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Free convection-radiation interaction from an isothermal plate inclined at a small angle to the horizontal

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Summary. The interaction of free convection with thermal radiation in boundary layer flow from an inclined isothermal plate is studied numerically. Introducing appropriate transformations the equations governing the flow are expressed in the form of local nonsimilarity equations valid near the leading edge as well as in the downstream region. A group of transformations is also introduced such that the flow near the leading edge and far downstream can be described. Heated upward facing plates with positive and negative inclination angles are investigated. When the inclination is negative the boundary layer separates from the surface and the numerical solutions can be extended downstream past the point of separation. From the present investigation it may be concluded that the position of the separation point moves away from the leading edge with the increase of either of the thermal radiation parameter or the surface temperature parameter of the heated surface.

1 Introduction

Heat transfer by natural convection in laminar boundary layer flows has been analyzed extensively for semi-infinite flat plates in vertical, horizontal, and inclined orientations. Typical studies can be found, for example in the works of Ostrach [1], Pera and Gebhart [2], Hasan and Eichhorn [3], and Chen and Tzuoo [4]. On the other hand, heat transfer by simultaneous natural convection and thermal radiation in a participating fluid has not been received much attention; this is required since thermal radiation plays significant role in the overall surface heat transfer in a situation where convective heat transfer coefficients are small, as is the case in natural convection confined to the case of vertical semi-infinite plate (see e.g., Cess [5], Arpaci [6], Cheng and Ozisik [7], Hasegawa et al. [8], Bankston et al. [9], Sparrow and Cess [10], Viskanta and Grosh [11] and Cess [12]). Ali et al. [13] first investigated the boundary layer flow over a semi-infinite horizontal plate considering gray-gas that emit and absorb but do not scatter thermal radiation. Recently, Hossain and Takhar [14] have analyzed the effect of radiation using the Rosseland diffusion approximation which leads to nonsimilarity solutions for the forced and free convection flow of an optically dense viscous incompressible fluid past a heated vertical plate with uniform free stream velocity and surface temperature. Using a group of transformations, the boundary layer equations governing the flow were reduced to local nonsimilarity equations valid both in the forced convection and free convection regimes. The resulting equations were solved using the implicit finite difference method.

On the other hand, very recently, an attempt has been made by Hossain et al. [15] to predict heat transfer in the boundary layer free convection flow of an electrically conducting fluid without thermal radiation effect over an upper surface of a semi-infinite heated plate which is inclined at a small angle $\phi = O(Gr^{-1/5})$ to the horizontal, where Gr is suitably defined Grashof number, in the presence of a uniform magnetic field.

In the present paper it is proposed to investigate the effect of free convection-radiation interaction on the boundary layer flow of an optically dense viscous incompressible fluid along the heated surface which is inclined at a small angle to the horizontal. In fact the problem that was considered by Jones [16] for viscous incompressible fluid in absence of thermal radiation showed that the problem can be formulated in terms of a regular and inverse series expansion of a characterizing coordinate that essentially provides a link between the similarity states appropriate at the leading edge and far downstream. In particular, it was noted that the correct form of expansions required the introduction of logarithmic terms if consistency is to be maintained. Accordingly, an estimate of indeterminacy could be obtained via a reconciliation of the asymptotic expansions and the numerical integration of the governing boundary layer equations using the 'selected points' technique of Lanczos [17], for which solutions were presented by series of Chebychev polynomials. But here we introduce a significant improvement of the numerical solutions employed to solve the problem considered in [16]. The known limiting forms of the boundary layer equations are linked via a continuous transformation in the characterizing coordinate as proposed by Hunt and Wilks [18]. Accordingly, a complete integration is achieved in the context of a single set of equations. This set of equations is solved numerically using the implicit finite difference method together with the Keller box elimination technique of Keller [19]. Both the positive and negative inclinations of the plate are considered. When the inclination of the plate is negative, the numerical integrations are continued through the point at which the boundary layer separation takes place at the surface of the plate.

2 Basic equations

Consider the steady free convection boundary layer flow of a viscous incompressible fluid over a semi-infinite flat plate inclined at an angle ϕ to the horizontal. The surface temperature of the plate is maintained at a uniform temperature T_w which is greater than the ambient fluid temperature, T_∞. The fluid is assumed to be a gray, emitting and absorbing, but non-scattering medium. The physical coordinates $(\bar{x}, \bar{y})$ are chosen such that $\bar{x}$ is measured from the leading edge along the surface of the plate and $\bar{y}$ is measured normal to the surface of the plate. Further assumptions are made:

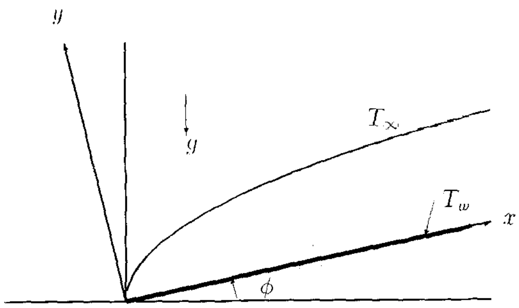

Fig. 1. Co-ordinate system and physical model

(i) that the radiative heat flux in the $\bar{x}$ direction is considered negligible in comparison with that in the $\bar{y}$ direction,

(ii) the viscous dissipation and axial heat conduction effects are negligible and

(iii) that the variation of fluid properties are limited to density variation which affects the buoyancy terms only.

Under these assumptions along with the Boussinesq approximation, the conservation equations for the steady two-dimensional flow problem under consideration can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \nabla^2 u + (Gr \tan \phi)\, \theta \tag{2}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \nabla^2 v + Gr\theta \tag{3}$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial}{\partial y} \left[\left\{ 1 + \left(1 + \frac{4}{3} R_d(\theta_w - 1)\, \theta \right)^3 \right\} \frac{\partial \theta}{\partial \psi} \right] \tag{4}$$

where ∇^2 is the two-dimensional Laplacian and $Gr = g\beta \Delta T L^3 \cos \phi / v^2$ is the appropriately defined Grashof number, $R_d = 4\sigma T_\infty^3 / k a_s$ is inverse of radiation-conduction parameter and $\theta_w = T_w / T_\infty$ is the surface temperature parameter for heated surface (i.e. $T_w > T_\infty$). Equations (1)–(4) are subject to the boundary conditions

$$\begin{aligned} &u = v = 0, \quad \theta = 1 \quad \text{at} \quad y = 0 \\ &u, \theta, p \to 0 \quad \text{as} \quad y \to \infty . \end{aligned} \tag{5}$$

In these equations the non-dimensional variables are defined as

$$x = \frac{\bar{x}}{L}, \quad y = \frac{\bar{y}}{L}, \quad u = \frac{\bar{u}L}{v}, \quad v = \frac{\bar{v}L}{v}$$

$$p = \frac{L^2}{\varrho v^2} \{\Delta \bar{p} p + Lg(x \sin \phi + y \cos \phi)\} \tag{6}$$

$$\theta = \left(\frac{T - T_\infty}{\Delta T} \right), \quad \Delta T = T_w - T_\infty, \quad \Delta p = \bar{p} - p_\infty$$

where L is the streamwise length of the plate, g is the gravitational acceleration, Pr is the Prandtl number, ϱ, β and v are, respectively, the density, thermal expansion coefficient, and the kinematic viscosity of the fluid.

We now introduce the following transformations into the above equations:

$$\tilde{y} = Gr^{1/5} y, \quad \tilde{u} = Gr^{-2/5} u, \quad \tilde{v} = Gr^{-1/5} v, \quad \tilde{p} = Gr^{-4/5} p \tag{7}$$

and ignoring the terms which are of the order $Gr^{-2/5}$ relative to those retaining in the limit $Gr \to \infty$, Eqs. (1)–(4) take the form:

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = -\frac{\partial p}{\partial x} = -\frac{\partial p}{\partial x} + \frac{\partial^3 \psi}{\partial y^3} + \Lambda\theta, \tag{8}$$

$$0 = -\frac{\partial p}{\partial y} + \theta, \tag{9}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial}{\partial y}\left[\left\{1 + \left(1 + \frac{4}{3}R_d(\theta_w - 1)\,\theta\right)^3\right\}\frac{\partial \theta}{\partial y}\right], \tag{10}$$

where ψ is the stream function which satisfies the equation of continuity (1), and Λ is the inclination parameter defined by

$$\Lambda = Gr^{1/5}\tan\phi.$$

The boundary conditions (5) now take the form

$$\begin{aligned} &\psi = \frac{\partial \psi}{\partial y} = 0, \quad \theta = 1 \quad \text{at} \quad y = 0, \\ &\frac{\partial \psi}{\partial y} \to 0, \quad \theta \to 0, \quad p \to 0 \quad \text{as} \quad y \to \infty. \end{aligned} \tag{11}$$

It should be noted that the range of inclination angle ϕ considered here is that Λ is $O(1)$ so that the buoyancy force term is formally comparable with induced pressure gradient along the plate. It may further be noted that the horizontal plate problem corresponds to $\Lambda = 0$, while the vertical flat plate problem corresponds to $\Lambda \to \infty$, in which case the scalling (7) becomes inappropriate.

In the situation, where the wall temperature T_w is not very much different from the free stream temperature T_∞ the energy equation (10) takes the following form (see Ali [13]):

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\left(1 + \frac{4}{3}R_d\right)\frac{\partial^2 \theta}{\partial y^2}. \tag{12}$$

3 Solutions

We first transform Eqs. (8)–(10) into the equations valid near the leading edge of the plate (small x) and far downstream (large x), respectively. However, to obtain a solution which is valid for all x, equations (8)–(10) are solved numerically using the implicit finite difference method along with Keller box scheme of Keller [19], in combination with the method of continuous transformation devised by Hunt and Wilks [18].

Solution for small x

We expect that, near the leading edge of the plate, the structure of the boundary layer to be similar to that associated with non-radiative flow along a semi-infinite horizontal plate. Consequently, we employ following transformation (used by Stewartson [20]):

$$\begin{aligned} &\psi = x^{3/5} f(x, \eta), \quad p = x^{2/5} h(x, \eta), \\ &\theta = \theta(x, \eta), \quad \eta = y/x^{2/5}. \end{aligned} \tag{13}$$

This gives $\theta = h'$, from Eq. (9), and then substituting (13) into (8) and (10) we obtain

$$f''' + \frac{3}{5} f f'' - \frac{1}{5} f'^2 - \frac{2}{5}(h - \eta h') + \Lambda x^{3/5} h' = x\left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} + \frac{\partial h}{\partial x}\right) \tag{14}$$

$$\frac{1}{Pr}\left[\left\{1 + \left(1 + \frac{4}{3} R_d(\theta_w - 1) h'\right)^3\right\} h''\right]' + \frac{3}{5} f h'' = x\left(f' \frac{\partial h'}{\partial x} - h'' \frac{\partial f}{\partial x}\right). \tag{15}$$

Corresponding boundary conditions (11) become

$$\begin{aligned} &f(x, 0) = f'(x, 0), \quad h'(x, 0) = 1, \\ &f'(x, \infty) = h(x, \infty) = h'(x, \infty) = 0, \end{aligned} \tag{16}$$

where primes denote differentiations with respect to η.

In the situation where the difference between the surface temperature and the free stream temperature is very small, i.e., $\theta_w \sim 1$ the energy equation reduces to

$$\frac{1}{Pr}\left(1 + \frac{4}{3} R_d\right) h''' + \frac{3}{5} f h'' = x\left(f' \frac{\partial h'}{\partial x} - h'' \frac{\partial f}{\partial x}\right). \tag{17}$$

Solution for large x when $\phi > 0$

At the distance from the leading edge, since the development of the boundary layer is chiefly influenced by the component of the buoyancy force parallel to the plate, the approximate transformation for the asymptotic form of the solution when the plate is at the positive angle of inclination is suggested by Bejan [21]

$$\begin{aligned} &\psi = x^{3/4} \tilde{f}(x, \tilde{\eta}), \quad p = x^{1/4} \tilde{h}(x, \tilde{\eta}), \\ &\theta = \tilde{\theta}(x, \tilde{\eta}), \quad \tilde{\eta} = y/x^{1/4}, \end{aligned} \tag{18}$$

so that $\tilde{\theta} = \tilde{h}'$ and Eqs. (8)–(10) transform into

$$\tilde{f}''' + \frac{3}{4} \tilde{f} \tilde{f}'' - \frac{1}{2} \tilde{f}'^2 - \frac{1}{4} x^{-3/4}(\tilde{h} - \tilde{\eta} \tilde{h}') + \Lambda \tilde{h}' - x^{1/4} \frac{\partial \tilde{h}}{\partial x} = x\left(\tilde{f}' \frac{\partial \tilde{f}'}{\partial x} - \tilde{f}'' \frac{\partial \tilde{f}}{\partial x}\right) \tag{19}$$

$$\frac{1}{Pr}\left[\left\{1 + \left(1 + \frac{4}{3} R_d(\theta_w - 1) \tilde{h}'\right)^3\right\} \tilde{h}''\right]' + \frac{3}{4} \tilde{f} \tilde{h}'' = x\left(\tilde{f}' \frac{\partial \tilde{h}'}{\partial x} - \tilde{h}'' \frac{\partial \tilde{f}}{\partial x}\right) \tag{20}$$

and the corresponding boundary conditions take the form

$$\begin{aligned} &\tilde{f}(x, 0) = \tilde{f}'(x, 0) = 0, \quad \tilde{h}'(x, 0) = 1, \\ &\tilde{f}'(x, \infty) = \tilde{h}(x, \infty) = \tilde{h}(x, \infty) = 0. \end{aligned} \tag{21}$$

As before, when the difference between the surface temperature and the free stream temperature is very small, i.e., $\theta_w \sim 1$ the energy equation reduces to

$$\frac{1}{Pr}\left(1 + \frac{4}{3} R_d\right) \tilde{h}''' + \frac{2}{4} \tilde{f} \tilde{h}'' = x\left(\tilde{f}' \frac{\partial \tilde{h}'}{\partial x} - \tilde{h}'' \frac{\partial \tilde{f}}{\partial x}\right). \tag{22}$$

Solution for all values of x

Of the governing boundary layer equations (8)–(10), the original numerical algorithm involved a switch between the leading edge and far downstream transformed forms (14)–(15) and (20)–(21), respectively. Although, instrumental in providing a background for analytic investigation, transformations (13) and (16) need not necessarily be the most appropriate for numerical integration. In fact the switch feature of the algorithm may be avoided by the introduction of the continuous transformation in x, linking the limiting solutions (13) and (18) as follows:

$$\psi = x^{3/5}(1+x)^{3/20} F(x, Y), \qquad p = x^{2/5}(1+x)^{-3/20} H(x, Y) \tag{23}$$

where

$$Y = y(1+x)^{3/20}/x^{2/5} \tag{24}$$

so that $\theta = H'$ and Eqs. (8) and (10) transform into

$$F''' + \frac{12+15x}{20(1+x)} FF'' - \frac{2+5x}{10(1+x)} F'^2 + \Lambda \left(\frac{x}{1+x}\right)^{3/5} H'$$
$$- \frac{1}{(1+x)^{3/4}} \left\{ \frac{8+5x}{20(1+x)} (H - \bar{\eta} H') + x \frac{\partial H}{\partial x} \right\} = x \left(F' \frac{\partial F'}{\partial x} - F'' \frac{\partial F}{\partial x} \right) \tag{25}$$

$$\frac{1}{Pr} \left[\left\{ 1 + \left(1 + \frac{4}{3} R_d (\theta_w - 1) H' \right)^3 \right\} H'' \right]' + \frac{12+15x}{20(1+x)} FH'' = x \left(F' \frac{\partial H'}{\partial x} - H'' \frac{\partial F}{\partial x} \right). \tag{26}$$

The boundary conditions are

$$\begin{aligned} &F(x,0) = F'(x,0) = 0, \qquad H'(x,0) = 1, \\ &F'(x,\infty) = H(x,\infty) = H'(x,\infty) = 0. \end{aligned} \tag{27}$$

where, primes now denote differentiation with respect to Y. It is worth pointing out that in the limits $x \to 0$ and $x \to \infty$ the governing equations (14)–(15) and (20)–(21) for (f, h) and $(\tilde{f}, \tilde{h})$ can easily be recovered from (25)–(26).

Finally, when $\theta_w \sim 0$ the energy equation reduces to

$$\frac{1}{Pr} \left(1 + \frac{4}{3} R_d \right) H''' + \frac{12+15x}{20(1+x)} FH'' = x \left(F' \frac{\partial H'}{\partial x} - H'' \frac{\partial F}{\partial x} \right). \tag{28}$$

The quantities of physical interest in this problem are the skin-friction, and the heat transfer coefficients defined as

$$C_f = \frac{\tau_w}{\varrho (\nu/L)^2}, \qquad Nu = \frac{L q_w}{\varkappa \Delta T} \tag{29}$$

where, τ_w and q_w are the dimensional skin-friction and heat transfer at the plate which are given by

$$\tau_w = \mu \left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}, \quad q_w = -k\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0} - \frac{4\sigma}{3a_s}\left(\frac{\partial T^3}{\partial \bar{y}}\right)_{\bar{y}=0}. \tag{30}$$

Using Eqs. (24)–(25), we obtain

$$C_f/Gr^{3/5} = x^{-1/5}(1+x)^{9/20}F''(x,0) \tag{31}$$

$$Nu/Gr^{1/5} = -x^{-2/5}(1+x)^{3/20}\left(1+\frac{4}{3}R_d\theta_w{}^3\right)H''(\chi,0). \tag{32}$$

At the two limiting conditions, $x \to 0$ and $x \to \infty$ relations (31) and (32) take the forms

$$C_f/Gr^{3/5} = x^{-1/5}f''(x,0), \tag{33}$$

$$Nu/Gr^{1/5} = -x^{-2/5}\left(1+\frac{4}{3}R_d\theta_w{}^3\right)h''(x,0), \tag{34}$$

and

$$C_f/Gr^{3/5} = x^{1/4}\tilde{f}''(x,0), \tag{35}$$

$$Nu/Gr^{1/5} = -x^{-1/4}\left(1+\frac{4}{3}R_d\theta_w{}^3\right)\tilde{h}''(x,0). \tag{36}$$

Throughout, when the thermal radiation parameter $R_d = 0$, there is no radiation interaction and the problem reduces to that investigated by Jones [16] for pure natural convection flow along an inclined surface with small angle of inclination to the horizontal. However, solutions thus obtained are discussed in the following section. The sets of Eqs. (14)–(16), (18)–(20), and (23)–(25) are coupled nonlinear partial differential equations involving the parameters Pr, R_d, θ_w and Λ. Thus, these sets are to be solved simultaneously subjected to the given boundary conditions. In the present investigation all the set are solved numerically employing a very efficient finite difference method introduced by Keller [19], and which was used most efficiently by Cebeci and Bradshaw [22] and very recently by Hossain et al. [14]–[15].

4 Results and discussion

The numerical results for the heat transfer coefficient $C_f/Gr^{3/5}$ and the heat transfer rate coefficient $Nu/Gr^{1/5}$ are obtained for representative values of the pertinent parameters R_d and θ_w for both positively ($\Lambda > 0$) and negatively ($\Lambda < 0$) inclined surface with Pr equals unity. Throughout we have considered the case of heated surface. It should be noted here that for both CO_2 in the temperature range $100 \sim 650\,°F$ (with corresponding Prandtl number range $0.76 \sim 0.6$) and NH_3 vapor in the temperature range $120 \sim 400\,°F$ (with corresponding Prandtl number range $0.88 \sim 0.84$) at 1 atm, the value of R_d ranges approximately from 10 to 30, whereas for water vapor in the temperature range $220 \sim 900\,°F$ (with corresponding Prandtl number Pr ~ 1.0) the R_d values lie between 30 to 200 (see Cess [12]). In Figs. 2 to 5, the numerical values obtained for $C_f/Gr^{3/5}$ and $Nu/Gr^{1/5}$ from the solutions of the representative equations for

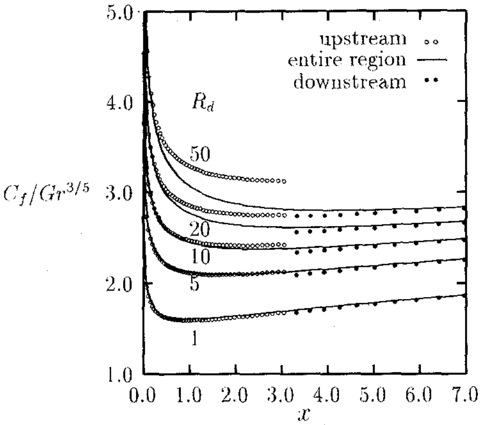

Fig. 2. Values of skin-friction coefficient against x for different values of R_d with $Pr = 1$, $\Lambda = 1.0$ and $\theta_w = 1.1$

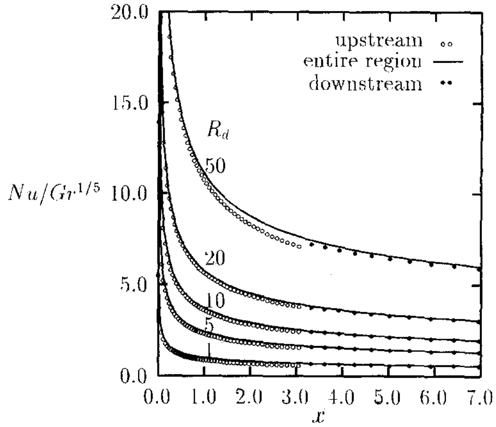

Fig. 3. Values of Nusselt number coefficient against x for different values of R_d with $Pr = 1$, $\Lambda = 1.0$ and $\theta_w = 1.1$

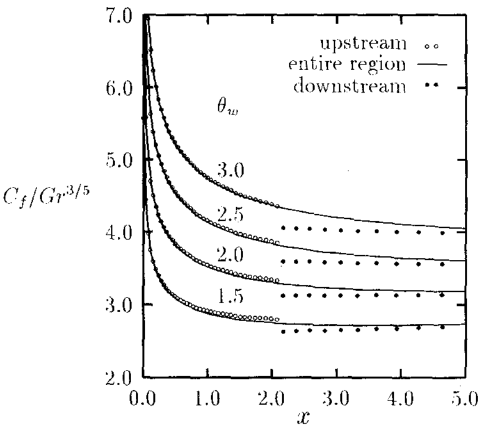

Fig. 4. Values of skin-friction coefficient against x for different values of θ_w with $Pr = 1$, $\Lambda = 1.0$ and $R_d = 10.0$

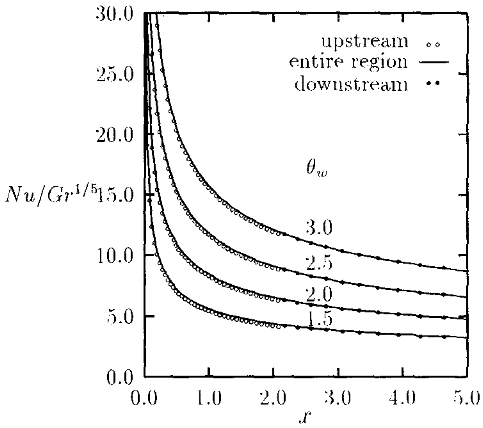

Fig. 5. Values of Nusselt number coefficient against x for different values of θ_w, with $Pr = 1$, $R_d = 10.0$ and $\Lambda = 1.0$

upstream, downstream and entire regime are compared with unit value of Λ and Pr. In these figures the circled curves, the bulleted curves and the solid curves represent respectively for the cases upstream, downstream and entire region. It can be seen that the solutions obtained for the cases upstream and downstream region are in excellent agreement with those of the entire region. Undoubtedly these solutions are better than the series solution. In Figs. 2 and 3 the curves represent, respectively the values of $C_f/Gr^{3/5}$ and $Nu/Gr^{1/5}$ against x for values of R_d equals 1.0, 5, 10, 20 and 50 with the temperature parameter $\theta_w = 1.1$. From these figures it is observed that both $C_f/Gr^{3/5}$ and $Nu/Gr^{1/5}$ increase owing to increases in the thermal radiation parameter R_d. The values of $C_f/Gr^{3/5}$ and $Nu/Gr^{1/5}$ against x for values of the surface temperature parameter θ_w equals 1.1, 2.0, 2.5 and 3.0 with $R_d = 10.0$ are shown in Figs. 3 and 5. It can be seen that an increase of the surface temperature also leads to increase in the values of $C_f/Gr^{3/5}$ and $Nu/Gr^{1/5}$. In the previous figures, since we have seen excellent agreement between the solutions from the equations valid in the upstream and downstream regimes with those of the entire regime, here in Figs. 6 and 7, we represent the values of $C_f/Gr^{3/5}$ and $Nu/Gr^{1/5}$ against x for values of Λ equals

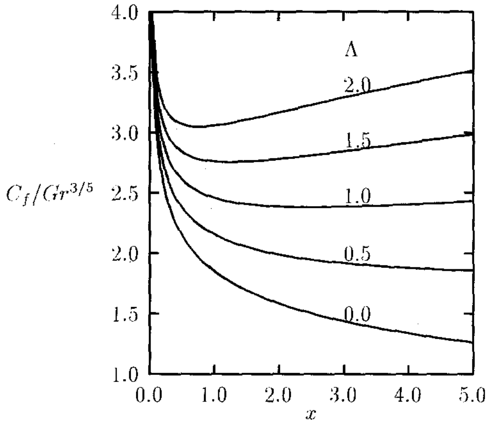

Fig. 6. Values of skin-friction coefficient for different values of $A > 0$ with $Pr = 1$, $R_d = 10.0$ and $\theta_w = 1.1$

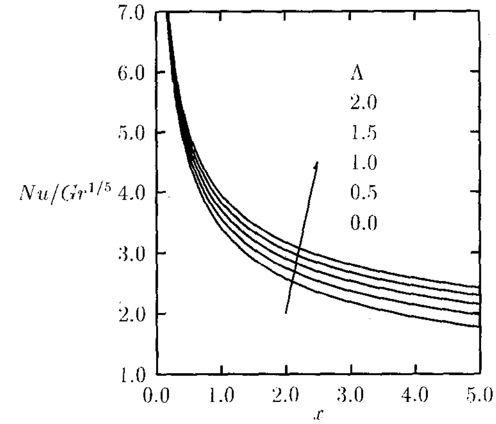

Fig. 7. Values of Nusselt number coefficient for different values of $A > 0$ with $Pr = 1$, $R_d = 10.0$ and $\theta_w = 1.1$

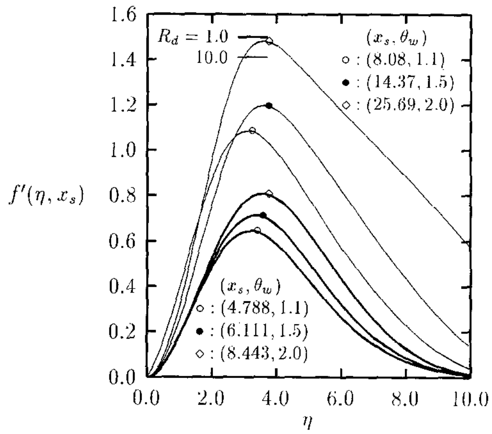

Fig. 8. Velocity profiles at the point of separation x_s for $R_d = 1.0$ and 10.0 and $\theta_w = 1.1$, 1.5, and 2.0 with $Pr = 1$ and $A = -1.0$

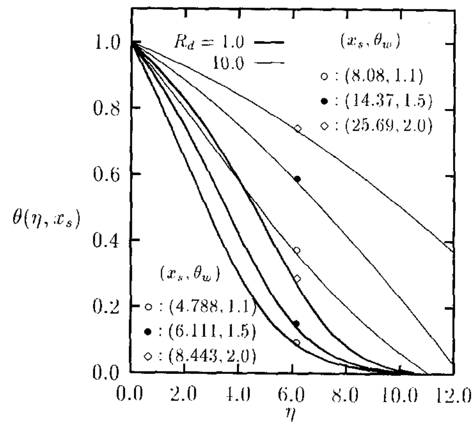

Fig. 9. Temperature distribution at the point of separation x_s for $R_d = 1.0$ and 10.0 and $\theta_w = 1.1$, 1.5, and 2.0 with $Pr = 1$ and $A = -1.0$

0.0, 0.5, 1.0, 1.5 and 2.0 with $R_d = 10.0$ and $\theta_w = 1.1$ obtained only for the equations valid in the case of entire region. Here again, we see that increase in the inclination to the horizontal leads to increase in both the values of $C_f/Gr^{3/5}$ and $Nu/Gr^{1/5}$. We further observe that owing to increase in the inclination there is an increase in the boundary layer thickness.

When the plate is at a negative angle to the horizontal, the separation of the boundary layer would occur downstream from the leading edge of the plate as the opposed buoyancy force and induced pressure gradient are of comparable magnitude. Here we have calculated the position of separations point, x_s, for $A = -1$, when R_d equals 1 and 10 with values of θ_w equals 1.1, 1.5 and 20. The numerical values of these separation points are entered in Figs. 8 and 9. In these figures, the velocity and temperature profiles are presented at the corresponding separation points. It can be seen that at an increase in both the radiation parameter and the surface temperature parameter leads to increase both in velocity as well as in the temperature profiles. We also notice that the position of the separation point moves away from the leading edge with the increase of either of the radiation-convection interaction parameter or the surface temperature parameter of the heated surface.

5 Conclusions

Effect of radiation-convection interaction on natural convection flow along an isothermal flat plate inclined at small angle to the horizontal has been investigated by integrating the transformed boundary layer equations for small and large distances from the leading edge of the plate numerically using implicit finite difference method. Novel variables are also proposed such that the numerical solution is effective by simulation of a single set of equations which incorporate the equations at the upstream and downstream regimes. In case of the negative inclination at $\Lambda = -1$, the separation points are accurately determined since the solutions at these points behaved in regular manner. From the present investigation it may also be concluded that the position of the separation point moves away from the leading edge with the increase of either of the radiation-convection interaction parameter or the surface temperature parameter of the heated surface. It is hoped that experimental data will be available in near future to very the results of the present investigation.

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