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The Effect of Time-Periodic Surface Temperature Oscillations on Free Convection from a Vertical Surface in a Porous Medium

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Abstract. In this paper, we consider the unsteady free convection boundary layer flow which is induced by time-periodic variations in the surface temperature of a vertical surface embedded in a porous medium. The basic steady flow is that of a power-law distribution where the surface temperature varies as the nth power of the distance from the leading edge. Small-amplitude time-periodic disturbances are added to this basic distribution. Both the low- and high-frequency limits are considered separately, and these are compared with a full numerical solution obtained by using the Keller-box method. Attention is restricted to the cases $n \leqslant 1$; when $n = 1$, the flow is locally self-similar for any prescribed frequency of modulation.

Key words: unsteady, natural convection, vertical surface.

Nomenclature

A_t	amplitude.
g	acceleration due to gravity.
i	$\sqrt{-1}$.
K	permeability of the porous medium.
L	characteristic length of the plate.
n	surface temperature exponent.
Nu	local Nusselt number.
Ra	Rayleigh number.
Ra_x	local Rayleigh number.
t	nondimensional time.
T	temperature of the fluid.
T_{ref}	reference temperature.
ΔT	temperature difference.
u, v	nondimensional velocity components along x, y-axes.
x	nondimensional vertical coordinate.
y	nondimensional horizontal coordinate.

Greek letters

α	effective thermal diffusivity.
β	coefficient of thermal expansion.

ϵ small constant.
η similarity variable.
ζ transformed coordinate.
θ dimensionless temperature.
ν kinematic viscosity.
ξ frequency variable or parameter.
σ ratio of heat capacity of saturated porous medium to that of fluid.
ϕ phase angle.
ψ nondimensional streamfunction.
ω nondimensional frequency.

Superscripts
$-$ dimensional variables.
$'$ differentiation with respect to either η or ζ.

Subscripts
0 mean steady condition.
w wall condition.
∞ ambient condition.

1. Introduction

Convective heat transfer through porous media has been a subject of great interest for the last three decades. An upsurge in research activities in this field has been accelerated because of a broad range of applications in various disciplines, such as geophysical, thermal and insulation engineering, the modelling of packed sphere beds, the cooling of electronic systems, groundwater hydrology, chemical catalytic reactors, grain storage devices, fiber and granular insulation, petroleum reservoirs, coal combustors, and nuclear waste repositories.

Since the pioneering work of Cheng and Minkowycz (1977) on boundary-layer free convection from a vertical flat plate embedded in a fluid-saturated porous medium this configuration model has been progressively refined to incorporate various boundary conditions, inertial effects, conjugate heat transfer effects, layering, etc. The work of Cheng and Minkowycz (1977) and Johnson and Cheng (1978) were especially noteworthy as they introduced the mathematical technique of boundary-layer theory into the subject and identified similarity solutions of the governing equations. The existence and identification of similarity solutions have been central to a number of further developments, particularly in the examination of free convection resulting from the use of Darcy's law. Several comprehensive reviews and books of the literature pertinent to this area are due to Cheng (1978), Bejan (1987), Tien and Vafai (1990), Nakayama (1995), Kimura *et al.* (1996) and Nield and Bejan (1998).

In unsteady free convection boundary-layer theory of a viscous (clear) fluid along a vertical heated surface, one area of study which has received much attention in the past is the response of the boundary-layer to imposed time-periodic oscillations (Nanda and Sharma, 1963; Merkin, 1967; Kelleher and Yang, 1968;

and Yang *et al.*, 1974). Since non-uniform surface temperature variations are more likely to occur physically than are steady surface temperatures, it is important to determine the extent to which unsteady mean surface temperatures affect the boundary-layer flow. In the present paper, a linearized theory is used to study how the porous medium vertical free convection boundary-layer responds to small-amplitude time-dependent surface temperature oscillations. We use a power-law distribution of surface temperature as the basic steady temperature profile about which we introduce time-dependent perturbations. Detailed numerical solutions are presented for cases with a power-law exponent, $n < 1$; in these cases the flow is non-similar and numerical solutions are obtained using the Keller-box method. These solutions are supplemented by asymptotic solutions for both small and large distances from the leading edge (which are equivalent to low and high frequency limits, respectively). When $n = 1$ the unsteady response is locally self-similar and solutions in this case are obtained from solving the governing ordinary differential equations with the distance from the leading edge (or frequency) as a parameter.

2. Basic Equations

Consider a vertical heated surface with variable wall temperature $T_w(\bar{x}, \bar{t})$ embedded in a fluid-saturated porous medium of uniform ambient temperature T_∞. A rectangular Cartesian coordinate system is chosen with the origin fixed at the leading edge of the surface, such that the $\bar{x}$-axis is directed upwards along the wall and the $\bar{y}$-axis is measured normal to the surface into the porous medium. With the usual boundary-layer and Darcy–Boussinesq approximations, the unsteady thermal and velocity fields adjacent to the surface are described by the following equations (see Ingham *et al.*, 1982):

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u} = \frac{gK\beta}{\nu}(T - T_\infty), \tag{2}$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \alpha\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}, \tag{3}$$

where $\bar{u}$, $\bar{v}$ are the Darcian velocity components along the $\bar{x}$ and $\bar{y}$ direction; g is the acceleration due to gravity; K is the the permeability of the porous medium; ν is the kinematic viscosity; β is the thermal expansion coefficient of the fluid; T is the temperature of the fluid and the porous medium which are in local thermal equilibrium and α is the thermal diffusivity.

The boundary conditions are given as below:

$$\begin{aligned} &t = 0: \quad \bar{v} = 0, \quad T = T_\infty \quad \text{for } \bar{y} \geqslant 0, \\ &y = 0: \quad \bar{v} = 0, \quad T = T_w(\bar{x}, \bar{t}), \\ &y \to \infty: \quad u = 0, \quad T = T_\infty. \end{aligned} \tag{4}$$

Now we introduce the following nondimensional variables:

$$
\begin{aligned}
&t = \left(\frac{\alpha \mathrm{Ra}}{\sigma L^2}\right)\bar{t}, \quad x = \frac{\bar{x}}{L}, \quad y = \frac{\mathrm{Ra}^{1/2}\bar{y}}{L}, \\
&\psi = \frac{\bar{\psi}}{\alpha \mathrm{Ra}^{1/2}}, \quad \theta = \frac{T - T_\infty}{\Delta T},
\end{aligned} \tag{5}
$$

where θ is the nondimensional temperature, and ψ is the nondimensional stream-function which is defined in the usual way, namely, $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, with u and v being the nondimensional velocity components along the x- and y-axes, respectively; $\Delta T = T_{\mathrm{ref}} - T_\infty$ is the temperature difference and $\mathrm{Ra} = gK\beta\Delta T L/\alpha\nu$ is the Rayleigh number for porous medium flows.

Introducing the above transformations into the Equations (1)–(3) we get

$$
\frac{\partial\psi}{\partial y} = \theta, \tag{6}
$$

$$
\frac{\partial^2\psi}{\partial t\partial y} + \frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = \frac{\partial^3\psi}{\partial y^3}. \tag{7}
$$

The corresponding boundary conditions given in (4) then become

$$
\begin{aligned}
&y = 0\colon \quad \psi = 0, \quad \theta(x,t) = \frac{\partial\psi}{\partial y} = \theta_{\mathrm{w}}(x)\mathrm{Re}\left[1 + \epsilon\exp(\mathrm{i}\omega t)\right], \\
&y \to \infty\colon \quad \frac{\partial\psi}{\partial y} \to 0, \quad \theta \to 0,
\end{aligned} \tag{8}
$$

where $\theta_{\mathrm{w}}(x)$ is an as yet unspecified real function of x, ϵ is a small constant and ω is the nondimensional frequency.

Based on the linearized theory for small ϵ (i.e. a Taylor series expansion in ϵ), solutions to Equations (6) and (7) are taken to be of the form

$$
\psi = \psi_0(x,y) + \epsilon\psi_1(x,y)\exp(\mathrm{i}\omega t) + \cdots \tag{9}
$$

where ψ_0 is the steady solution when the plate temperature is maintained at $\theta_{\mathrm{w}}(x)$. When (9) is substituted in Equation (7), and terms of O(1) and O(ϵ) collected together, we obtain

$$
\frac{\partial\psi_0}{\partial y}\frac{\partial^2\psi_0}{\partial x\partial y} - \frac{\partial\psi_0}{\partial x}\frac{\partial^2\psi_0}{\partial y^2} = \frac{\partial^3\psi_0}{\partial y^3} \tag{10}
$$

subject to

$$
\begin{aligned}
&y = 0\colon \quad \psi_0 = 0, \quad \frac{\partial\psi_0}{\partial y} = \theta_{\mathrm{w}}(x) \\
&y \to \infty\colon \quad \frac{\partial\psi_0}{\partial y} \to 0.
\end{aligned} \tag{11}
$$

and

$$\mathrm{i}\omega\frac{\partial\psi_1}{\partial y}+\frac{\partial\psi_0}{\partial y}\frac{\partial^2\psi_1}{\partial x\partial y}-\frac{\partial\psi_0}{\partial x}\frac{\partial^2\psi_1}{\partial y^2}+\frac{\partial\psi_1}{\partial y}\frac{\partial^2\psi_0}{\partial x\partial y}-\frac{\partial\psi_1}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\frac{\partial^3\psi_1}{\partial y^3} \qquad (12)$$

along with

$$\begin{aligned} y=0: &\qquad \psi_1=0, \quad \frac{\partial\psi_1}{\partial y}=\theta_{\mathrm{w}}(x) \\ y\to\infty: &\qquad \frac{\partial\psi_1}{\partial y}\to 0. \end{aligned} \qquad (13)$$

It is known from Cheng and Minkowycz (1977) that similarity solutions for Equation (6) are possible when the surface temperature takes the power-law form:

$$\theta_{\mathrm{w}}(x)=x^n. \qquad (14)$$

The case, $n=0$, corresponds to a uniform surface temperature and $n=1/3$ represents the case of uniform surface heat flux. However, the case, $n=1/3$, does not correspond to uniform heat flux at $\mathrm{O}(\epsilon)$. The leading order similarity solutions are given by

$$\psi_0=2^{1/2}x^{(n+1)/2}F(\eta), \qquad \eta=2^{-1/2}x^{(n-1)/2}y, \qquad (15)$$

where F satisfies the ordinary differential equation

$$F'''+(n+1)FF''-2nF'^2=0, \quad \text{for } n>-1 \qquad (16)$$

with the boundary conditions

$$F(0)=0, \quad F'(0)=1, \quad F'(\infty)=0, \qquad (17)$$

and primes denote differentiation with respect to η. Solutions of Equation (16) satisfying the boundary conditions (17) are entered in Table I for different values of the exponent n.

The transformation (15) suggests that the most convenient form for ψ_1 is

$$\psi_1=2^{1/2}x^{(n+1)/2}f(\xi,\eta), \qquad (18)$$

where

$$\xi=(2\omega)x^{1-n}, \qquad (19)$$

Table I. Values of $-F''(0)$ for different values of n

n	0.0	0.25	0.5	0.75	1.0
$-F''(0)$	0.6277	0.8861	1.0894	1.2618	1.4142

may be regarded either as a scaled frequency variable or as a scaled distance from the leading edge. Thus, when $n < 1$, small values of ξ correspond either to low frequencies or small distances from the leading edge. Substituting (18) and (19) into Equation (12), we obtain

$$f''' + (n+1)Ff'' - \left(\mathrm{i}\xi + 4nF'\right) f' + (n+1)F''f \\ = 2(1-n)\xi \left(F' \frac{\partial f'}{\partial \xi} - F'' \frac{\partial f}{\partial \xi} \right), \tag{20}$$

subject to

$$f(\xi, 0) = 0, \quad f'(\xi, 0) = 1, \quad f'(\xi, \infty) = 0. \tag{21}$$

3. Solution for $n = 1$ at any Prescribed Frequency

In this case, the independent variable ξ in (19) reduces to a constant which depends only on the frequency ω. Thus, Equation (20) reduces to

$$f''' + 2Ff'' - (\mathrm{i}\xi + 4F')f' + 2F''f = 0, \tag{22}$$

subject to

$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0, \tag{23}$$

which is a complex ordinary differential equation with the real independent variable η. Also, for $n = 1$, Equation (16) has the solution

$$F(\eta) = \frac{1}{\sqrt{2}} \left(1 - \mathrm{e}^{-\sqrt{2}\eta}\right), \tag{24}$$

which gives $F''(0) = -\sqrt{2}$ and this agrees precisely with the numerical solution of (12); see Table I.

Equation (22) appears not to have an analytical solution and it has therefore been integrated numerically for a wide range of values of ξ. We note that when ξ is large, the solutions of (18) decay exponentially with leading order behaviour $f' \propto 1 - \exp[-(1+\mathrm{i})2^{-1/2}\xi^{1/2}\eta]$ for fixed values of ξ. Thus, the leading order time-dependent solution is confined to a thinning region near the heated surface as ξ is increased.

4. Solutions for $n < 1$ at any Prescribed Frequency

The partial differential Equation (16) can be solved using the local nonsimilarity methods which have been employed successfully in a similar oscillating flow problem presented by Hossain *et al.* (1998). However, the convergence of this method usually becomes increasingly difficult for larger values of ξ. Therefore, we have integrated Equation (16) for $n = 0$, 1 and 0.5 using the Keller-box method (Keller, 1978), a method that has been shown to be particularly useful for a wide variety of parabolic equations.

4.1. SERIES SOLUTION FOR LOW FREQUENCIES AND $n < 1$

In this case, we assume that f can be described in terms of a power series in ξ of the form

$$f(\xi, \eta) = \sum_{m=0}^{\infty} (2\mathrm{i}\xi)^m f_m(\eta). \tag{25}$$

Substituting (25) into (20) and equating coefficients of like powers of $(2\mathrm{i}\xi)$, we get the following ordinary differential equations:

$$\begin{aligned} &f_0''' + (n+1)Ff_0'' - 4nF'f_0' + (n+1)F''f_0 = 0, \\ &f_0(0) = 0, \quad f_0'(0) = 1, \quad f_0'(\infty) = 0 \end{aligned} \tag{26}$$

and

$$\begin{aligned} &f_m''' + (n+1)Ff_m'' + [2m(n-1) - 4n]F'f_m' + \\ &\quad + [(n+1) - 2m(n-1)]F''f_m = \tfrac{1}{2}f_{m-1}', \\ &f_m(0) = f_m'(0) = f_m'(\infty) = 0, \end{aligned} \tag{27}$$

where $m = 1, 2, 3, \ldots$. Solutions of the above set are obtained using the Runge–Kutta method and terms up to $m = 6$ have been used. The resulting solutions are shown in Figures 1 and 2 in terms of amplitude and phase of rate of the heat transfer (defined in (41), below) at the surface of the plate.

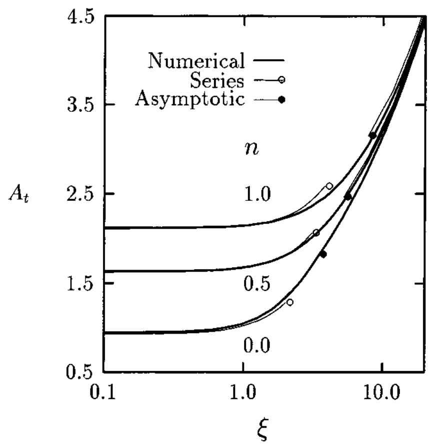

Figure 1. Amplitude of rate of heat transfer for different values of n.

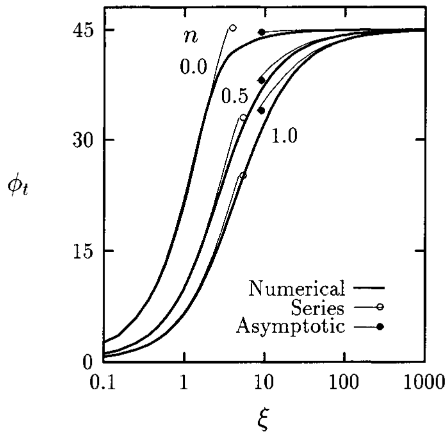

Figure 2. Phase angle of rate of heat transfer for different values of n.

We note that the solution of Equation (26) is given by

$$f_0 = \tfrac{1}{2}(F + \eta F'). \tag{28}$$

When the frequency of oscillation is small ($\xi \ll 1$) the flow is given quite accurately by the quasi-steady-state theory. Indeed, it is straightforward to show that the right-hand side of (28) is precisely the $\mathrm{O}(\delta)$ part of the solution of (16) subject to the boundary conditions, $F(0) = 0$, $F'(0) = 1 + \delta$, $F'(\infty) = 0$, when expanded for small δ.

4.2. ASYMPTOTIC SOLUTION FOR HIGH-FREQUENCY AND $n < 1$

When the frequency of surface-temperature oscillation becomes very high, the boundary-layer response should be confined to a very thin region adjacent to the surface. Indeed, as the frequency approaches infinity, the solution tends to be independent of x similar to the shear-wave solution in the corresponding forced-flow problem for a viscous (clear) fluid, and the thickness of the unsteady region is approximately the same as the thermal penetration depth in a periodically heated solid, that is the thermal equivalent to a Stokes layer. To obtain the asymptotic solution for large frequencies we introduce the transformation

$$f = \xi^{-1/2} G(\xi, \zeta), \quad \zeta = \xi^{1/2}\eta. \tag{29}$$

From (15) and (19) we can see that ζ is proportional to y for any fixed frequency. Equation (16) thus becomes

$$\begin{aligned} &G''' - \mathrm{i}G' + \xi^{-1/2}\left[(n+1)FG'' - 4nF'G' + 2nF''G\right] \\ &\quad = 2(n-1)\left[\xi^{1/2}\left(f'G'_\xi - F''G_\xi\right) + \tfrac{1}{2}\xi^{-1/2}\zeta\left(f'G'' - F''G'\right)\right], \end{aligned} \tag{30}$$

subject to the boundary conditions

$$\begin{aligned} &\zeta = 0: \qquad G = 0, \quad G' = 1, \\ &\zeta \to \infty: \quad G' \to 0, \end{aligned} \tag{31}$$

where primes now denote differentiation with respect to ζ.

Since high frequencies are considered here, only the region immediately next to the surface is affected. Consequently, the function F can be represented in this region as

$$\begin{aligned} F &= \eta + a_1\eta^2 + a_2\eta^3 + a_3\eta^4 + \cdots \\ &= \xi^{-1/2}\zeta + a_1\xi^{-1}\zeta^2 + a_2\xi^{-3/2}\zeta^3 + a_3\xi^{-2}\zeta^4 + \cdots , \end{aligned} \tag{32}$$

where according to Equations (16) and (17), the constants $a_1, a_2, \ldots$ are given by

$$a_1 = (\tfrac{1}{2})F''(0), \quad a_2 = \tfrac{1}{3}n, \quad a_3 = \tfrac{1}{12}(3n-1)F''(0). \tag{33}$$

Based on the presence of $\xi^{-1/2}$ terms in (30) and (32), the solution of Equation (30) may be obtained in the form

$$G = \sum_{m=0}^{\infty} \zeta^{-m/2} G_m(\zeta). \tag{34}$$

On substituting (34) into Equation (30) and collecting terms of like powers of ξ, we obtain

$$\begin{aligned} &G_0''' - \mathrm{i}G_0' = 0, \qquad G_1''' - \mathrm{i}G_1' = 0, \qquad G_2''' - \mathrm{i}G_2' = 4nG_0' - 2n\zeta G_0'', \\ &G_3''' - \mathrm{i}G_3' = (1-3n)a_1\zeta^2 G_0'' - 2(1-5n)a_1\zeta G_0' - 4na_1G_0 - \\ &\qquad\qquad -2n\zeta G_1'' - (1-5n)G_1' \end{aligned} \tag{35}$$

with the boundary conditions

$$\begin{aligned} &G_0(0) = 0,\ G_0'(0) = 1,\ G_0'(\infty) = 0, \\ &G_m(0) = G_m'(0) = G_m'(\infty) = 0, \end{aligned} \tag{36}$$

for $m = 1, 2, 3 \ldots$. The closed-form solutions of these equations are

$$\begin{aligned} G_0' &= \exp\left(-\frac{1+\mathrm{i}}{\sqrt{2}}\zeta\right), \\ G_1' &= 0, \\ G_2' &= -\frac{1}{2}\left[\frac{5n(1-\mathrm{i})}{\sqrt{2}}\zeta + n\zeta^2\right]\exp\left(-\frac{1+\mathrm{i}}{\sqrt{2}}\zeta\right), \\ G_3' &= \frac{a_1}{2}\left[\frac{3(7n-1)\mathrm{i}}{2}\zeta - \frac{(13n-3)(1-\mathrm{i})}{2\sqrt{2}}\zeta^2 - \frac{3n-1}{3}\zeta^3\right] \times \\ &\quad \times \exp\left(-\frac{1+\mathrm{i}}{\sqrt{2}}\zeta\right). \end{aligned} \tag{37}$$

5. Results and Discussion

As previously indicated, numerical solutions for $n = 1$ of Equations (22) and (23) have been obtained for a wide range of the frequency parameter ξ. Also, numerical solutions of Equations (16) and (20) have been obtained for $n = 0$, 0.5.

Once these results are known, it is then a simple matter to determine the response characteristics of the laminar boundary-layer in the present problem. The determination of surface rate of heat transfer is of primary importance and it may be conveniently expressed in terms of the local Nusselt number as

$$\mathrm{Nu} = -\frac{\bar{x}}{\Delta T}\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0} \tag{38}$$

Therefore, we obtain the expressions,

$$\frac{\mathrm{Nu}}{(2\mathrm{Ra}_x)^{1/2}} = -\begin{cases} F''(0) + \epsilon f''(\xi, 0)\exp(\mathrm{i}\omega t)\ n = 1, \quad \text{any} \quad \xi \\[2ex] F''(0) + \epsilon\left[\displaystyle\sum_0^\infty (2\mathrm{i}\xi)^m f''_m(0)\right]\exp(\mathrm{i}\omega t), \quad n < 1, \quad \text{small} \quad \xi \\[2ex] F''(0) - \epsilon\left[\dfrac{1+\mathrm{i}}{\sqrt{2}}\xi^{1/2} + \dfrac{5n(1-\mathrm{i})}{2\sqrt{2}}\xi^{-1/2} - \right. \\ \quad \left. - \dfrac{3(7n-1)\mathrm{i}}{4}a_1\xi^{-1}\right]\exp(\mathrm{i}\omega t), n < 1, \quad \text{large} \quad \xi, \end{cases} \tag{39}$$

where $\mathrm{Ra}_x = g\kappa\beta\Delta T\bar{x}^{n+1}/\alpha\nu$ is the local Rayleigh number.

It is convenient to rewrite expression (39) collectively as follows:

$$\frac{\mathrm{Nu}}{(2\mathrm{Ra}_x)^{1/2}} = -F''(0) + \epsilon\{A_\mathrm{r}(\xi) + \mathrm{i}A_\mathrm{i}(\xi)\}\exp(\mathrm{i}\omega t), \tag{40}$$

where $A_\mathrm{r}(\xi)$ and $A_\mathrm{i}(\xi)$ are respectively the real and imaginary parts of $f''(\xi, 0)$. Thus, the amplitude $A_t(\xi)$ and phase angle $\phi_t(\xi)$ of the rate of heat transfer can easily be calculated from

$$A_t = \sqrt{A_\mathrm{r}^2 + A_\mathrm{i}^2}, \qquad \phi_t = \tan^{-1}\left(\frac{A_\mathrm{i}}{A_\mathrm{r}}\right). \tag{41}$$

Results are presented graphically in Figures 1 and 2. These figures display the results for amplitude and phase obtained for both the low and high frequency analyses. These results compare very favourably with the numerical solutions obtained using the Keller-box method. The results correspond to the following values of the temperature exponent, $n = 0$, 0.5, 1. From Figure 1 we observe that the amplitude of the local Nusselt number increases in the low frequency range as n increases. The amplitude increases to a single asymptote for large ξ for all values of the exponent.

Figure 2 illustrates the variation of the phase angle of the local Nusselt number for the different values of n. It is seen that in the intermediate range of values of ξ, the phase angle of the local Nusselt number decreases due to increases in the value of the temperature exponent n. We further observe that it reaches the asymptotic value $\pi/4$ as the frequency parameter ξ becomes asymptotically large.

6. Conclusions

In this paper, we have determined the heat transfer response of a laminar free convection boundary-layer along a vertical heated surface embedded in a fluid saturated porous medium to time-periodic surface temperature oscillations, when the mean surface temperature varies as a power n of distance from the leading edge. Detailed small-ξ and large-ξ analyses have been described and a full numerical solution presented which covers the flow at intermediate values of ξ. When $n = 1$ the leading order steady state solution is given analytically, whereas, for other values of ξ the self-similar solutions have to be obtained numerically. We can make the following general observations from the detailed results:

1. At intermediate values of ξ the amplitude of the local Nusselt number increases as the surface temperature exponent increases, but the phase of the Nusselt number decreases.
2. There is always a phase lead for all frequencies of oscillation. But the phase lead always increases as ξ increases and approaches the common asymptotic value of $\pi/4$. The amplitude A_t becomes proportional to $\xi^{1/2}$.

Finally, we note that our analysis has been concerned with flow from a semi-infinite surface and the characteristics described here are in some respects unlike those found in cavity flows. Lage and Bejan (1993) discussed the effects of a pulsating heat flux on a sidewall of a square cavity, and found that a resonance effect arises which maximises the heat flux through the central part of the cavity itself. This effect would seem to be dependant on having other bounding surfaces in order to obtain a basic recirculating flow in the absence of pulsations. Such a phenomenon is unlikely to arise in the present problem which describes flow in an infinite domain.

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