stagnation point of a cylindrical surface embedded in a porous medium are

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2}$$

$$\bar{u} = \frac{g_0 K\beta}{\nu}(T - T_\infty)\left[1 + a\sin(\pi\bar{\omega}\bar{t})\right]S(\bar{x}) \tag{3}$$

$$\sigma\frac{\partial T}{\partial \bar{t}} + \bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\frac{\partial^2 T}{\partial \bar{y}^2} \tag{4}$$

where for our model we have

$$S(\bar{x}) = \frac{\bar{x}}{\ell} \tag{5}$$

where ℓ is some length scale. We shall solve Eqs. (2)–(4) subject to the initial and boundary conditions

$$\begin{array}{ll} \bar{t} = 0: & \bar{u} = \bar{v} = 0,\ T = T_\infty \quad \text{for } (\bar{x}, \bar{y}) > 0 \\ \bar{t} > 0: & \bar{v} = 0,\ T = T_{\mathrm{w}} \quad \text{on } \bar{y} = 0, \bar{x} > 0 \\ & \bar{u} \to \infty,\ T \to T_\infty \quad \text{as } \bar{y} \to \infty, \bar{x} > 0 \end{array} \tag{6}$$

Further, we make Eqs. (2)–(6) non-dimensional using the scalings

$$x = \bar{x}/\ell, \quad y = Ra^{1/2}(\bar{y}/\ell), \quad t = (U_{\mathrm{c}}/\sigma\ell)\bar{t}$$

$$u = \bar{u}/U_{\mathrm{c}}, \quad y = Ra^{1/2}(\bar{v}/U_{\mathrm{c}}), \quad \theta = (T - T_\infty)/\Delta T$$

$$\omega = (\sigma\ell/U_{\mathrm{c}})\bar{\omega}, \quad \Delta T = T - T_\infty \tag{7}$$

where U_{c} and Ra are the characteristic velocity and Rayleigh number, which are defined as

$$U_{\mathrm{c}} = \frac{g_0 K\beta\Delta T}{\nu}, \quad Ra = \frac{g_0 K\beta\Delta T\ell}{\nu\alpha} \tag{8}$$

This leads to the equations, on using Eq. (7),

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

$$u = x\left[1 + a\sin(\pi\omega t)\right]\theta \tag{10}$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} \tag{11}$$

We can combine Eqs. (9)–(11) by writing

$$\psi = xf(y, t), \quad \theta = \theta(y, t) \tag{12}$$

where ψ is the stream function defined in the usual way, namely $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ to give the following equations for our g-jitter model of the stagnation point flow as

$$f' = \left[1 + a\sin(\pi\omega t)\right]\theta \tag{13}$$

$$\theta'' + f\theta' = \frac{\partial \theta}{\partial t} \tag{14}$$

where primes denote differentiation with respect to y.

The initial and boundary conditions (6) become

$$\begin{array}{ll} t = 0: & f = \theta = 0 \quad \text{for } y > 0 \\ t > 0: & f = 0, \theta = 1 \quad \text{on } y = 0 \\ & \theta \to 0 \quad \text{as } y \to \infty \end{array} \tag{15}$$

It seems more convenient to rescale t by writing $\tau = \omega t$, so that Eqs. (13) and (14) become

$$f' = \left[1 + a\sin(\pi\tau)\right]\theta \tag{16}$$

$$\theta'' + f\theta' = \omega\frac{\partial \theta}{\partial \tau} \tag{17}$$

subject to the boundary conditions (15).

We characterize our solution in terms of a (non-dimensional) mean rate of heat transfer Q defined as

$$Q = \frac{1}{2}\int_0^2 \theta'(0, \tau)\,\mathrm{d}\tau \tag{18}$$

3. Results and discussion

Eqs. (16) and (17) were solved numerically using the well-known Keller-box method first introduced by Keller and Cebeci [17]. After reduction of these equations to first-order form and the application of central difference approximation based midway between grid points, the resulting non-linear system of algebraic equations were solved using a multi-dimensional Newton–Raphson iteration scheme. In all our results, we used uniform grids with 201 points in the range $0 \leq y \leq 10$, and a constant stepsize 0.05 in the t-direction. As we are interested in determining the ultimate time-periodic state of the flow, we allow the Keller-box code to continue running until a periodic state is attained. This is adjudged to have happened when

$$|\theta'(y = 0, \tau) - \theta'(y = 0, \tau - 2)| < 10^{-6} \tag{19}$$

over the entire period, i.e. from $\tau = 2n - 2$ to $2n$, where n is an integer denoting how many periods of gravity variation have occurred since the beginning of the numerical simulation. At each timestep, convergence is deemed to have taken place when the maximum absolute correction is less than 10^{-6}.

There are two parameters to vary in this problem, the amplitude, a, and frequency, ω, of the g-jitter effect. In Figs. 1–3, we display the surface rate of heat transfer, $\theta'(0, \tau)$, keeping ω fixed at the values 5, 1 and 0.2, respectively, but varying a between 0 and 1 in each figure. We do not allow a to take values above 1 since this is equivalent to having the perceived gravity reverse its direction over part of the g-jitter cycle. When $\omega = 5$, shown in Fig. 1, the effect of increasing a is to give an almost proportional increase in the local response. The peak in the acceleration felt by the system occurs at $t = 0.5$ in this figure, but the peak in the response is just before where the acceleration has reduced back to its mean value. For such a large value of ω, $\partial\theta/\partial\tau$ is quite small (see Eqs. (16) and (17)), and therefore, we would expect the response to lag, quite substantially, behind that of the perceived gravitational acceleration.

In Fig. 2, we display how the surface rate of heat transfer varies when $\omega = 1$, a lower value than that in Fig. 1. Here, the peak in the response is closer to $\tau = 0.5$ where the acceleration reaches its peak. The overall range of the heat transfer response is much greater than for $\omega = 5$. This trend continues in Fig. 3 where we set $\omega = 0.2$, a fairly low value, and one which corresponds to a fairly rapid response to changes in acceleration, as may be seen by the fact the peak rate of heat transfer is now quite close to $\tau = 0.5$. Moreover, we can see that this effect is slightly greater for larger values of a.

In Fig. 4, we have set $a = 0.5$ and considered different values of ω. The trends described in relation to Figs. 1–3 may be seen easily in this figure: the increase in the amplitude of response as ω decreases, and the decreasing lag in the response behind that of the perceived gravitational acceleration. Indeed, when ω reaches the values of 0.01 or below, the heat transfer response is almost exactly in phase with the gravitational acceleration, and therefore, we could assume

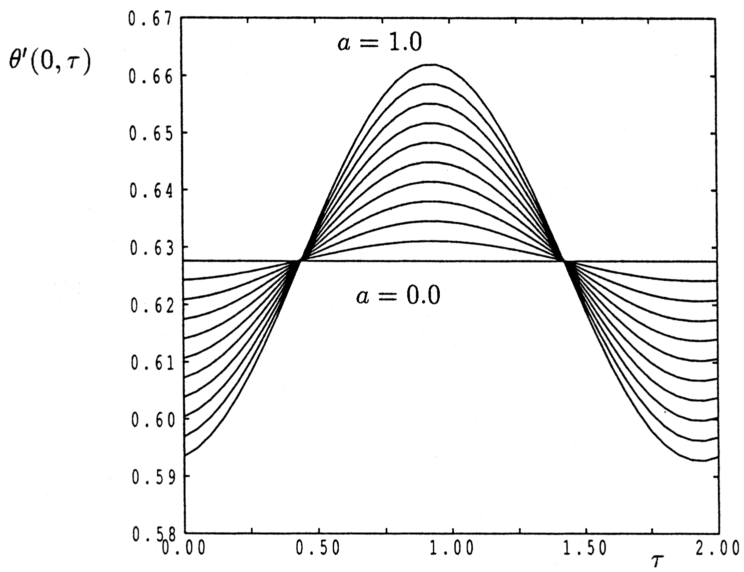

Fig. 1. The rate of heat transfer, $\theta'(0, \tau)$, for $\omega = 5$ with $a = 0.0, 0.1, \ldots, 1.0$. The solutions represent the periodic flow after the decay of transients.

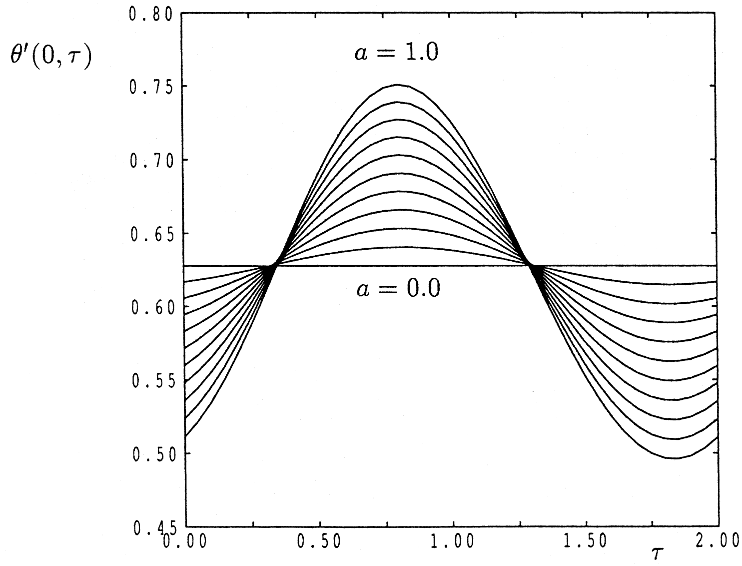

Fig. 2. The rate of heat transfer, $\theta'(0, \tau)$, for $\omega = 1$ with $a = 0.0, 0.1, \ldots, 1.0$. The solutions represent the periodic flow after the decay of transients.

that the flow is quasi-static. On the other hand, for large values of ω, the numerical evidence seems to suggest that the flow is unaffected by the g-jitter effect at leading order since the overall variation in $\theta'(0, \tau)$ as τ varies is very small. Both these observations may be used to devise asymptotic theories for large and small values of ω, we hope to report on this in due course.

We have already mentioned that large values of ω correspond to relatively sluggish responses and large values of ω to fast responses (i.e. quickly decaying transients). In Table 1, we display the value of τ at

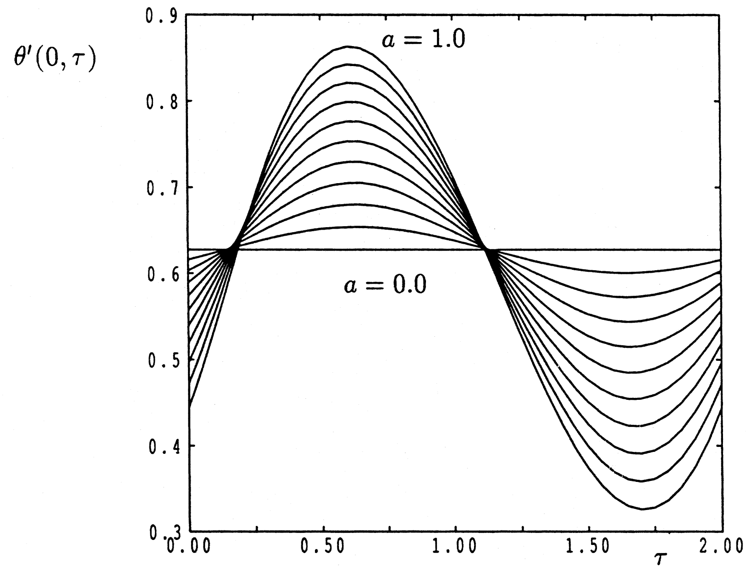

Fig. 3. The rate of heat transfer, $\theta'(0, \tau)$, for $\omega = 0.2$ with $a = 0.0, 0.1, \ldots, 1.0$. The solutions represent the periodic flow after the decay of transients.

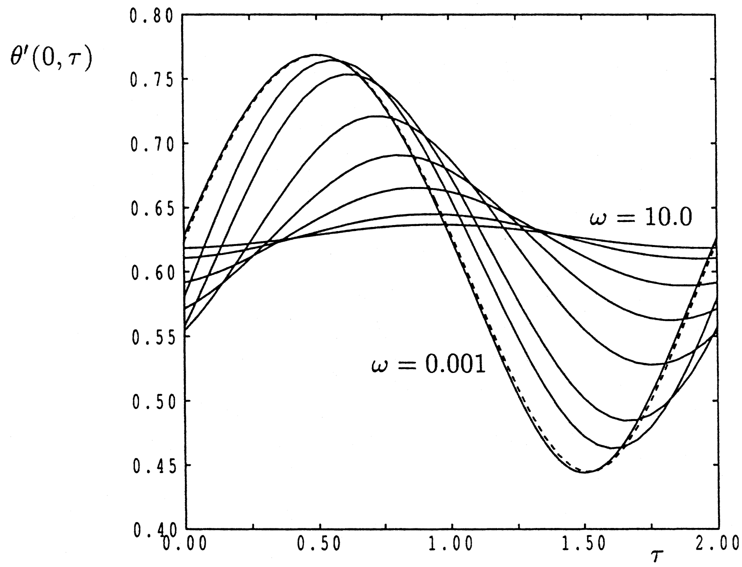

Fig. 4. The rate of heat transfer, $\theta'(0, \tau)$, for $a = 0.5$ with $\omega = 0.001$, 0.01, 0.1, 0.2 0.5, 1.0, 2.0, 5.0 and 10.0. The solution for $\omega = 0.01$ is displayed as a dashed line. The solutions represent the periodic flow after the decay of transients.

which the periodicity convergence criterion given in Eq. (19) was satisfied.

Table 2 displays the values Q which were computed using the Trapezium rule for the data shown in Figs. 1–3.

From Table 2, we see that the mean rate of heat transfer hardly varies at all when ω is relatively large, whereas it displays a distinct decrease in size as a increases when ω is relatively small. Even with the large local responses shown in Fig. 3 when $a = 1$, the mean rate of heat transfer changes only by a factor of only 65 from that given by the uniform $a = 0$ case.

4. Conclusions

We have investigated in this paper how the free convection boundary-layer flow near the forward stagnation point of a cylindrical body embedded in a fluid-saturated porous medium is modified by time-periodic variations in the gravitational acceleration. The transformed equations are solved numerically using the Keller-box method to investigate the parametric dependence of the forcing amplitude a and the forcing frequency ω on the local heat transfer and the mean rate of heat transfer. It was found that for small values of a, the heat transfer response is almost exactly in phase with the gravitational acceleration so that we could assume that the flow is quasi-static. It is also shown that the gravitational modulation is more effective in investigating a transition from conductive to a convective temperature field at higher frequency ω.

Table 1
Values of τ at which the convergence criterion given in Eq. (19) is satisfied

ω	τ
13.20	4
13.21	4
13.22	6
13.23	8
14	14
15	20
16	32
17	44

Table 2
Values of Q for various values of a and ω

a	$\omega = 0.2$	$\omega = 1$	$\omega = 5$
0.0	0.6276	0.6276	0.6276
0.1	0.6270	0.6273	0.6275
0.2	0.6258	0.6267	0.6274
0.3	0.6240	0.6261	0.6273
0.4	0.6215	0.6253	0.6272
0.5	0.6184	0.6243	0.6271
0.6	0.6145	0.6231	0.6269
0.7	0.6100	0.6219	0.6267
0.8	0.6047	0.6204	0.6266
0.9	0.5987	0.6188	0.6264
1.0	0.5918	0.6171	0.6262

The persistance of the velocity and temperature oscillations even when the thermal field is conductive, could be of importance in mass transport processes in the presence of impurities.

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