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Combined heat and mass transfer in natural convection flow from a vertical wavy surface

M. A. Hossain, Dhaka, Bangladesh, and **D. A. S. Rees,** Bath, United Kingdom

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Summary. In the present paper, effects of combined buoyancy forces from mass and thermal diffusion by natural convection flow from a vertical wavy surface have been investigated using the implicit finite difference method. Here we have focused our attention on the evolution of the surface shear stress, $f''(0)$, rate of heat transfer, $g'(0)$, and surface concentration gradient, $h'(0)$ with effect of different values of the governing parameters, such as the Schmidt number Sc ranging from 7 to 1500 which are appropriate for different species concentration in water (Pr = 7.0), the amplitude of the waviness of the surface ranging from 0.0 to 0.4 and the buoyancy parameter, w, ranging from 0.0 to 1.

Notation

C	species concentration in the boundary layer
C_∞	species concentration of the ambient fluid
C_w	species concentration at the surface
D	chemical molecular diffusivity
f	dimensionless stream function
g	acceleration due to gravity
Gr_x	local modified Grashof number
N	ratio of the buoyancy forces due to the temperature difference and the concentration difference
p	pressure of the fluid
T	temperature of the fluid in the boundary layer
T_∞	temperature of the ambient fluid
T_w	temperature at the surface
u, v	the x- and y-components of the velocity field
x, y	axis in the direction along and normal to the plate

Greek symbols

α	thermal diffusivity
β_T	volumetric coefficient of thermal expansion
β_C	volumetric coefficient of expansion with concentration
Ψ	stream function
η	nondimensional similarity variable
ξ	x/L
ϱ_∞	density of the ambient fluid
ν	kinematic coefficient of viscosity
Ψ	stream function
τ	dimensionless skin friction
μ	fluid viscosity

1 Introduction

In nature and in many engineering applications such as in oceanography, geophysics, metallurgy and chemical engineering there are many transport processes which are governed by the joint action of the buoyancy forces from both thermal and mass diffusion. Representative applications of interest include: solidification of binary alloy and crystal growth, dispersion of dissolved materials or particulate water in flows drying and dehydration operation in chemical and food processing plants, combustion of atomized liquid fuels, cyclone evaporation, drying and flash drying, to name a few. Because of the coupling between the velocity field and the diffusive scalar fields, the double diffusive convection is more complex than the convection associated with a single diffusive scalar, and many different behaviors may be expected.

Simultaneous heat and mass transfer in laminar free convection boundary layer flows for plates with various orientations has been extensively investigated in the past. More information on this subject can be found in the monograph by Gebhart et al. [1] and in the papers by Khair and Bejan [2], Lin and Wu [3], [4], and Mongruel et al. [5]. However, it appears that the effects of combined buoyancy forces from mass and thermal diffusion by mixed convection flows have not been studied yet.

The objective of the present paper is to study the effects of combined buoyancy forces from mass and thermal diffusion by natural convection flow from a vertical wavy surface, since surfaces are sometimes roughened intentionally in order to enhance the heat transfer and mass transfer. Roughened surfaces are encountered in several heat transfer devices such as flat plate solar collectors and flat plate condensers in refrigerators. Large scale surface nonuniformities are encountered in cavity wall insulting systems and grains storage containers. The only papers to date which study the effects of such nonuniformities, strictly on thermal boundary layer flow of Newtonian fluid and in the absence of species concentration are those of Moulic and Yao [6], and Yao [7]. Hossain and Pop [8] investigated the magnetohydrodynamic boundary layer flow and heat transfer from a continuous moving wavy surface, and the problem of free convection from a wavy vertical surface in the presence of a transverse magnetic field has been studied by Hossain et al. [9]. On the other hand, Rees and Pop [10] investigated the free convection induced by a horizontal wavy surface in porous media.

In this investigation the focus is on the boundary layer regime promoted by the combined events near the wavy surface when the surface is at a uniform temperature and a uniform mass diffusion which differ from those of the flowing fluid. The analysis is confined to mass diffusion processes with low concentration levels. The transformed boundary layer equations are solved numerically using the implicit finite difference method with the Keller-box technique (Keller [11]). Consideration is given to the situation where the buoyancy forces assist the natural convection flow for various combinations of the buoyancy ratio parameter ω and Schmidt number Sc with the Prandtl number $\mathrm{Pr} = 7.0$. The results allow us to predict the different behavior that can be observed when the relevant parameters are varied.

2 Formulation of the problem

Consider the steady laminar boundary layer flow of a viscous incompressible fluid along a semi-infinite vertical plate with a wavy surface which is driven only by thermal and solutal buoyancy forces. The surface of the plate can be described by

$$\hat{y} = \hat{\sigma}(\hat{x}) . \tag{1}$$

The characteristic length associated with the wavy surface is L. The temperature of the surface is held uniform at T_w which is higher than the ambient temperature T_∞. The species concentration at the surface is also maintained uniform at C_w which is also higher or lower than the ambient concentration C_∞. The analysis outlined below is of arbitrary $\hat{\sigma}$.

The dimensionless governing equations are the Navier-Stokes equations, the energy equation and the equation for the chemical diffusion of species concentration in two-dimensional Cartesian coordinates $(\hat{x}, \hat{y})$ (see Fig. 1).

Under the usual Boussinesq approximation the flow is governed by the following boundary layer equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mathrm{Gr}^{1/4}\sigma_x \frac{\partial p}{\partial y}\,(1+\sigma_x{}^2)\,\frac{\partial^2 u}{\partial y^2} + \frac{\theta + N\phi}{1+N}, \tag{3}$$

$$\sigma_{xx}u^2 + \sigma_x\left(\frac{\theta + N\phi}{1+N}\right) = \sigma_x \frac{\partial p}{\partial x} - \mathrm{Gr}^{1/4}\,(1+\sigma_x{}^2)\,\frac{\partial p}{\partial y}, \tag{4}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \frac{1}{\mathrm{Pr}}\,(1+\sigma_x{}^2)\,\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

$$u\,\frac{\partial C}{\partial x} + v\,\frac{\partial C}{\partial y} = \frac{1}{\mathrm{Sc}}\,(1+\sigma_x{}^2)\,\frac{\partial^2 T}{\partial y^2}, \tag{6}$$

where

$$x = \frac{\hat{x}}{L}, \qquad y = \frac{\hat{y} - \sigma_x{}^2}{L}\mathrm{Gr}^{1/4}, \qquad p = \frac{L^2}{\varrho\nu^2}\,\mathrm{Gr}^{-3/4}\hat{p}, \qquad u = \frac{L}{\nu}\,\mathrm{Gr}^{1/2}\hat{u},$$
$$v = \frac{L}{\nu}\,\mathrm{Gr}^{1/4}(\hat{v} - \sigma_x\hat{u}), \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \qquad \sigma_x = \frac{d\hat{\sigma}}{d\hat{x}} = \frac{d\sigma}{dx}; \tag{7}$$

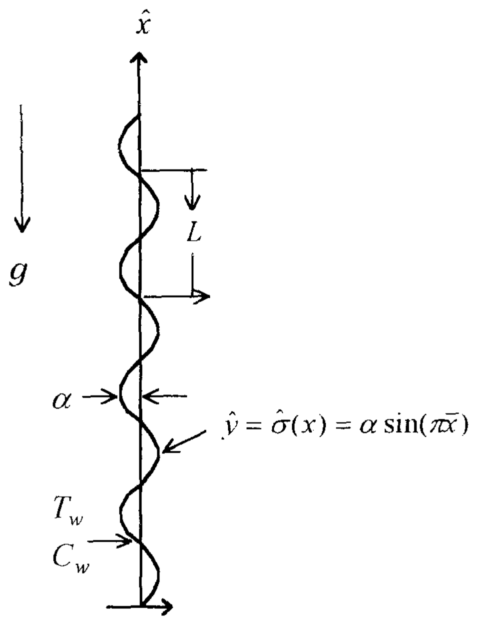

Fig. 1. Physical model and the coordinates

and

$$\mathrm{Gr}_{L,T} = \frac{g\beta_T(T_w - T_\infty)}{\nu^2}, \qquad \mathrm{Gr}_{L,C} = \frac{g\beta_C(C_w - C_\infty)}{\nu^2},$$
$$\mathrm{Gr} = \mathrm{Gr}_{L,T} + \mathrm{Gr}_{L,C}, \qquad N = \frac{\mathrm{Gr}_{L,C}}{\mathrm{Gr}_{L,T}} \tag{8}$$

are, respectively, the Grashof number due to variations in temperature and species concentration, the modified Grashof number, and N measures the relative importance of chemical and thermal diffusion in causing the density difference which derives the flow. We may further observe that N is zero for no species diffusion, infinite for no thermal diffusion, positive for both effects combining to drive the flow and negative for the opposed flow (cooled plate).

The (x, y) are not orthogonal, but a regular rectangular computational grid can be easily fitted in the transformed coordinates. It is also worthwhile to point out that (u, v) are the velocity components parallel to (x, y) which are not parallel to the wavy surface. The convection induced by the wavy surface is described in Eq. (1). Equation (4) indicates that the pressure gradient along the y-direction is $O(\mathrm{Gr}^{-1/4})$, which implies that the lowest order pressure gradient along x-direction can be determined from the inviscid-flow solution. For the present problem this pressure gradient is zero. Equation (4) also shows that $\mathrm{Gr}^{-1/4}\ \partial p/\partial y$ is $O(1)$ and is determined by the left-hand side of the equation. Elimination of $\partial p/\partial y$ between the Eqs. (3) and (4) leads to

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = (1+\sigma_x{}^2)\,\frac{\partial^2 u}{\partial y^2} + \left(\frac{1}{1+\sigma_x{}^2}\right)\frac{\theta + N\phi}{1+N} + \frac{\sigma_x\sigma_{xx}}{(1+\sigma_x{}^2)}\,u^2. \tag{9}$$

Now we introduce the further transformations into the Eqs. (9) and (5), (6) as described below:

$$\Psi = x^{3/4} f(x,\eta), \qquad \eta = x^{-1/4} y, \qquad \theta = \theta(x,\eta), \qquad \phi = \phi(x,\eta) \tag{10}$$

to get

$$(1+\sigma_x{}^2) f''' + \frac{3}{4} f f'' - \left(\frac{1}{2} + \frac{x\sigma_x\sigma_{xx}}{1+\sigma_x{}^2}\right) f'^2 + \left(\frac{1}{1+\sigma_x{}^2}\right)[(1-w)\,\theta + w\phi] = x\left\{f'\,\frac{\partial f'}{\partial x} - f''\,\frac{\partial f}{\partial x}\right\} \tag{11}$$

$$\frac{1}{\mathrm{Pr}}(1+\sigma_x{}^2)\,\theta'' + \frac{3}{4} f\theta' = x\left(f'\,\frac{\partial\theta}{\partial x} - \theta'\,\frac{\partial f}{\partial x}\right), \tag{12}$$

$$\frac{1}{\mathrm{Sc}}(1+\sigma_x{}^2)\phi'' + \frac{3}{4} f\phi' = x\left(f'\,\frac{\partial\phi}{\partial x} - \phi'\,\frac{\partial f}{\partial x}\right). \tag{13}$$

In Eq. (11), w is termed as the conjugate buoyancy parameter and is defined by $w = N/(1+N)$. We also see that for $N = 0$, $w = 0$ and as $N \to \infty$, $w = 1$.

The boundary conditions to be satisfied are

$$f(x,0) = f'(x,0) = 0, \quad \theta(0,x) = \phi(x,0) = 1, \quad f'(x,\infty) = \theta(0,\infty) = \phi(x,\infty) = 0. \tag{14}$$

The quantities of physical interest are surface shear stress, the rate of heat transfer and the rate of transfer of species concentration at the surface which may be obtained in terms of the local skin friction, C_{fx}, the local Nusselt number, $\mathrm{Nu_x}$, and the local Sherwood number, $\mathrm{Sh_x}$, respectively, from the relations given below:

$$C_{fx}\mathrm{Gr}_x{}^{-1/4} = (1+\sigma_x{}^2) f''(0,\xi), \tag{15}$$

$$\mathrm{Nu_x Gr_{x,T}^{-1/4}} = -\sqrt{1+\sigma_x{}^2}\,\theta'(0,\xi), \tag{16}$$

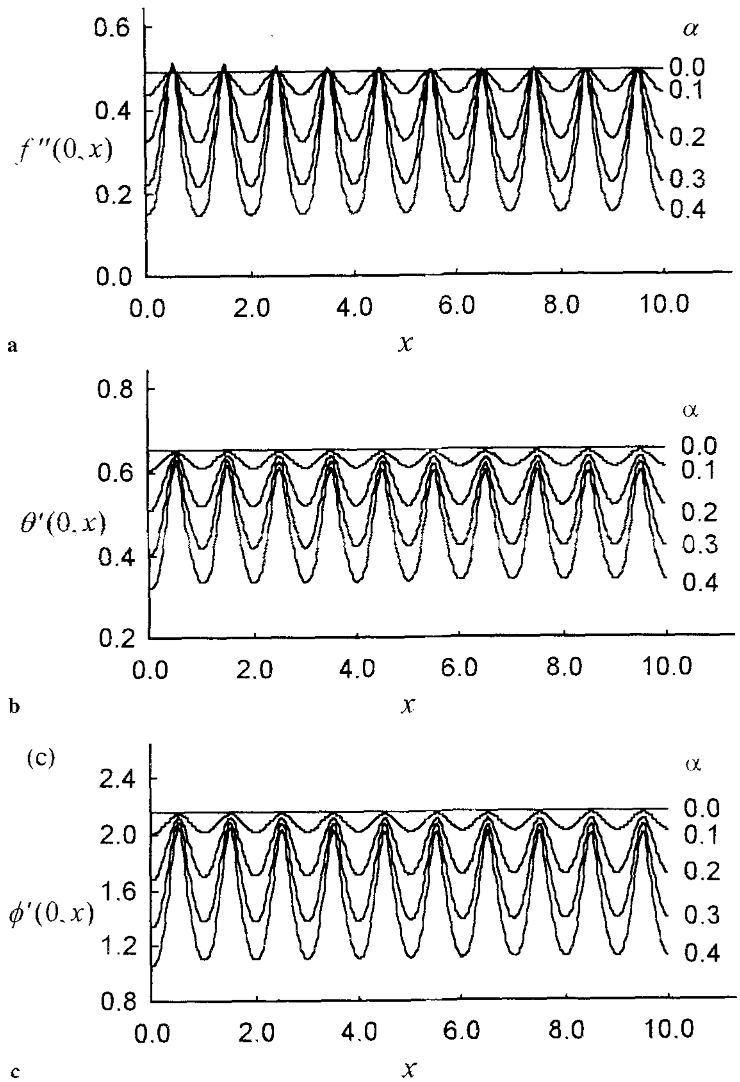

Fig. 2. **a** Skin friction coefficient, **b** Heat transfer coefficient, **c** Mass transfer coefficients for different α while $\mathrm{Sc} = 150$ and $w = 0.5$

and

$$\mathrm{Sh_x Gr_{x,C}^{-1/4}} = -\frac{1}{\mathrm{w}}\sqrt{1+\sigma_x{}^2}\,\phi'(0,\xi). \tag{17}$$

In the present investigation we have considered the form of $\sigma = \alpha \sin(\pi x)$. It should be noted here that, in absence of the species concentration in the flow field (i.e., $N = 0$), solutions of Eqs. (24)–(28) were obtained by Yao [7] considering the form of $\sigma = \alpha \sin(\pi x)$. And for the flat plate ($\alpha = 0$) the problem has been discussed by Gebhard and Pera [1] for different values of the Schmidt number Sc, while $\mathrm{Pr} = 0.71$ and 7.0 which represent air and water at 20°C and 1 atmosphere.

3 Results and discussions

In this Section we describe briefly the numerical results obtained from Eqs. (11)–(14) governing the flow using the implicit finite difference method together with Keller box scheme, which has most recently been applied successfully by Hossain et al. [8], [9].

Given that there are four parameters to vary we have focussed attention on the values $\alpha = 0.2$, $w = 0.5$, $\mathrm{Pr} = 7$ and $\mathrm{Sc} = 7.0$, 50, 150, 500 and 1500, and investigate the effects of separate variations of each parameter in turn. As the equations governing the diffusion of

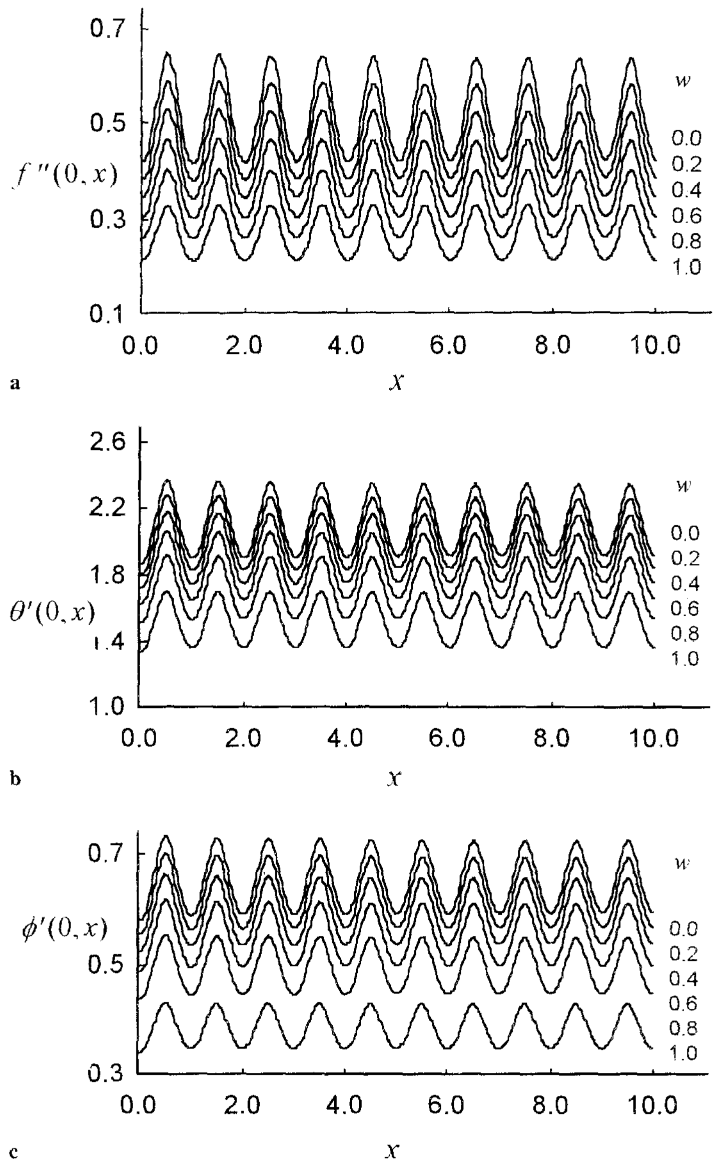

Fig. 3. **a** Skin friction coefficient, **b** Heat transfer coefficient, **c** Mass transfer coefficients for different w while $\mathrm{Sc} = 150$ and $\alpha = 0.2$

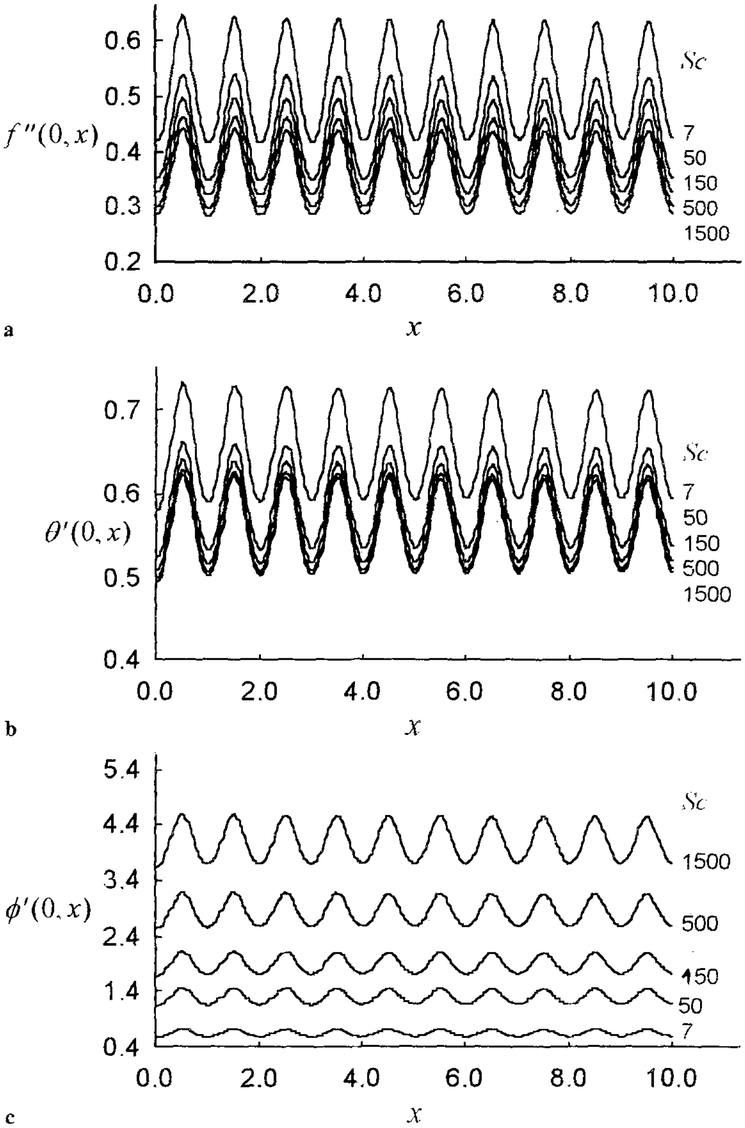

Fig. 4. **a** Skin friction coefficient, **b** Heat transfer coefficient, **c** Mass transfer coefficients for different Sc while $w = 0.5$ and $\alpha = 0.2$

heat and mass are identical in form, we have not sought to vary the Prandtl number, but the chosen value corresponds to that of water.

Figures 2–4 deal, respectively, with variations in α, w and Sc. In each figure the resulting flow is represented by the evolution of the surface shear stress, $f''(0)$, rate of heat transfer, $g'(0)$, and surface concentration gradient, $h'(0)$.

In these figures we see immediately that the flow settles very quickly indeed to a state which is periodic in space. In this regard the flow is very much like that described in Rees and Pop [10] which deals with the convection induced by a wavy surface with uniform heat flux embedded in a porous medium. The asymptotic behavior of the present flow and of that described in Rees and Pop seems to involve all the terms in the nonsimilar governing equations, rather than only some of them, and therefore an asymptotic analysis may not be possible in either case.

Figure 2 shows effect of variations in the surface amplitude, α, for $\mathrm{Pr} = 7$. In general, increasing values of α decrease the local values of f'', $-g'(0)$ and $-h'(0)$. This may be traced to the fact that the buoyancy forces along the surface decrease as the surface is increasingly misaligned with the vertical direction. At such points diffusion plays a greater role and therefore the boundary layer thickens, thereby decreasing the gradients.

Figure 3 shows how variations in w affect the flow. When $w = 0$ the flow is induced entirely by thermal effects and the concentration field is passive, whereas when $w = 1$ the roles are reversed. Given that $\mathrm{Pr} < \mathrm{Sc}$, the respective boundary layers are of substantially different thickness, the thicker being that corresponding to $w = 1$; this is reflected by the fact that the magnitude of the concentration gradient when $w = 1$ is smaller than the magnitude of the temperature gradient when $w = 0$. The increase in the values of f'', $-g'(0,x)$ and $-h'(0,x)$ as w increases cannot be explained with ease since, when $w = 0$, the detailed concentration field is computed as a forced convection problem, the basic flow being a thermally induced free convection boundary layer, whereas when $w = 1$, it is the concentration field which induces the boundary layer flow.

Variations in the Schmidt number, Sc, are considered in Fig. 4. The surface concentration gradient, $h'(0)$, increases with increasing values of Sc, as expected. This is accompanied by a slight decrease in the rate of heat transfer as the near-surface flowfield adjusts slightly to the changing value of Sc. When $\mathrm{Sc} = 7$ (not shown) the values of $g'(0,x)$ and $h'(0,x)$ are identical.

4 Conclusions

We have sought to discover the effect of a wavy surface profile on double diffusive convection from a vertically-aligned uniform temperature surface in an otherwise still fluid. In general, the presence of surface waves serves to thicken the boundary layer and reduce the surface rate of heat transfer, concentration gradient and shear stress. Increasing values of w give rise to increasing values of the surface parameters, although a simple explanation of the reason for this is difficult due to the changing qualitative nature of the flow (by which we mean the change from free to forced convective regimes). Variations in the Schmidt number cause opposite changes in the values of $g'(0,x)$ and $h'(0,x)$; the reason for this is due to the changing near-wall flow field.

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Authors' address: Prof. M. A. Hossain, Department of Mathematics, University of Dhaka, Dhaka-1000, Bangladesh; Dr. D. A. S. Rees, Department of Mechanical Engineering, University of Bath, Bath BA7 2AY, U.K.