Pergamon

Int. Comm. Heat Mass Transfer, Vol. 27, No. 3, pp. 415–424, 2000

0735-1933/00/$–see front matter

PII S0735-1933(00)00122-6

THE EFFECT OF G-JITTER ON VERTICAL FREE CONVECTION BOUNDARY-LAYER FLOW IN POROUS MEDIA

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(Communicated by J.P. Hartnett and W.J. Minkowycz)

ABSTRACT

We consider how the boundary-layer flow induced by a constant temperature vertical surface embedded in a porous medium is modified by time-periodic variations in the gravitational acceleration. The amplitude of these variations is assumed to be small compared with the mean acceleration. An amplitude expansion is used to determine the detailed effect of such g-jitter, and the expansion is carried through to fourth order. It is found that the mathematical problem has no free parameters when Darcy-flow is assumed; the resulting nonsimilar boundary-layer equations are solved using the Keller-box technique. The numerical and asymptotic solutions show that the g-jitter effect is eventually confined to a thin layer embedded within the main boundary-layer, but it becomes weak at increasing distances from the leading edge.

Introduction

The subject of thermal convection in porous media has attracted considerable attention in the last three decades and is now considered to be an important field of study in the general areas of fluid mechanics and heat transfer in view of its importance in various engineering applications, such as heat transfer associated with storage of nuclear waste, exothermic reaction in packed-bed reactors, heat removal from nuclear fuel debris, heat recovery from geothermal systems and particularly in the field of large storage systems of agricultural products, to name a few applications. The exhaustive volume of work devoted to this area is amply documented by the most recent books by Nield and Bejan [1], and Ingham and Pop [2].

As it is well accepted, the free convection is driven by buoyancy forces resulting from both temperature gradient and gravity field. Such forces can arise in a number of ways, for example when a system with density gradients is subjected to vibrations. The resulting buoyancy forces, which are produced by the interaction of density gradients with the acceleration field, known as g-jitter, have a complex spatio-temporal structure depending on both the nature of density gradients and the spatial and frequency distribution of the vibration-induced acceleration field. There is a growing literature which tries to characterize the g-jitter environment, and the review articles by Alexander [3] and Nelson [4] give a good summary of the earlier work on the subject. Also papers by Amin [5], Biringen and Peltier [6], Biringen and Danabasoglu [7], Alexander et al. [8], Farooq and Homsy [9,10], Li [11], and Pan and Li [12] investigate the effect of g-jitter on convective fluid motion for a viscous (non-porous) fluid. These studies showed that convection in microgavity is related to the magnitude of g-jitter and to the alignment of the gravity field with respect to the growth or the direction of the temperature gradient. Several numerical models have been used to estimate the adverse effects of time varying g-jitter. The obtained results have been used to study the effects associated with both idealized single- and multiple- frequency g-jitter modulations and realistic g-jitter data calculated by accelerometers during real flight experiments. It was found that the frequency, amplitude and spatial orientation of the residual gravity vector all play an important role in determining the convective flow behaviour of the system.

However, there have been very limited efforts in the study of g-jitter effects on the convective flow in porous media. Malashetty and Padmavathi [13] have recently investigated the stability of free convection in a horizontal porous layer heated from below assuming that the gravitational field has a simple form of the following type:

$$\underline{g} = g_{\mathrm{mean}}\big(1 + \epsilon\cos(\Omega t)\big)\underline{k} \tag{1}$$

where $\underline{g}$ denotes the time-dependent gravitational field, g_{mean} is the mean gravitational field, t is the time, Ω is the frequency of the single-harmonic component of oscillation, ϵ is the amplitude of the modulation and $\underline{k}$ is the unit vector pointing vertically downward. The Brinkman-Forchheimer flow model with effective viscosity larger than the viscosity of the fluid has been considered to give a more general theoretical results. A method based on small amplitude of the modulation has been used to compute the critical values of the Rayleigh number and wave number. It was found that the low-frequency g-jitter can have a significant effect on the stability of the system.

The scope of the present paper is to study the behaviour of the g-jitter induced free convection in the boundary-layer over a vertical flat plate embedded in a fluid-saturated porous medium. Our objective is to develop a basic understanding of the oscillating flow associated with a gravitational field given by Eq. (1) while considering the simple Darcy flow model. We use a small-amplitude modulation, ϵ, by considering a regular perturbation expansion of the solutions in ϵ, up to fourth order, to determine the detailed effect of the g-jitter on the flow and heat transfer characteristics.

Governing Equations and Analysis

The nondimensional equations of motion for two-dimensional convective boundary-layer flow in a porous medium are given by

$$\psi_{yy} = \theta_y, \qquad \theta_t = \theta_{yy} + \psi_x\theta_y - \psi_y\theta_x; \tag{2a, b}$$

where Darcy's law and the Boussinesq approximation are both assumed to be valid. All the variables in (2a) and (2b) have been nondimensionalised as in Riley and Rees [14]. Here, x is the coordinate along the upward-facing heated horizontal surface which is maintained at a nondimensional temperature $\theta = 1$. Far from the heated surface $\theta \longrightarrow 0$ and the nondimensional streamfunction is taken to be zero on the heated surface $y = 0$. The basic steady boundary layer flow was first given by Cheng & Minkowycz [15] in terms of a similarity solution. They found that

$$\psi \sim x^{1/2} f(\eta), \qquad \theta \sim g(\eta), \tag{3}$$

where the similarity variable, η, is defined as $\eta = y/x^{1/2}$.

In this paper we assume that the force due to gravity varies sinusoidally in time but with a small relative amplitude, as given by Eq. 1. Thus Eq. (2a) must be replaced by

$$\psi_{yy} = \big(1 + \epsilon \cos \omega t\big)\theta_y, \tag{4}$$

where ω is a nondimensional frequency, and, given the time-dependent forcing, we set

$$\psi \sim x^{1/2} f(\eta, t), \qquad \theta \sim g(\eta, t), \tag{5}$$

where f and g satisfy the equations

$$f'' = g'\big(1 + \epsilon\cos(\omega t)\big), \qquad g'' + \tfrac{1}{2} f g' = x(f' g_x - f_x g') + x g_t. \tag{6a, b}$$

Here primes denote derivatives with respect to η and the t and x subscripts represent derivatives with respect to those variables. The boundary conditions required to complete the specification of the mathematical problem are that

$$f = 0, \quad g = 1 \quad \text{at } \eta = 0 \qquad \text{and} \qquad f', g \to 0 \quad \text{as} \quad \eta \to \infty. \tag{7}$$

We note that Eq. (6a) may be integrated once with respect to η to reduce the system to third-order.

The solution of Eqs. (6) and (7) may be found by first using the transformations,

$$\tau = \omega t, \qquad \xi = \omega x, \tag{8}$$

which removes ω from the governing equations. Now f is expanded in the following power series:

$$f = f_0 + \epsilon[f_{1c}\cos\tau + f_{1s}\sin\tau] + \epsilon^2[f_2 + f_{2c}\cos 2\tau + f_{2s}\sin 2\tau] \\ + \epsilon^3[f_{3c1}\cos\tau + f_{3s1}\sin\tau + f_{3c3}\cos 3\tau + f_{3s3}\sin 3\tau] + \epsilon^4[f_4 + \ldots] + \ldots, \tag{9}$$

together with one of identical form for g. All the f–terms in (9) are functions of η and ξ only. After substitution of this power series into the governing equations we obtain the following

$$f_0' = g_0 \qquad f_{1c}' = g_{1c} + g_0 \qquad f_{1s}' = g_{1s} \tag{10a, b, c}$$

$$f_2' = g_2 + \tfrac{1}{2}g_{1c} \qquad f_{2c}' = g_{2c} + \tfrac{1}{2}g_{1c} \qquad f_{2s}' = g_{2s} + \tfrac{1}{2}g_{1s} \tag{10d, e, f}$$

$$f_{3c1}' = g_{3c1} + g_2 + \tfrac{1}{2}g_{2c} \qquad f_{3s1}' = g_{3s1} + \tfrac{1}{2}g_{2s} \tag{10g, h}$$

$$f_{3c3}' = g_{3c3} + \tfrac{1}{2}g_{2c} \qquad f_{3s3}' = g_{3s3} + \tfrac{1}{2}g_{2s} \tag{10i, j}$$

$$f_4' = g_4 + \tfrac{1}{2}g_{3c1} \tag{10k}$$

$$g_0'' + \tfrac{1}{2}fg_0' = 0, \tag{11a}$$

$$g_{1c}'' + \mathcal{L}_{1c}^0 + \mathcal{L}_0^{1c} = \xi g_{1s} \tag{11b}$$

$$g_{1s}'' + \mathcal{L}_{1s}^0 + \mathcal{L}_0^{1s} = -\xi g_{1c} \tag{11c}$$

$$g_2'' + \mathcal{L}_2^0 + \tfrac{1}{2}(\mathcal{L}_{1c}^{1c} + \mathcal{L}_{1s}^{1s}) + \mathcal{L}_0^2 = 0, \tag{11d}$$

$$g_{2c}'' + \mathcal{L}_{2c}^0 + \tfrac{1}{2}(\mathcal{L}_{1c}^{1c} - \mathcal{L}_{1s}^{1s}) + \mathcal{L}_0^{2c} = 2\xi g_{2s}, \tag{11e}$$

$$g_{2s}'' + \mathcal{L}_{2s}^0 + \tfrac{1}{2}(\mathcal{L}_{1s}^{1c} + \mathcal{L}_{1c}^{1s}) + \mathcal{L}_0^{2s} = -2\xi g_{2c}, \tag{11f}$$

$$g_{3c1}'' + \mathcal{L}_{3c1}^0 + \tfrac{1}{2}(\mathcal{L}_{2c}^{1c} + \mathcal{L}_{2s}^{1s}) + \mathcal{L}_2^{1c} + \tfrac{1}{2}(\mathcal{L}_{1c}^{2c} + \mathcal{L}_{1s}^{2s}) + \mathcal{L}_{1c}^2 + \mathcal{L}_0^{3c1} = \xi g_{3s1}, \tag{11g}$$

$$g_{3s1}'' + \mathcal{L}_{3s1}^0 + \tfrac{1}{2}(\mathcal{L}_{2s}^{1c} - \mathcal{L}_{2c}^{1s}) + \mathcal{L}_2^{1s} - \tfrac{1}{2}(\mathcal{L}_{1s}^{2c} + \mathcal{L}_{1c}^{2s}) + \mathcal{L}_{1s}^2 + \mathcal{L}_0^{3s1} = -\xi g_{3c1}, \tag{11h}$$

$$g_{3c3}'' + \mathcal{L}_{3c3}^0 + \tfrac{1}{2}(\mathcal{L}_{2c}^{1c} - \mathcal{L}_{2s}^{1s} + \mathcal{L}_{1c}^{2c} - \mathcal{L}_{1s}^{2s}) + \mathcal{L}_0^{3c3} = 3\xi g_{3s3}, \tag{11i}$$

$$g_{3s3}'' + \mathcal{L}_{3s3}^0 + \tfrac{1}{2}(\mathcal{L}_{2s}^{1c} + \mathcal{L}_{2c}^{1s} + \mathcal{L}_{1s}^{2c} + \mathcal{L}_{1c}^{2s}) + \mathcal{L}_0^{3s3} = -3\xi g_{3c3}, \tag{11j}$$

$$g_4'' + \mathcal{L}_4^0 + \tfrac{1}{2}(\mathcal{L}_{3c1}^{1c} + \mathcal{L}_{3s1}^{1s}) + \mathcal{L}_2^2 + \tfrac{1}{2}(\mathcal{L}_{2c}^{2c} + \mathcal{L}_{2s}^{2s} + \mathcal{L}_{1c}^{3c1} + \mathcal{L}_{1s}^{3s1}) + \mathcal{L}_0^4 = 0. \tag{11k}$$

In Eqs. (10) and (11) the notation $\mathcal{L}$ is defined according to

$$\mathcal{L}_b^a = \tfrac{1}{2}f_a g_b' - \xi\Big(f_a'\frac{\partial g_b}{\partial \xi} - \frac{\partial f_a}{\partial \xi}g_b'\Big), \tag{12}$$

and we note that Eq. (11a) may also be written in the form, $g_0'' + \mathcal{L}_0^0 = 0$, although the solutions at $O(1)$ are independent of ξ.

Of most interest to engineers is the time–averaged rate of heat transfer, and therefore the sinusoidal components of the solution for g are not relevant for this purpose. Thus the time–averaged rate of heat transfer may be obtained from

$$\left.\frac{\partial g}{\partial \eta}\right|_{\eta=0} = g_0(0) + \epsilon^2 g_2(0,\xi) + \epsilon^4 g_4(0,\xi) + O(\epsilon^6). \tag{13}$$

From this expression we can determine by how much the g–jitter effect has changed the overall rate of heat transfer. If we define $Q(\xi)$ as a scaled global rate of heat transfer according to

$$Q(\xi) = \frac{\omega^{1/2}}{2}\int_0^\xi \left.\frac{\partial \theta}{\partial y}\right|_{y=0} dx \tag{14a}$$

then it may be shown that

$$Q(\xi) = \frac{1}{2}\int_0^\xi \left(\frac{g_0' + \epsilon^2 g_2' + \epsilon^4 g_4'}{x^{1/2}}\right)_{\eta=0} dx \equiv Q_0(\xi) + \epsilon^2 Q_2(\xi) + \epsilon^4 Q_4(\xi), \tag{14b}$$

where the terms Q_0, Q_2 and Q_4 are defined by comparing like coefficients of ϵ. Clearly $Q_0 = -0.44376x^{1/2}$, and both Q_2 and Q_4 need to be computed.

Numerical Results

The solutions of Eqs. (10a) and (11a) are now very well known, and the solution curves may be found in [15]. The whole system (10) and (11) comprises a set of parabolic partial differential equations which were solved using the Keller–box method [16]. Details of the method may be found in many recent publications, and here we have used the semi–automatic procedure outlined in Rees [17]. All the results quoted here were obtained using uniform grids in both the ξ and η directions; we took 400 intervals to cover both $0 < x < 40$ and $0 < \eta < 20$. We found that the accuracy of the solutions at increasing powers of ϵ became poorer and therefore this level of grid refinement was essential to be able to compute the solutions correctly to at least four significant figures.

As expected, the leading order solutions are independent of ξ and we obtain the leading order rate of surface heat transfer $g_0'(0) = -0.44376$, which is in error only in the fifth decimal place. The variation of the $O(\epsilon)$ rates of heat transfer, $g_{1c}'(0)$ and $g_{1s}'(0)$, are given in Fig. 1. We see quite a marked variation in heat transfer with ξ at positions fairly close to the leading edge, but the heat transfer decays in an oscillatory fashion as ξ increases. This situation is similar to that described by Hossain et al [18] where forced convection with an temporally oscillating free stream was shown to yield solution curves exhibiting decaying spatial oscillations. A detailed examination of the $O(\epsilon)$ profiles (not shown for the sake of brevity) indicates that the main contribution to the temperature field takes

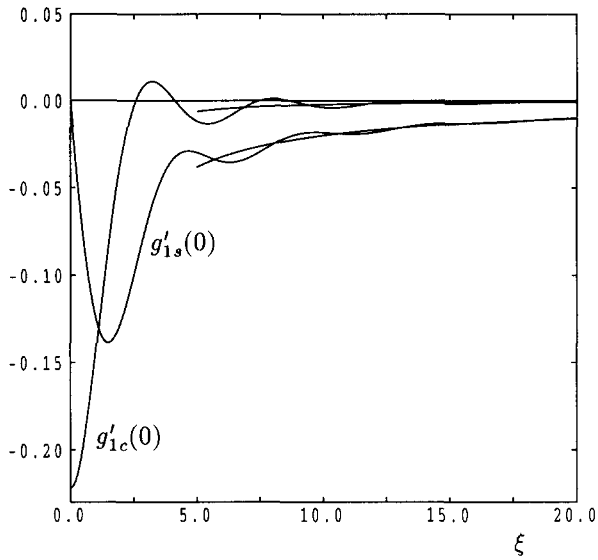

FIG. 1.
The computed $O(\epsilon)$ rates of heat of transfer, $g'_{1c}(0)$ and $g'_{1s}(0)$, as functions of ξ, together with the corresponding asymptotic solutions.

place within a narrow region close to the surface as ξ becomes large; this observation forms the basis of the asymptotic analysis summarised in the next section.

At $O(\epsilon^2)$, and at higher orders, the detailed rate of heat transfer curves exhibit the same type of behaviour in that they display an overall decay with decaying oscillations superimposed. As the functions g_{2c2}, g_{2s2}, and all the $O(\epsilon^3)$ functions represent variations which average out to zero over time (see eq. (9)), we omit presentation of these for the sake of brevity. Therefore, in Figs. 2 and 3, we display the respective variations with ξ of the time-independent terms, $g'_2(0)$ and $g'_4(0)$, as these provide the mean correction to rate of heat transfer in the absence of the g-jitter effect. Once more, these values decay as ξ increases, although the oscillatory component is more complicated for the fourth order term than those shown in Fig. 1 due to the nonlinear interaction of the $O(\epsilon)$ and higher order solutions.

Asymptotic analysis

Here we provide a brief analysis of the behaviour of the $O(\epsilon)$ solutions at large distances from the leading edge. It proves convenient firstly to cast equations (10b), (10c), (11b) and (11c) in complex form by setting $f_1 = f_{1c} + if_{1s}$ and $g_1 = g_{1c} + ig_{1s}$. We obtain

$$f'_1 = g_1 + g_0, \qquad g''_1 + \tfrac{1}{2}(f_0 g'_1 + f_1 g'_0) = \xi(f'_0 g_{1\xi} - f_{1\xi} g'_0) - i\xi g_1, \qquad (15a,b)$$

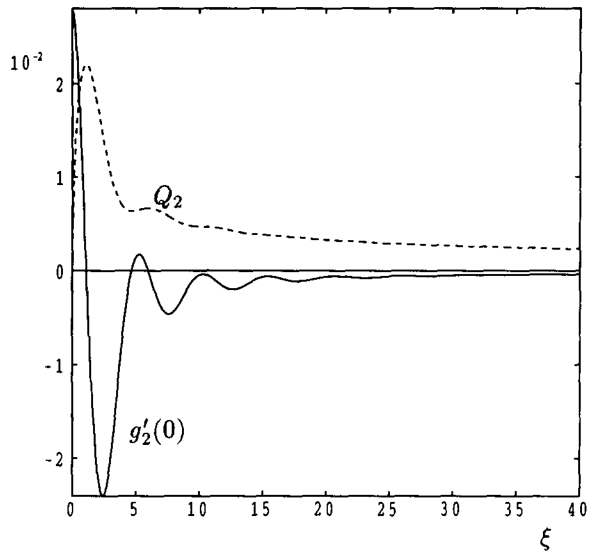

FIG. 2.
The computed $O(\epsilon^2)$ local rate of heat of transfer, $g_2'(0)$ (solid line) and global rate of heat transfer, Q_2 (dashed line), as functions of ξ.

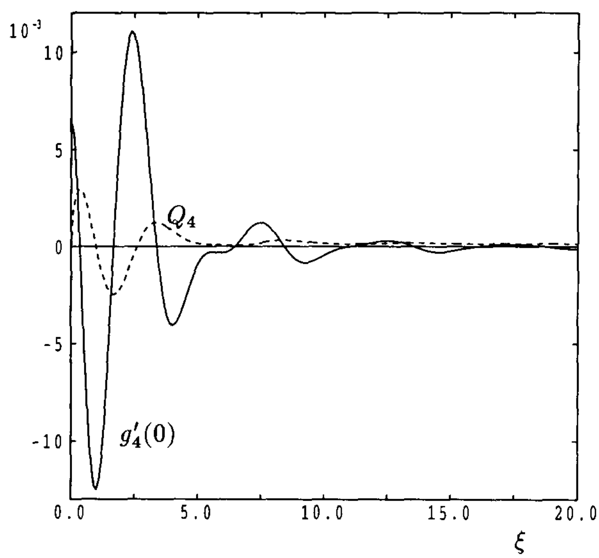

FIG. 3.
The computed $O(\epsilon^4)$ local rate of heat of transfer, $g_4'(0)$ (solid line) and global rate of heat transfer, Q_4 (dashed line), as functions of ξ.

subject to $f_1 = g_1 = 0$ at $\eta = 0$ and $g_1 \to 0$ as $\eta \to \infty$. The presence of a thin near-wall layer reflects the increasing size of the final term in Eq. (15b) and the need to balance its magnitude with the second derivative term. Mathematically this leads to $\eta = O(\xi^{-1/2})$ in the near-wall layer, which, given the definition of η, means that y is the natural variable to use and that the near-wall layer is of uniform thickness. In the outer (i.e. main) and inner (near-wall) layers we expand the solutions using series of the form,

$$f_1 = \sum_{n=0}^{\infty} F_n(\eta)\xi^{-n/2}, \qquad f_1 = \sum_{n=0}^{\infty} \mathcal{F}_n(y)\xi^{-n/2}, \tag{16,17}$$

respectively. Given constraints of space we omit the details of the analysis, which utilises the method of matched asymptotic expansions and which is straightforward though lengthy. We obtain the following solutions for the outer region,

$$\begin{aligned} &F_0 = f_0 \qquad F_1 = 0 \qquad F_2 = i(a - f_0'') \qquad F_3 = 0 \\ &F_4 = \tfrac{1}{2}a[f_0' - 1] - \int_0^\eta f_0''(\zeta)f_0''(\zeta)\,d\zeta \qquad F_5 = \frac{a^2}{2\sqrt{2}}(1+i) \\ &F_6 = i\Big[\tfrac{3}{4}a[f_0' - 1] - f_0''f_0''' + \tfrac{3}{2}f_0'\int_0^\eta f_0''(\zeta)f_0''(\zeta)\,d\zeta - 3\int_0^\eta f_0'(\zeta)f_0''(\zeta)f_0''(\zeta)\,d\zeta\Big] \end{aligned} \tag{18}$$

and for the inner region,

$$\begin{aligned} &\mathcal{F}_0 = 0 \qquad \mathcal{F}_1 = y \qquad \mathcal{F}_2 = \tfrac{1}{2}ay^2 \qquad \mathcal{F}_3 = 0 \qquad \mathcal{F}_4 = \frac{ai}{4}y^2 - \frac{a}{48}y^4 \\ &\mathcal{F}_5 = -\frac{a^2}{2}y + \frac{ia^2}{12}y^3 - \frac{a^2}{240}y^5 + \frac{a^2(1+i)}{2\sqrt{2}}\Big[1 - e^{-(1-i)y/\sqrt{2}}\Big] \\ &\mathcal{F}_6 = -\frac{ia}{32}y^4 + \frac{a}{960}y^6 \end{aligned} \tag{19}$$

In (18) and (19) the constant a is defined as

$$a = -0.44376 = g_0'(0). \tag{20}$$

The surface rate of heat transfer may now be obtained using the solutions given in (19) and we find that

$$\frac{\partial^2 f_1}{\partial \eta^2}\Big|_{\eta=0} = \sum_{n=0}^{\infty} \mathcal{F}_n''\Big|_{y=0}\xi^{-n/2+1} = a + \frac{ai}{2}\xi^{-1} - \frac{a^2(1-i)}{2\sqrt{2}}\xi^{-3/2} + O(\xi^{-5/2}). \tag{21}$$

Given that $g_1'(\eta = 0) = f_1''(\eta = 0) - a$, then we obtain

$$\frac{\partial g_1}{\partial \eta}\Big|_{\eta=0} = \frac{ai}{2}\xi^{-1} - \frac{a^2(1-i)}{2\sqrt{2}}\xi^{-3/2} + O(\xi^{-5/2}). \tag{22}$$

This formula is compared with the numerical solutions shown in Fig. 1 and is found to be in excellent agreement if allowance is made for the decaying oscillations.

It is clear from Fig. 1 that further work should be undertaken to determine how the numerical solution at $O(\epsilon)$ approaches the asymptotic solution. A careful numerical assessment of the difference between the numerical and the asymptotic solutions indicates that the waviness corresponds to the presence of an complex exponential solution of the homogeneous form of the $O(\epsilon)$ equations; it is hoped to report on this aspect in further work.

Discussion

We have considered how vertical free convection in a porous medium is affected by small-amplitude variations in the force of gravity about its mean value. The nonsimilar boundary layer equations were solved to fourth order in ϵ in order to determine the mean corrections to the surface rate of heat transfer. It was found that these corrections are significant relatively close to the leading edge, but that their effect wanes further downstream. An asymptotic analysis was also provided which supplements the $O(\epsilon)$ numerical solution.

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Received February 6, 2000