# Radiation–Conduction Interaction on Mixed Convection Flow Along a Slender Vertical Cylinder

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## Nomenclature

$a$ = Rosseland mean absorption coefficient
$C_p$ = specific heat at constant pressure
$f$ = dimensionless stream function
$Gr$ = Grashof number
$g$ = acceleration because of gravity
$Nu$ = Nusselt number
$Pr$ = Prandtl number
$Q_w$ = surface heat flux
$R_d$ = Planck number
$r, x$ = radial and axial coordinates
$T$ = temperature of the fluid
$T_w$ = temperature of the heated surface
$T_\infty$ = temperature of the ambient fluid
$u, v$ = velocities in the $x$ and $r$ directions
$\alpha$ = coefficient of thermal diffusivity
$\beta$ = coefficient of cubical expansion
$\zeta$ = scaled streamwise coordinate
$\eta$ = similarity variable
$\theta$ = scaled temperature
$\theta_w$ = surface temperature ratio to the ambient fluid
$\kappa$ = coefficient of thermal conductivity
$\lambda$ = curvature parameter
$\nu$ = kinematic viscosity
$\xi$ = nondimensional axial coordinate
$\rho$ = density of the fluid
$\sigma$ = Stefan–Boltzmann constant
$\sigma_s$ = scattering coefficient

## I. Introduction

SEBAN and Bond[1] were the first to study the axisymmetric boundary-layer flow from a heated vertical cylinder. They considered the case relatively close to the leading edge where the boundary layer is thin relative to the cylinder radius, and solutions were obtained using an approximate series solution method. Chen and Mucoglu[2] extended this work by investigating the corresponding mixed convective flow. Free convection induced by horizontal cylinders and axisymmetric bodies of arbitrary cross sections have been investigated.[3–7] Such studies of convection along or about vertical and horizontal cylinders are important in the fields of geothermal power generation and drilling operations, where the freestream and buoyancy-induced fluid velocities are of roughly the same order of magnitude. In the context of space technology and in processes involving high temperatures, the effects of radiation are of vital importance.

The inclusion of radiation–conduction effects in the energy equation leads to a more highly nonlinear partial differential equation. The majority of studies concerned with the interaction of thermal radiation and natural convection were made by Refs. 8–13 for the case of a vertical semi-infinite plate. Soundalgekar and Takhar[14] studied radiation effects on free convection flow of a gas past a semi-infinite flat plate using the Cogley–Vincentine–Giles equilibrium model (Cogley et al.[15]). Hossain and Takhar[16] analyzed the effect of radiation using the Rosseland diffusion approximation that leads to nonsimilar boundary-layer equations governing the mixed convective flow of an optically dense viscous incompressible fluid past a heated vertical plate with a uniform freestream velocity and surface temperature. Only recently have problems of natural convection–radiation interaction on boundary-layer flow from a cylinder with the Rosseland diffusion approximation been studied.[17,18]

In the present paper we investigate the effect of radiation–conduction interaction in mixed convective flow of a viscous, incompressible and optically dense gray gas along a slender impermeable vertical cylinder. The Rosseland diffusion approximation is assumed here; more detailed analyses would require further modeling of surface radiation (see Özişik[19] for further details). The governing boundary-layer equations form a nonsimilar parabolic system, whose solution is obtained using the well-known Keller-box method. Results are expressed in terms of the local skin friction and the local heat transfer rate for a range of values of the physical parameters: $R_d$, $\theta_w$, and $Pr$, where we restrict attention to unit values of $\lambda$.

## II. Mathematical Formulation

Equations (1–3) represent the steady two-dimensional, laminar mixed convective boundary-layer flow of a viscous incompressible and optically dense gray gas along a long vertical cylinder of radius $r_0$ with a constant freestream velocity $u_\infty$. The surface temperature of the cylinder is maintained at the constant value $T_w$, which is higher than that of $T_\infty$. The governing equations for the present problem are

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + g\beta(T - T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left[\left(\frac{16\sigma T^3}{3\kappa(a + \sigma_s)} + 1\right) r\frac{\partial T}{\partial r}\right] \tag{3}$$

In writing these equations the Boussinesq approximation has been assumed. Radiation effects are considered by using the Rosseland diffusion approximation. The appropriate boundary conditions for the present problem are

$$\begin{aligned} u = v = 0, \quad & T = T_w \quad \text{at} \quad r = r_0 \\ u \to U_\infty, \quad & T \to T_\infty \quad \text{as} \quad r \to \infty \end{aligned} \tag{4}$$

Now we introduce the following group of transformations that are suitable for the mixed convection regime:

$$\psi(x, r) = r_0(\nu U_\infty x)^{1/2}(1 + \xi)^{1/4} f(\xi, \eta)$$
$$\frac{T - T_\infty}{T_w - T_\infty} = \theta(\xi, \eta), \qquad \eta = \frac{r^2 - r_0^2}{2r_0}\left(\frac{U_\infty}{\nu x}\right)^{1/2}(1 + \xi)^{1/4} \tag{5}$$
$$\xi = \frac{g\beta(T_w - T_\infty)x}{U_\infty^2}$$

where $\psi$ is defined in the usual way to allow the equation of continuity [Eq. (1)], to be satisfied. These transformations are motivated by the forms of the free convection and forced convection similarity solutions of the equivalent convection prob-

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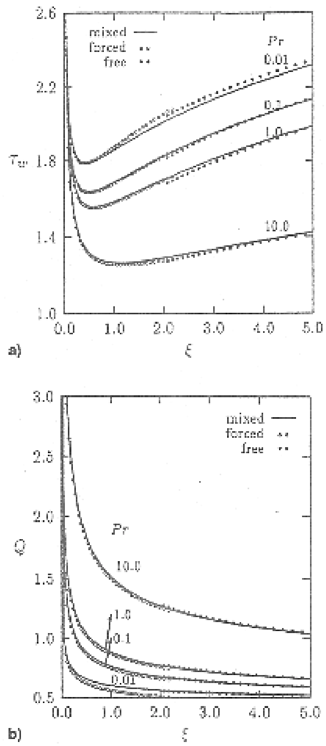


**Fig. 1 Values of a) shear stress and b) rate of heat transfer against $\xi$ for different $Pr$ with $R_d = 0.0$ and $\lambda = 1$.**

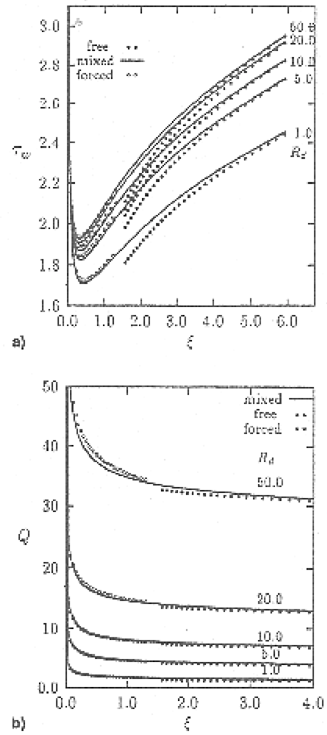


**Fig. 2 Values of a) shear stress and b) rate of heat transfer against $\xi$ for different $R_d$ with $Pr = 0.7$, $\theta_w = 1.1$, and $\lambda = 1$.**

lem over a flat vertical surface. In Eqs. (2) and (3), the substitution of Eq. (5) results in the following equations:

$$\{[1 + \Lambda(\xi)\eta]f''\}' + P_1(\xi)ff'' - P_2(\xi)f'f' + P_3(\xi)\theta = \xi\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right) \quad (6)$$

$$\frac{1}{Pr}\left([1 + \Lambda(\xi)\eta]\left\{1 + \frac{4}{3}R_d\,[1 + (\theta_w - 1)\theta]^3\right\}\theta'\right)' + P_1(\xi)f\theta' = \xi\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right) \quad (7)$$

where

$$\Lambda(\xi) = \frac{2(\xi/\lambda)^{1/2}}{(1+\xi)^{3/4}}, \qquad P_1(\xi) = \frac{2 + 3\xi}{4(1+\xi)}$$
$$P_2(\xi) = \frac{\xi}{2(1+\xi)}, \qquad P_3(\xi) = \frac{\xi}{1+\xi} \quad (8)$$

$\lambda = (Gr_0/Re_0)$ is the curvature parameter, and $Re_0$ and $Gr_0$ are, respectively, the Reynolds and Grashof numbers based on $r_0$ and are defined by

$$Gr_0 = g\beta(T_w - T_\infty)r_0^3/\nu^2, \qquad Re_0 = U_\infty r_0/\nu$$

$Pr$, $R_d$, and $\theta_w$ are defined as follows:

$$Pr = \frac{\nu}{\kappa}, \qquad R_d = \frac{4\sigma T_\infty^3}{\kappa(a + \sigma_s)}, \qquad \theta_w = \frac{T_w}{T_\infty}$$

The corresponding boundary conditions transform into

$$f(\xi, \infty) = f'(\xi, \infty) = 0, \qquad \theta(\xi, 0) = 1$$
$$f(\xi, \eta) \to (1 + \xi)^{-1/2}, \qquad \theta(\xi, \eta) \to 0 \qquad \text{as } \eta \to \infty \quad (9)$$

The nondimensional surface shear stress $\tau_w$ and rate of surface heat transfer $Q$ are

$$\tau_w = (1 + \xi)^{3/4}\xi^{-1/2}f''(\xi, 0) \quad (10)$$

$$Q = -\left(1 + \frac{4}{3}R_d\theta_w^3\right)\frac{(1+\xi)^{1/4}}{\xi^{1/2}}\theta'(\xi, 0) \quad (11)$$

Convection along a flat plate corresponds to $r_0 \to \infty$, or, equivalently, to $\lambda \to \infty$; this flat plate problem has been investigated by Hossain and Takhar.[16]

In the forced convection regime, relatively near the leading edge ($\xi << 1$), the functions given in Eq. (8) take the following form:

$$\Lambda(\xi) \sim 2(\xi/\lambda)^{1/2}, \quad P_1(\xi) \sim \tfrac{1}{2}, \quad P_2(\xi) \sim 0, \quad P_3(\xi) \sim \xi \quad (12)$$

The local values of $\tau_w$ and $Q$ at the surface of the cylinder reduce to the following expressions:

$$\tau_w = \xi^{-1/2}f''(\xi, 0) \quad (13)$$

$$Q = -(1 + \tfrac{4}{3}R_d\theta_w^3)\xi^{1/2}\theta'(\xi, 0) \quad (14)$$

In the free convection-dominated regime, i.e., at large $\xi$, the functions given in Eq. (8) take the following forms:

$$\Lambda(\xi) \sim \frac{2\xi^{1/4}}{\lambda^{1/2}}, \qquad P_1(\xi) \sim \frac{3}{4}, \qquad P_2(\xi) \sim \frac{1}{2}, \qquad P_3(\xi) \sim 1 \tag{15}$$

In this case the local shear stress $\tau_w$ and the local rate of heat transfer $Q$ at the surface of the cylinder take the following forms:

$$\tau_w = \xi^{3/4} f''(\zeta, 0) \tag{16}$$

$$Q = -(1 + \tfrac{4}{3} R_d \theta_w^3)\xi^{-1/4}\theta'(\zeta, 0) \tag{17}$$

where $\zeta = \xi^{1/4}/\lambda^{1/2}$.

Equations (6–8) constitute a system of nonlinear partial differential equations with parameters $Pr$, $\lambda$, $R_d$, and $\theta_w$, which are solved using the Keller-box method.[20] Further details of the computational procedures have been detailed by Hossain et al.[16–18] For the present problem a wide range of numerical results have been derived using these methods, but just a small selection has been presented here.

## III. Results and Discussion

Numerical results for the local skin friction coefficient $\tau$ and the rate of heat transfer coefficient $Q$ have been obtained for representative values of $Pr$, $R_d$, and $\theta_w$. We note that for $CO_2$ in the 100–650°F temperature range (with the corresponding Prandtl number range of 0.76–0.6) and for $NH_3$ vapor in the 120–400°F temperature range (with $Pr$ in the range 0.88–0.84) at 1 atm, the value of $R_d$ ranges from ~10–30. On the other hand, for water vapor in the 220–900°F temperature range (with $Pr$ = 1.0), the $R_d$ values lie between 30 and 200.[9]

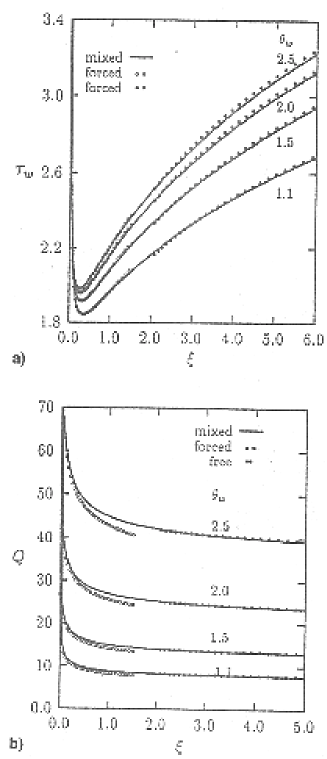


Fig. 3 Values of a) shear stress and b) rate of heat transfer against $\xi$ for different $\theta_w$ with $Pr$ = 0.7, $R_d$ = 10.0 and $\lambda$ = 1.

In Figs. 1–3 the numerical values obtained for $\tau$ and $Q$ in the upstream ($\xi \ll 1$), downstream ($\xi \gg 1$), and intermediate regimes are presented and compared for different values of $Pr$, $R_d$, and $\theta_w$. In these figures $\lambda$ has been assumed to be unity to reduce the number of nondimensional parameters to three. Here the open circles, the filled circles, and the curves represent, respectively, solutions in the upstream, downstream, and intermediate regions. In all cases it can be seen that the large and small $\xi$ asymptotic solutions are in excellent agreement with the Keller-box simulations.

Figure 1 shows how $\tau$ and $Q$ vary with $\xi$ for different values of the Prandtl number in the absence of radiation ($R_d$ = 0). Figure 1 also shows the effect of varying the Prandtl number on the values of $\tau$ and $Q$.

In Fig. 2 we see how $\tau$ and $Q$ are affected by changes in $R_d$, and where the surface temperature parameter is set at $\theta_w$ = 1.1 with $Pr$ = 0.7. It is observed that both $\tau$ and $Q$ increase as $R_d$ increases. The corresponding effect of varying $\theta_w$ with $R_d$ = 10 and $Pr$ = 0.7 are shown in Fig. 3, where the values of $\theta_w$ are taken to be 1.1, 1.5, 2.0, and 2.5. Here it can be seen that an increase in the surface temperature also leads to an increase in the values of the local skin friction and the rate of heat transfer.

## IV. Conclusions

The effect of radiation–conduction interaction on natural convection flow along an isothermal vertical slender cylinder has been investigated by means of a boundary-layer theory. Solutions have been obtained in the small-, large-, and intermediate-$\xi$ regimes. The transformations used to generate the nonsimilar flow are such that the forced and free convection limits follow naturally from the resulting equations. The solutions of the equations in the large- and small-$\xi$ asymptotic limits show excellent agreement with the simulations obtained using the Keller-box method. It is hoped that experimental data will be available in the near future to verify the results of the present investigation.

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