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Convective Wall Plume in Micropolar Fluids

A boundary layer analysis is presented to study the steady-state free convection flow arising from a line thermal source located at the leading edge of a vertical adiabatic surface embedded in a micropolar fluid. Nonsimilar solutions based on the finite difference method are presented for the velocity, angular velocity, and temperature fields. One case is identified where similarity solutions are possible. The results are presented for Prandtl number Pr = 1 *and* 10, *and the micropolar parameters* Δ *ranging from 0.5 to 5 and* $n = 0, 0.5, 1$.

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Notation

C_f skin friction coefficient
C_p specific heat at constant pressure
g acceleration due to gravity
Gr Grashof number
I dimensionless heat input by the thermal source
L reference length
n ratio between microrotation component and fluid shear stress
N dimensionless microrotation velocity
p dimensionless pressure
Pr Prandtl number
Q rate of heat released from the line source
Re Reynolds number
T temperature
T_R reference temperature
T_∞ ambient temperature
u dimensionless velocity in x-direction
U reference velocity
v dimensionless velocity in y-direction
x, y dimensionless coordinate parallel and normal to the plate, respectively
X, Y boundary layer variables

Greek letters

β coefficient of thermal expansion
Δ dimensionless material parameter
ξ, η transformed similarity variables
ϱ density
μ dynamic viscosity
ν kinematic viscosity
θ dimensionless temperature
τ_w skin friction
$\varkappa$ microrotation viscosity
ψ dimensionless stream function

Superscripts

$-$ dimensional variables
$'$ differentiation with respect to η

1. Introduction

The free convection flow of a Newtonian fluid resulting from a horizontal line source of heat placed at the base of a vertical surface was first studied by Zimin and Luikov [1] by means of a similarity hypothesis. A good review of this work was provided by Ingham and Pop [2]. This configuration, also referred to as the wall plume, is a mathematical model of considerable interest in several engineering applications such as hot-wire anemometry, flows that arise in fire studies, and the cooling of electronic circuits. A problem frequently encountered in electrical circuitory is that of devices mounted on an unheated surface which dissipates heat energy at a constant rate. The removal of this energy is often only by natural convection, and it is important to determine the nature of heat transfer and of the resulting flow. These considerations relate to the arrangement of electric components and circuit boards for the effective removal of the dissipation heat energy. Since much of the restriction in the close packing of circuitory is due to the heat transfer considerations, it is important to determine the downstream effects of a heated body located on an unheated surface. Similar considerations are also important in several manufacturing processes in which local heating gives rise to a constant thermal source on an unheated surface. Jaluria [3] and Joshi [4] indicated that such flows are also encountered in the boundary layer regimes in transport enclosures.

Jaluria and Gebhart [5] presented numerical solutions for the problem of a convective wall plume for a wide range of Prandtl numbers, varying from 0.01 to 100. Afzal [6] derived higher order solutions of this problem based upon the method of matched asymptotic expansions. Pop et al. [7–8] recently analyzed the convective wall plume in power-law fluids.

The classical Navier-Stokes theory does not describe adequately the flow properties of either polymeric fluids or certain naturally occurring fluids such as animal blood. The theory of micropolar fluids, first proposed by Eringen [9], is capable of describing such fluids. In this theory, the local effects arising from the microstructure and the intrinsic motion of the fluid elements are taken into account. Micropolar fluids consist of a suspension of small, rigid, cylindrical elements such as large dumbbell-shaped molecules. Eringen [10] has also developed the theory of thermo-micropolar fluids by extending the theory of micropolar fluids. This theory has generated a considerable amount of interest and many problems have also been studied (see Pop and Na [11]).

The present research was undertaken in order to investigate the free convection flow of a micropolar fluid arising from a line thermal source positioned at the leading edge of a vertical adiabatic surface for large values of the generalized Grashof number, employing the boundary layer approximation. The boundary layer equations are transformed into nonsimilar form, and numerical solutions are presented which illustrate the effects of the varying material parameters of the fluid on the velocity and temperature fields. The particular solution for a Newtonian fluid is compared with the published results and to the best of our knowledge this problem has not been reported prior to the present analysis.

2. Governing equations

Consider the problem of steady, laminar, free convection from a line source of heat positioned at the leading edge of a vertical adiabatic surface immersed in an unbounded micropolar fluid. We choose $(\bar{x}, \bar{y})$ as Cartesian coordinates, with the $\bar{x}$-axis measured along the wall in the upward direction and the $\bar{y}$-axis normal to it. The temperature T_∞ of the ambient fluid is assumed to be a constant. The basic equations in non-dimensional form can be written under the Boussinesq approximation as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{1}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mathrm{Gr}^{-1}(1+\varDelta)\left\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right\} + \varDelta\,\mathrm{Gr}^{-1}\,\frac{\partial N}{\partial y} + \mathrm{Gr}^{-1}\,\theta\,, \tag{2}$$

$$u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \mathrm{Gr}^{-1}(1+\varDelta)\left\{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right\} - \varDelta\,\mathrm{Gr}^{-1}\,\frac{\partial N}{\partial x}\,, \tag{3}$$

$$u\,\frac{\partial N}{\partial x} + v\,\frac{\partial N}{\partial y} = -2\varDelta\,\mathrm{Gr}^{-1}\,N + \varDelta\,\mathrm{Gr}^{-1}\left\{\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right\} + \mathrm{Gr}^{-1}\left\{1 + \frac{\varDelta}{2}\right\}\left\{\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right\}, \tag{4}$$

$$u\,\frac{\partial \theta}{\partial x} + v\,\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\,\mathrm{Gr}^{-1}\left\{\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right\}. \tag{5}$$

The boundary conditions for these equations are given by

$$\begin{aligned} &y = 0\colon && u = v = 0\,, && N = -n\,\frac{\partial u}{\partial y}\,, && \frac{\partial \theta}{\partial y} = 0\,,\\ &y \to \infty\colon && u \to 0\,, && N \to 0\,, && \theta \to 0\,. \end{aligned} \tag{6}$$

To these boundary conditions we have to add the condition which expresses the fact that there is a uniform heat flux from the line source, which can be written in non-dimensional form as

$$\int\limits_0^\infty u\theta\,\mathrm{d}y = I\,, \tag{7}$$

where

$$I = \frac{Q}{\delta C_p U T_R L}\,, \qquad U = \frac{g\beta T_R L^2}{\nu}\,, \qquad \mathrm{Gr} = \frac{g\beta T_R L^3}{\nu^2}\,.$$

The non-dimensional variables are defined as

$$\begin{aligned} &x = \bar{x}/L\,, \quad y = \bar{y}/L\,, \quad u = \bar{u}/U\,, \quad v = \bar{v}/U\,, \quad p = \bar{p}/\varrho U^2\,,\\ &N = \bar{N}/(U/L)\,, \quad \theta = (T - T_\infty)/T_R\,, \quad \text{and} \quad \varDelta = k/\mu\,, \end{aligned}$$

and we now introduce the boundary layer variables

$$x = \mathrm{Gr}\,X\,, \qquad y = Y\,, \qquad u = \frac{\partial \psi}{\partial y}\,, \qquad v = -\mathrm{Gr}^{-1}\,\frac{\partial \psi}{\partial x}\,, \qquad p = \mathrm{Gr}^{-1}\,P\,. \tag{8}$$

Substituting (8) in (2)–(5) and letting $\mathrm{Gr} \to \infty$, under the boundary layer approximation, we obtain the following:

$$\frac{\partial \psi}{\partial Y}\,\frac{\partial^2 \psi}{\partial X\,\partial Y} - \frac{\partial \psi}{\partial X}\,\frac{\partial^2 \psi}{\partial Y^2} = (1+\varDelta)\,\frac{\partial^3 \psi}{\partial Y^3} + \varDelta\,\frac{\partial N}{\partial Y} + \theta\,, \tag{9}$$

$$\frac{\partial P}{\partial Y} = 0\,, \tag{10}$$

$$\frac{\partial \psi}{\partial Y}\,\frac{\partial N}{\partial X} - \frac{\partial \psi}{\partial X}\,\frac{\partial N}{\partial Y} = \left\{1 + \frac{\varDelta}{2}\right\}\frac{\partial^2 N}{\partial Y^2} - \varDelta\left\{2N + \frac{\partial^2 \psi}{\partial Y^2}\right\}, \tag{11}$$

$$\frac{\partial \psi}{\partial Y}\,\frac{\partial \theta}{\partial X} - \frac{\partial \psi}{\partial X}\,\frac{\partial \theta}{\partial Y} = \frac{1}{\mathrm{Pr}}\,\frac{\partial^2 \theta}{\partial Y^2}\,, \tag{12}$$

and the boundary conditions (6) are now given by

$$
\begin{aligned}
&Y = 0: \qquad \psi = 0\,, \qquad \frac{\partial \psi}{\partial Y} = 0\,, \qquad N = -n\,\frac{\partial^2 \psi}{\partial Y^2}\,, \qquad \frac{\partial \theta}{\partial Y} = 0\,,\\
&Y \to \infty: \quad \frac{\partial \psi}{\partial Y} = 0\,, \qquad N = 0\,, \qquad \theta = 0\,.
\end{aligned}
\tag{13}
$$

Also, the integral constraint (7) reduces to

$$
\int\limits_0^\infty \frac{\partial \psi}{\partial Y}\,\theta\;\mathrm{d}Y = 1\,. \tag{14}
$$

We now transform Eqs. (9)–(14) to a form amenable for numerical integration, and to do this we define the following transformations:

$$
\begin{aligned}
&\psi(X, Y) = X^{3/5} F(\xi, \eta)\,, \qquad N(X, Y) = X^{-1/5} G(\xi, \eta)\,,\\
&\theta(X, Y) = X^{-3/5} H(\xi, \eta)\,, \qquad \eta = Y/X^{2/5}\,, \qquad \xi = X^{4/5}\,.
\end{aligned}
\tag{15}
$$

The system of equations (9) to (12) can now be written in the following form:

$$
(1 + \Delta)\,F''' + \frac{3}{5}\,FF'' - \frac{1}{5}\,F'^2 + \Delta G' + H = \frac{4}{5}\,\xi\left\{F'\,\frac{\partial F'}{\partial \xi} - F''\,\frac{\partial F}{\partial \xi}\right\}, \tag{16}
$$

$$
\left(1 + \frac{\Delta}{2}\right) G'' + \frac{3}{5}\,FG' + \frac{1}{5}\,F'G - \Delta\xi(2G + F'') = \frac{4}{5}\,\xi\left\{F'\,\frac{\partial G}{\partial \xi} - G'\,\frac{\partial F}{\partial \xi}\right\}, \tag{17}
$$

$$
\frac{1}{\mathrm{Pr}}\,H'' + \frac{3}{5}\,FH' + \frac{3}{5}\,F'H = \frac{4}{5}\,\xi\left\{F'\,\frac{\partial H}{\partial \xi} - H'\,\frac{\partial F}{\partial \xi}\right\} \tag{18}
$$

subject to the boundary conditions given by

$$
\begin{aligned}
&F(\xi, 0) = 0\,, \qquad F'(\xi, 0) = 0\,, \qquad H'(\xi, 0) = 0\,, \qquad G(\xi, 0) = -nF''(\xi, 0)\,,\\
&F'(\xi, \infty) \to 0\,, \qquad H(\xi, \infty) \to 0\,, \qquad G(\xi, \infty) \to 0\,,
\end{aligned}
\tag{19}
$$

and the heat flux relation becomes

$$
\int\limits_0^\infty F'H\;\mathrm{d}\eta = 1\,, \tag{20}
$$

where the primes denote differentiation with respect to η.

It is worth mentioning that when $n = 0$, we obtain $G(\xi, 0) = 0$, which represents the case of concentrated particle flow in which the microelements close to the wall are not able to rotate. The case corresponding to $n = 1/2$ results in the vanishing of the antisymmetric part of the stress tensor and represents weak concentrations. Ahmadi [12] suggested that the particle spin is equal to the fluid vorticity at the boundary for fine particle suspensions. The case corresponding to $n = 1$ is representative of turbulent boundary layer flows. Thus, for $n = 0$ particles are not free to rotate near the surface, but as n increases to 0.5 and 1, the microrotation term is augmented and induces flow enhancement.

At this stage, it is worthwhile to draw attention to a case for which equations (16)–(20) are satisfied by similarity solutions. This is obtained when $\Delta = 0$, and the resulting equations may be written as

$$
F''' + \frac{3}{5}\,FF'' - \frac{1}{5}\,(F')^2 + H = 0\,, \tag{21}
$$

$$
G'' + \frac{3}{5}\,FG' + \frac{1}{5}\,F'G = 0\,, \tag{22}
$$

$$
\frac{H''}{\mathrm{Pr}} + \frac{3}{5}\,FH' + \frac{3}{5}\,F'H = 0 \tag{23}
$$

with boundary conditions

$$
\begin{aligned}
&F(0) = F'(0) = H'(0) = 0\,, \qquad G(0) = -nF''(0)\,,\\
&F'(\infty) = G(\infty) = H(\infty) = 0\,,
\end{aligned}
\tag{24}
$$

and the heat flux relation

$$
\int\limits_0^\infty F'H\;\mathrm{d}\eta = 1\,. \tag{25}
$$

It is customary to present the flow characteristics by means of the skin friction coefficient

$$C_f = \frac{2\tau_w}{\varrho U^2} = \frac{2}{\mathrm{Re}}\,[1 + \Delta(1-n)]\,\xi^{-1/4} F''(\xi,\,0)\,, \tag{26}$$

where $\mathrm{Re} = \dfrac{UL}{\nu}$ is the Reynolds number and τ_w is the wall skin friction given by

$$\tau_w = \left[\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \varkappa\left(\frac{\partial u}{\partial y} + N\right)\right]_{y=0}. \tag{27}$$

3. Numerical scheme

The numerical scheme to solve the partial differential equations (16)–(18) adopted here is based on a combination of the following concepts:

(a) The boundary conditions for $\eta = \infty$ are replaced by

$$F'(\xi,\,\eta_{\max}) = 0\,, \qquad G(\xi,\,\eta_{\max}) = 0\,, \qquad H(\xi,\,\eta_{\max}) = 0\,, \tag{28}$$

where $\eta_{\max}$ is a sufficiently large value of η, for the boundary conditions (19) to be satisfied. We have set $\eta_{\max} = 10$ in the present work.

(b) The equations were transformed by means of the substitution $\xi^* = \Delta\xi$; this has the advantage that the evolution of the flow from $\xi = 0$ onwards has been virtually completed and the large-ξ asymptotic state has been obtained by $\xi^* = 10$ for all values of Δ.

(c) The two-dimensional domain of interest was discretised using a uniform mesh consisting of 100 intervals lying in the range $0 \le \xi^* \le 10$, and a non-uniform mesh of 58 intervals in the range $0 \le \eta \le 10$ – mesh points were concentrated towards $\eta = 0$.

(d) The equations were then discretised using central differences in both ξ^* and η directions. The resulting nonlinear algebraic equations were solved using a multi-dimensional Newton-Raphson scheme where the Jacobian matrix has block-tridiagonal structure; this is the Keller-box method [13]. The Jacobian matrix, being the Fréchet derivative of the nonlinear system being solved, was computed numerically within the code, rather than being specified explicitly by the programmer.

4. Results and discussion

Typical results for F, F', $F''(\xi,\,0)$, H, $H(\xi,\,0)$, and G are plotted in Figs. 1–6 for different values of the parameters involved, namely Δ (material parameter), n (ratio of microrotation and fluid stress), Pr (Prandtl number), and ξ (streamwise coordinate); these quantities are relevant, to the stream function, vertical velocity, skin friction, fluid temperature, wall temperature, and the microrotation velocity. In practical applications, the determination of the tem-

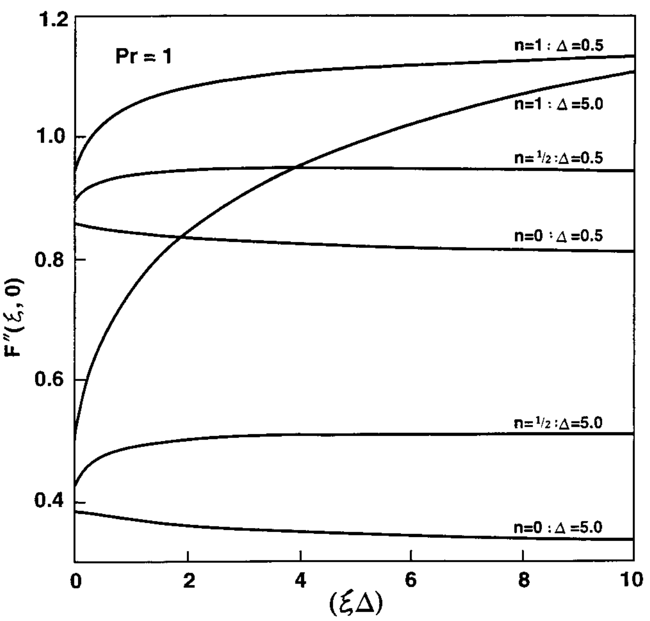

Fig. 1. Variation of the reduced skin friction coefficient versus ($\xi\Delta$)

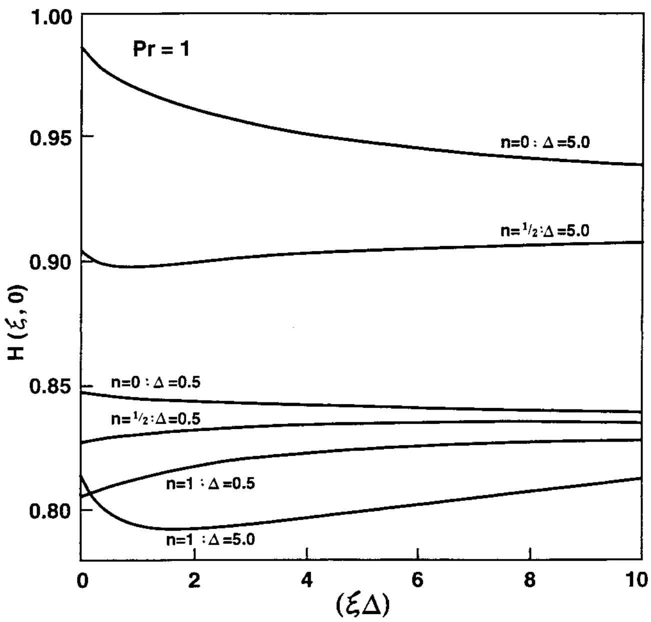

Fig. 2. Variation of the reduced wall temperature versus ($\xi\Delta$)

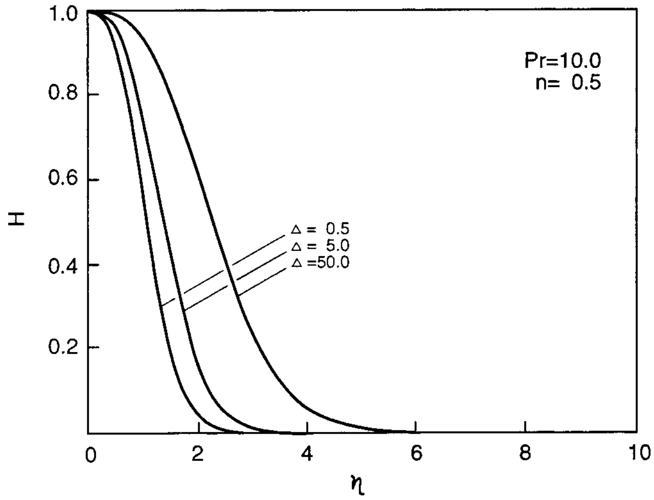

Fig. 3. Velocity profiles versus η

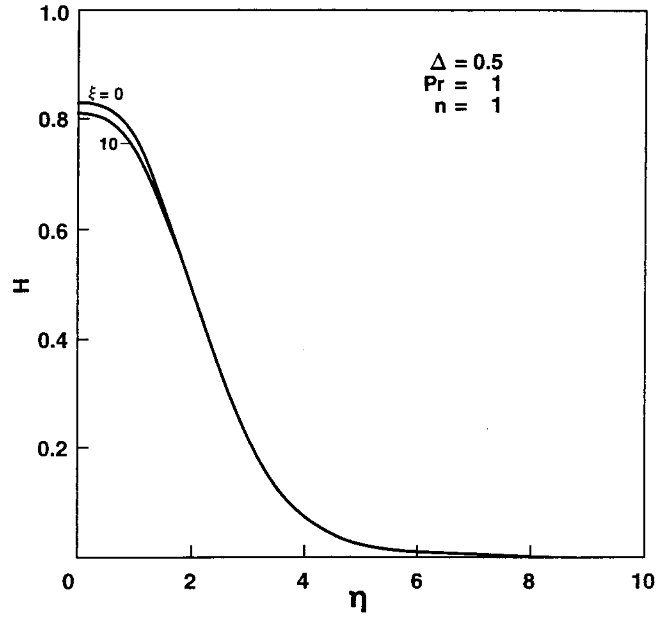

Fig. 4. Temperature profiles versus η

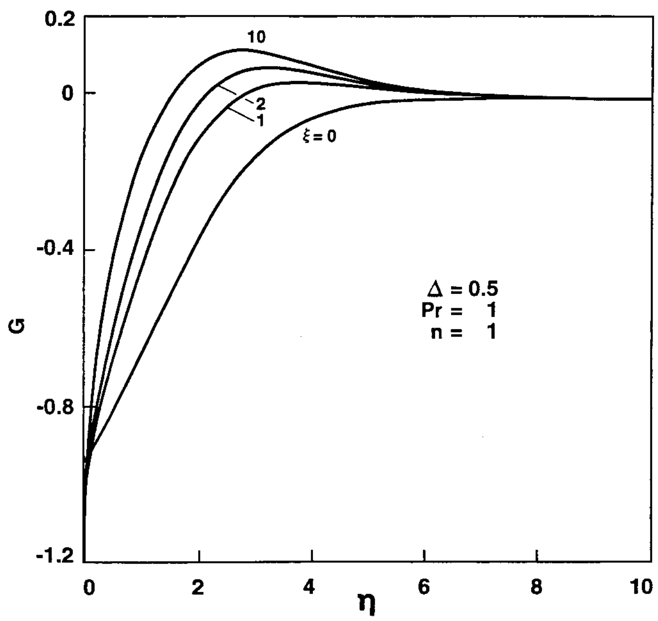

Fig. 5. Microrotation profiles versus η

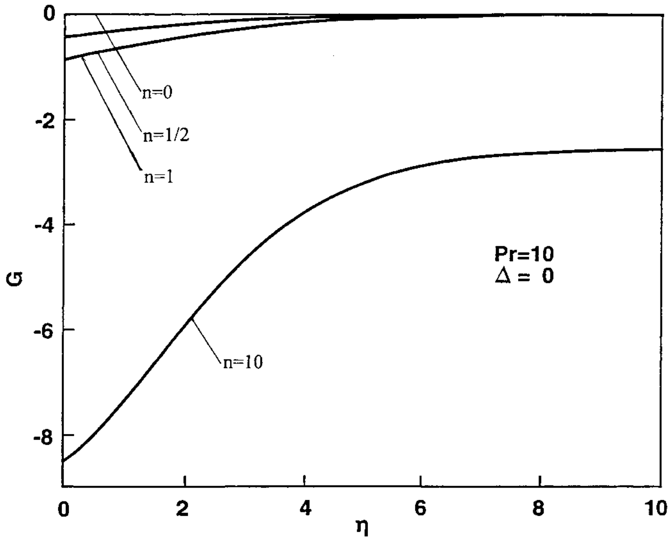

Fig. 6. Microrotation profiles versus η (similarity solution)

perature and flow fields downstream of the heated body is important. In order to assess the accuracy of the present numerical results, we compare the values of the reduced skin friction coefficient for a Newtonian fluid ($\Delta = 0$, $G = 0$), when the problem becomes similar to that obtained by Pop et al. [7]. The present results for $F''(0)$ are 2.62012 for $\mathrm{Pr} = 0.72$ (air) and 1.89965 for $\mathrm{Pr} = 6.7$ (water), while the corresponding values from [7] are 2.62012 for $\mathrm{Pr} = 0.72$ and 1.85964 for $\mathrm{Pr} = 6.7$, respectively. This shows that the agreement is excellent.

It is seen from Figs. 1 and 2, that for the same value of n, $F''(\xi, 0)$ decreases, while $H(\xi, 0)$ increases with increasing values of the material parameter Δ. This demonstrates the drag reducing nature of the micropolar fluids. It is also seen from these figures that the skin friction, $F''(\xi, 0)$, and the temperature at the wall, $H(\xi, 0)$, change significantly with the parameter, n.

Figs. 3 and 4 show that the maximum velocity and temperature profiles decrease with increasing ξ, the upstream distance from the leading edge of the wall. However, there is an overshoot in the microrotation profiles as ξ increases from zero (see Fig. 5). The profiles in Figs. 3 and 4 are found to be quite similar in form to those for natural convection flow arising from a line thermal source on an adiabatic vertical surface embedded in a Newtonian fluid (see Jaluria and Gebhart [5]). However there seems to be very little variation in the velocity and temperature profiles as the streamwise distance ξ increases from zero (similarity solutions) to a large value.

Finally, Fig. 6 displays results for the microrotation velocity profile variable G corresponding to the case $\Delta = 0$ when the problem of free convection wall plume in a micropolar fluid becomes similar. We notice from Eqs. (21)–(24) that the flow and heat transfer characteristics do not depend on the parameter n and are similar to those for a Newtonian fluid. However, the microrotation profiles take negative values and they increase in magnitude with increasing n. This is in agreement with the results reported by Rees and Bassom [14].

5. Concluding remarks

In this paper we have analysed the laminar free convection flow of a micropolar fluid generated by a line thermal source embedded at the leading edge of an adiabatic vertical flat plate. The nonsimilar boundary layer equations are solved numerically using a very efficient finite-difference method known as the Keller-box scheme. A wide selection of numerical results have been presented giving the evolution of the velocity, temperature, and microrotation profiles as well as the skin friction coefficient and the wall temperature. Our analysis makes it possible to derive also a similarity solution of the problem when $\Delta = 0$. The dependence of the present flow problem on the material parameters Δ and n is discussed, and it is shown that these parameters have a very pronounced effect on the flow and heat transfer characteristics. Some of the results presented in this paper were already known from past literature, in particular those concerning the skin friction coefficient in the free convection flow arising from a line thermal source located at the leading edge of a vertical adiabatic surface embedded in a Newtonian fluid ($\Delta = 0$, $n = 0$).

The results of the present study are important in practical applications, such as in the positioning of components dissipating energy on vertical circuit boards embedded in micropolar fluids. Heat transfer and free convective flow considerations are very important in this area and also in several frequently encountered situations in manufacturing processes.

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