MHD FORCED AND FREE CONVECTION BOUNDARY LAYER FLOW ALONG A VERTICAL POROUS PLATE

M. A. Hossain*, K. C. A. Alam
Department of Mathematics, University of Dhaka
Dhaka 1000, BANGLADESH

D. A. S. Rees
School of Mechanical Engineering, University of Bath
Bath BA2 7AY, U.K.

This paper details a study of the combined forced and free convection boundary layer flow of a viscous, incompressible and electrically conducting fluid along a vertical porous plate in the presence of transverse magnetic field. Numerical solutions of the local non-similar boundary layer equations governing the flow and energy are obtained by employing an implicit finite difference approximation together with the Keller box method. The results thus obtained are compared with perturbation solutions in terms of both the local rate of heat transfer and the local surface shear stress for different physical parameters. The dimensionless velocity and temperature functions in the flow are also presented.

Key words: magnetohydrodynamic, convection, transpiration, boundary layer.

1. Introduction

The problem considered here is that of the combined forced and free convection boundary layer flow of an electrically conducting and viscous incompressible fluid along vertical porous plate with a uniform surface temperature and vectored mass transfer in the presence of a transverse magnetic field of the form, $B_0 \propto x^{-1/4}$, where x measures the distance along the surface from the leading edge.

Extensive studies on mixed convection along vertical surfaces have been conducted by Sparrow *et al.* (1959), Sparrow and Gregg (1959), Merkin (1969),

* To whom correspondence should be addressed

Lloyd and Sparrow (1970) and Wilks (1973). It has been generally recognized that ξ (defined as Gr/Re^2 where Gr is the Grashof number and Re the Reynolds number) is the governing parameter for a vertical plate. Forced convection exists as a limit when ξ goes to zero which occurs at the leading edge, and the free convection limit can be reached at large values of ξ. Perturbation solutions have been developed in both cases, since both the forced convection and free convection limits admit similarity solutions. Empirical patching of two perturbation solutions has also been carried out to provide a uniformly valid solution by Rajn *et. al.* (1984) which covers the whole range of values of ξ. They obtained a finite difference solution applying an algebraic transformation $Z=l(1+\xi^2)$. Many solutions have been developed by considering the free convection effect as a perturbation quantity. Tingwei *et al.* (1982) have also studied the effect of forced and free convection along a vertical flat plate with uniform heat flux by considering the transpiration parameter ξ to be small.
Because of its application in nuclear engineering where convection aids the cooling of reactors, the natural convection boundary layer flow of an electrically conducting fluid up a hot vertical wall in the presence of a strong magnetic field has been studied by several authors, such as Singh and Cowling (1963), Sparrow and Cess (1961), Riley (1964) and Kuiken (1970). Later, Cramer and Pai (1974) presented a similarity solution for the above problem with a varying surface temperature. On the other hand, Wilks (1976) investigated the problem with uniform heat flux by formulating it in terms of both a regular and an inverse series expansion of a characterizing coordinate that provided a link between the similarity states close to and far from the leading edge. The combined effect of the forced and free convection with uniform heat flux in the presence of a strong magnetic field has been studied by Hossain and Ahmed (1984). Recently Hossain *et al.* (1996) have investigated the MHD free convection flow along a vertical porous flat plate with a power law surface temperature in the presence of variable transverse magnetic field by employing two different methods, namely: (i) perturbation methods for both small and large values of the scaled streamwise variable ξ ($= V_0\sqrt{(2x/\nu U_\infty)}$, where V_0 is the transpiration velocity) and (ii) a finite difference method (Hossain *et al.*; 1994; Keller, 1978). In the above analysis, solutions were obtained for a wide range of the values of the Prandtl number Pr and selected values of the magnetic field parameter. In the present paper we investigate the title problem by employing the same methods for a wide range of values of ξ and those values of the Prandtl number Pr which are appropriate for liquid metals. The resulting solutions to the present problem are compared and presented in terms of local surface shear stress and the local rate of heat transfer in both tabular and graphical forms. Representative nondimensional

velocity and temperature profiles are also presented graphically showing the effects of varying the different physical parameters.

2. Mathematical formulation

A steady two-dimensional, laminar mixed convection boundary-layer flow of a viscous incompressible and conducting fluid along a semi-infinite vertical porous heated plate is considered here. Let the temperature of the free stream be T_0 and its uniform velocity be U_∞. A magnetic field of strength $B_0(x)$ is considered to be applied parallel to the y-axis which is normal to the plate and is allowed to move past the plate with the fluid. Here we assume that the induced magnetic field produced by the motion of the electrically conducting fluid is negligible. This assumption is valid for sufficiently small values of the magnetic Reynolds number. Further, since no external electric field is applied and the effect of polarization of the ionized fluid is negligible, we may also assume that the electric field $E=0$. Under the usual Boussinesq approximation the boundary layer equations governing the flow along a vertical surface at which the temperature T_w is uniform (Cobble, 1979) are:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0\ , \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial^2 y}+g\beta(T-T_0)-\frac{\sigma B_0^2}{\rho}(u-U_\infty), \tag{2.2}$$

$$c_p\left(u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2} \tag{2.3}$$

where, u and υ are the velocity components associated with the direction of increasing coordinates x and y, respectively, ν - the kinetic coefficient of viscosity, ρ - the density of the ambient fluid, g - the acceleration due to gravity, β - the coefficient of volume expansion, σ - the electrical conductivity, k - the coefficient of thermal diffusivity, c_p - the specific heat at constant pressure and T is the temperature of the fluid. The boundary conditions for the present problem are:

$$\begin{aligned} &y=0: \quad u=0, \quad \upsilon=V_0, \quad T=T_w \\ &y\to\infty: \quad u=U_\infty, \quad T=T_0\ . \end{aligned} \tag{2.4}$$

Near the leading edge, the flow is caused mainly by the effect of the buoyancy force which arises from the difference in temperature between the plate and the ambient fluid. Therefore, the following group of transformations are introduced in (2.2) and (2.3):

$$\psi = \sqrt{2\nu U_\infty x}\left(f(\eta,\xi) \pm \frac{1}{2}\xi\right), \quad \theta(\eta,\xi) = \frac{T - T_0}{T_w - T_0} \tag{2.5}$$

where ψ is the stream function satisfying the equation of continuity (2.1), and:

$$\eta = \frac{V_0 y}{\nu \xi} \qquad \text{and} \qquad \xi = V_0 \sqrt{\frac{2x}{\nu U_\infty}} \, . \tag{2.6}$$

In Eqs. (2.5) the upper sign is taken throughout for suction and the lower sign corresponds to blowing or injection; in this paper we consider the case of suction only. Eqs. (2.1)÷(2.3) now become:

$$f''' + ff'' + \alpha\xi^2\theta + 2M(1 - f'^2) \pm \xi f'' = \xi\left\{f''\frac{\partial f}{\partial \xi} - f'\frac{\partial f'}{\partial \xi}\right\}, \tag{2.7}$$

$$Pr^{-1}\theta'' + f\theta' + \xi\theta' = \xi\left\{\theta'\frac{\partial f}{\partial \xi} - f'\frac{\partial \theta}{\partial \xi}\right\}. \tag{2.8}$$

The corresponding boundary conditions transform into:

$$\begin{aligned} &f = 0, \quad f' = 0, \quad \theta = 1 \quad \text{at} \quad \eta = 0, \\ &f' = 1, \quad \theta = 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{2.9}$$

where

$$Pr = \frac{\nu}{k}, \quad \alpha = 2\frac{G_L}{Re_L^2 S_L^2}, \quad G_L = \frac{g\beta\Delta T L^3}{\nu^2}, \quad Re_L = \frac{U_\infty L}{\nu} \tag{2.10}$$

$$B_0 = Bx^{-1/4}, \quad S_L = V_0\sqrt{\frac{2L}{\nu U_\infty}} \quad \text{and} \quad M = \frac{\sigma B^2}{\rho U_\infty}. \tag{2.11}$$

The physical quantities of interest are the local shear stress τ_w and the local rate of heat transfer Q. These are defined by:

$$\tau_w = \frac{\mu \upsilon U_0}{\nu}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \xi^{-1} f''(\xi,0) \ , \tag{2.12}$$

$$Q = \frac{\nu}{V_0(T_w - T_0)}\left(\frac{\partial u}{\partial y}\right)_{y=0} = -\xi^{-1}\theta'(\xi,0) \ . \tag{2.13}$$

3. Methods of solution

In many important practical situations similarity solutions do not exist and the resulting momentum and thermal boundary layers are not self-similar. In the present analysis, we investigate the problem governed by Eqs. (2.7) to (2.9) by employing the extended series solution method together with Pade's approximation (Gerald 1980), the implicit finite difference approximation known as the Keller box method (Keller 1978), and a large-ξ asymptotic analysis.

3.1. Perturbation solution

Forced convection exists as a limit when ξ goes to zero which occurs at the leading edge, and the free convection limit can be reached when ξ becomes large. In this section solutions have been developed for both limits of ξ.

Series solution (denoted - SS)

The series solution is valid for sufficiently small values of ξ. Accordingly, the functions $f(\xi,\eta)$ and $\theta(\xi,\eta)$ are expanded in a power series in ξ, that is, we take:

$$f(\xi,\eta) = \sum_{n=0}^{\infty} \xi^n f_n(\eta) \qquad \text{and} \qquad \theta(\xi,\eta) = \sum_{n=0}^{\infty} \xi^n \theta_n(\eta) \ . \tag{3.1}$$

Substituting the above expansion into Eqs. (2.7)÷(2.8) and equating the coefficients of various powers of ξ, we get the following equations:

$$f_0''' + f_0 f_0''' + 2M(1 - f_0') = 0 \ , \tag{3.2}$$

$$f_1''' + f_0 f_1'' - f_0' f_1' + 2 f_0'' f_1 + f_0'' - 2M f_1' = 0 \ , \tag{3.3}$$

$$\theta_0'' + Pr f_0 \theta_0' = 0 \ , \tag{3.4}$$

$$\theta_1'' + Pr(f_0 \theta_1 - f_0' \theta_1 + 2 f_1 \theta_0' + \theta_0) = 0 \ , \tag{3.5}$$

$$f_n''' + \sum_{r=0}^{n} [(r+1) f_r f_{n-r}'' - r f_r' f_{n-r}'] + \alpha \theta_{n-2} - 2F f_n' + f_{n-1}'' = 0 \ , \tag{3.6}$$

$$\theta_n'' + \Pr \sum_{r=0}^{n} [(r+1) f_r \theta_{n-r}' - r f_{n-r}' \theta_r] = 0, \quad (n \geq 2). \tag{3.7}$$

The corresponding boundary conditions are :

$$\begin{gathered} f_i(0) = f_i'(0) = \theta_i(0) = \theta_0(0) - 1 = 0, \\ f_i'(\infty) = \theta_i(\infty) = 0 \end{gathered} \tag{3.8}$$

for $i=0,\ldots,n$. Equations (2.13), (3.1), (3.2) and (3.3) are linear, but coupled, and may be solved independently pairwise one after another. The implicit Runge-Kutta- Butcher (Butcher, 1964) initial value solver together with Nachsheim-Swigert iteration scheme of Nachsheim and Swigert (1965) are employed to solve the first pair (2.11) and (2.12), and the other pairs up to $O(\xi^{10})$ are solved exploiting the method of superposition. Here Pade's approximant is used to obtain a more accurate approximation for the local skin-friction, τ_w, and the rate of heat transfer, Q, instead of using the direct sum.

Pade's approximant of a series is considered as the ratio of two polynomials. For example, Pade's approximant $[M,N]$ is the fraction P'/Q' where P' is a polynomial of degree M and Q' is a polynomial of degree N. The coefficients of these polynomials are obtained by imposing the condition that the expansion of Pade's approximant and the original series must agree to the order $M+N$, and $Q'(0)=1$ As remarked by Van Dyke (1975), the diagonal approximants where $M=N$ are most useful in improving the convergence. In the present problem, M and N are chosen as $M=N=5$, for which the coefficient of the series has to be taken up to $O(\xi^{10})$.

Large-ξ asymptotic expansion (denoted -ASY)

In this subsection attention shall be given to the behaviour of the solution to equations (2.7) and (2.8) when ξ is large. Given that we considering only the suction case, for which we take the positively signed terms in (2.7) and (2.8), an order-of-magnitude analysis of the various terms in these equations shows that the largest are f'', $\xi f''$ and $\alpha\xi^2\theta$ in (2.7) θ'' and $\xi\theta'$ in (2.8). In their respective equations, both terms have to be balanced in magnitude and the only way to do this is to assume that η is small, and hence η derivatives are large. Given that $\theta = O(1)$ as $\xi \to \infty$, it is necessary to find the appropriate scaling for f and η. On balancing the f'', θ, and $\xi f''$ terms in (3.7), it is found that $\eta = O(\xi^{-1})$ and $f = O(\xi^{-1})$ as $\xi \to \infty$. Therefore, the following substitutions are made:

$$f = \frac{1}{X}F(\xi,\zeta), \qquad \theta = \frac{1}{X}\Theta(\xi,\zeta) \quad \text{with} \quad \zeta = \eta\xi \quad \text{and} \quad X = \xi \ . \tag{3.9}$$

Equations (2.7) and (2.8) become:

$$F''' + F'' + \alpha\Theta = \frac{2M}{X^2}[F' - 1] + \frac{1}{X}\left[F'F'_X - F''F_X\right], \tag{3.10}$$

$$\frac{1}{Pr}\Theta'' + \Theta' = \frac{1}{X}[F'\Theta x - Fx\Theta] \ . \tag{3.11}$$

The boundary conditions are simply:

$$F = 0, \quad F' = 0, \quad \text{and} \quad \Theta = 1 \quad \text{at} \quad \zeta = 0 \ , \tag{3.12}$$

and

$$F' \to 0, \quad \Theta \to 0, \quad \text{as} \quad \zeta \to \infty. \tag{3.13}$$

Equations (3.5) and (3.7) can in fact be solved by using a straightforward perturbation expansion in X. Let:

$$F = F_0(\zeta) + X^{-2}F_2(\zeta) + X^{-4}F_4(\zeta) + \dots \ , \tag{3.14}$$

$$\Theta = \Theta_0(\zeta) + X^{-2}\Theta_2(\zeta) + X^{-4}\Theta_4(\zeta) + \dots \ . \tag{3.15}$$

By equating the various power of ξ, we obtain:

$$\Theta_0'' + Pr\Theta_0' = 0 \ , \tag{3.16}$$

$$F_0''' + F_0'' = -\alpha\Theta_0, \tag{3.17}$$

$$\Theta_2'' + Pr\Theta_2' = 0, \tag{3.18}$$

$$F_2''' + F_2'' = \alpha\Theta_2 + 2M(F_0' - 1), \tag{3.19}$$

$$\Theta_4'' + Pr\Theta_4' = 2F_0'\Theta_2 + 2F_2\Theta_0', \tag{3.20}$$

$$F_4''' + F_4'' = -\alpha\Theta_4 + 2MF_2' + 2(F_0''F_2 - F_0'F_2'). \tag{3.21}$$

The corresponding boundary conditions are:

$$\begin{aligned} &F_0 = F_0' = F_2 = F_2' = F_4 = F_4' = 0, \\ &\Theta_0 - 1 = \Theta_2 = \Theta_4 = 0 \qquad \text{at} \qquad \eta = 0, \\ &F_0' - 1 = F_2' = F_4' = 0, \quad \Theta_0 = \Theta_2 = \Theta_4 = 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{3.22}$$

Solutions of the equations for $Pr \neq 1$ are obtained as follows:

$$\Theta_0 = \exp(-Pr\zeta) \ , \tag{3.23}$$

$$F_0 = \frac{\alpha}{Pr^2(Pr-1)}\exp(-Pr\zeta) + \left\{1 - \frac{\alpha}{Pr(Pr-1)}\right\}\exp(-\zeta) + \frac{\alpha}{Pr^2} + \zeta - 1 \ , \tag{3.24}$$

$$\Theta_2 = 0 \ , \tag{3.25}$$

$$\begin{aligned} F_2 = 2M\Bigg[&\frac{\alpha}{Pr^3(Pr-1)^2}\exp(-Pr\zeta) - \left\{1 - \frac{\alpha}{Pr(Pr-1)}\right\}\zeta\exp(-\zeta) + \\ &-\left\{1 - \frac{\alpha}{Pr(Pr-1)} + \frac{\alpha}{Pr^3(Pr-1)^2}\exp(-Pr\zeta)\right\}\Bigg]. \end{aligned} \tag{3.26}$$

Higher order terms are not shown here because the expressions are too large. Hence, the expressions for F'' and Θ' at $\zeta=0$ may be obtained from Eq. (3.21)÷(3.24) together with the expressions given below:

$$\Theta_0' = -Pr,$$

$$F_0'' = \frac{\alpha + Pr}{Pr},$$

$$\Theta_2' = 0,$$

$$F_2'' = 2M\left(1 - \frac{\alpha}{Pr^2}\right),$$

$$\Theta_4' = 2M\frac{2Pr^3 - 2\alpha Pr - \alpha}{Pr(Pr+1)^2}.$$

In a similar manner, for $Pr = 1$, the corresponding expressions for F_i'' and Θ_i'
for $i = 1,2,3,4$ are given by:

$$\Theta_0' = -1, \qquad F_0'' = \alpha + 1,$$

$$\Theta_2' = 0, \qquad F_2'' = 2M(1-\alpha),$$

$$\Theta_4' = M(1 - 3/2\alpha).$$

3.2. Finite difference method (denoted - FDM)

In the present analysis, an efficient and accurate implicit finite difference method (the Keller box method) is employed to solve the system of Eqs. (2.7)÷(2.8). To begin with, Eqs. (2.7) and (2.8) are written in terms of a system of first order equations in η, which is then expressed in finite difference form by approximating the functions and their derivatives in terms of the central differences in both coordinate directions. Denoting the mesh points in the (ξ, η)-plane by ξ_i and η_j, where $i=1,2,3, \ldots, M$ and $j=1,2,3, \ldots, N$, central difference approximations are made such that those equations involving ξ explicitly are centered at $(\xi_{i-1/2}, \eta_{j-1/2})$ and the remainder at $(\xi_i, \eta_{j-1/2})$, where $\eta_{j-1/2} = \eta_j + \eta_{j-1}$, etc.. There results a set of nonlinear difference equations for the unknowns at ξ_i in terms of their values at ξ_{i-1}. These equations are then linearized by Newton's quasi-linearization technique and solved using a block tridiagonal algorithm, taking as the initial interate the converged solution at $\xi = \xi_{i-1}$. Now to initiate the process at $\xi = 0$, we first provide guess profiles for all five variables (arising from the reduction to first order form) and use the Keller box method to solve the governing ordinary differential equations. Having obtained the leading-edge solution it is possible to march step by step along the boundary layer. For a given value of ξ, the iterative procedure is stopped when the difference in computing the velocity and the temperature in the next iteration is less than 10^{-6}, i.e. when $\left|\delta f^i\right| \le 10^{-6}$, where the superscript i denotes an iteration number. The computations were not performed using a uniform grid in the η direction, but a non-uniform grid was used and defined by $\eta_j = \sinh(j/a)$, with $N = 251$ and $a=80$. The η_j values were chosen so that the outer boundary $\eta_e = 8$ which is sufficiently large, and they are such that the grid is sufficiently dense in the vicinity of the boundary to ensure accuracy; this is important because the boundary layer thins substantially as ξ increases, as shown in the asymptotic solution above. In the present numerical investigation the maximum values of ξ was chosen to be sufficiently large that the solutions compare well with the asymptotic values for the local skin-friction and the local rate of heat transfer. The numerical results thus obtained are shown graphically and in tabular form.

4. Results and discussion

In the present investigation we have obtained the solutions of the nonsimilar boundary layer equations governing the combined forced and free convection boundary layer flow of a viscous incompressible and electrically conducting fluid along a vertical porous plate in the presence of a transverse magnetic field. To this end we have employed a series perturbation method for small values of ξ, an asymptotic expantion for large values of ξ, and the implicit finite difference scheme known as the Keller box method for intermediate values of ξ. Numerical values of local shear stress and the rate of heat transfer for selected values of the magnetic field parameter, M, with $Pr=0.1$ and $\alpha=1.0$, as a function of the transpiration parameter ξ in the range 0.01 to 600.0 obtained by the above methods are compared in Table 1. From this table we see that, for smaller ξ, the solutions obtained by perturbation method (SS) are in good agreement with the numerical solution (FDM) up to ξ=0.1. On the other hand, the solutions obtained by the finite difference method approach those solutions obtained by asymptotic analysis as ξ increases. These observations lend support to the accuracy of the numerical method. From the same table, it is also observed that the rate of heat transfer obtained by the finite difference method converges towards the asymptotic solutions faster than the local shear stress. This can be explained by noting that $\Theta_2' = 0$, on the surface, whereas the second order expression for the shear stress given just below Eq. (3.39) is nonzero, in general. We further observe that, near the leading edge, the effect of changing the magnetic field parameter, M, gives substantial changes in both the surface shear stress and the rate of heat transfer, but this effect diminishes downstream where buoyancy and suction dominate the flow.

Table 1: Values of the local shear stress and the rate of heat transfer due to the three methods for different values of M with Pr =1.0 and α=1.0.

ξ	τ_w		Q	
	SS and ASY	FDM	SS and ASY	FDM
		$M=0$		
0.01	*47.5897	47.5782	*19.9858	20.0030
0.05	*10.0678	10.0623	* 4.0684	4.0721
0.10	* 5.4331	5.4279	* 2.0815	2.0840
0.20	* 2.5485	3.2037	*0 .7651	1.0941
0.50	* 3.5820	2.1428	*0 .5472	.5071
1.00		2.1649		.3135
10.00		6.6916		0.1202
100.00	**11.000	10.982	** 0.1000	0.1000
600.00	**11.000	10.990	** 0.1000	0.1000
		M =2.5		
0.01	*227.9433	227.9441	*23.1035	23.1150
0.05	*46.0139	46.0139	*4.6645	4.6670
0.10	*23.2926	23.2924	*2.3605	2.3618
0.20	* 8.2213	11.9653	*0.8273	1.2108
0.50	* 3.7249	5.2821	*0.3553	0.5250
1.00		3.2577		0.3036
10.00		5.7129		0.1191
100.00	** 7.4642	10.9310	**0.1000	0.1000
600.00	**10.1750	10.9886	**0.1000	0.1000
		M=5.0		
0.01	*319.4046	319.4060	*23.6319	23.6427
0.05	*64.2987	64.2987	*4.7684	4.7706
0.10	*32.4252	32.425	*2.4110	2.4122
0.20	*11.2324	16.5134	*0.8414	1.2340
0.50	* 4.7671	7.0498	*0.3560	0.5304
1.00		4.0450		0.3013
10.00		5.0698		0.1181
100.00	* *3.92857	10.8808	**0.1000	0.1000
600.00	* *9.35000	10.9873	**0.1000	0.1000

* for small ξ and ** for large ξ.

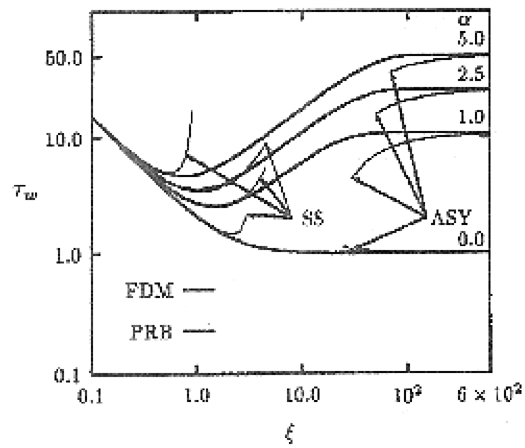

Fig. 1a. Local shear stress against ξ for different values of α with $M=1.0$ and $Pr=0.1$.

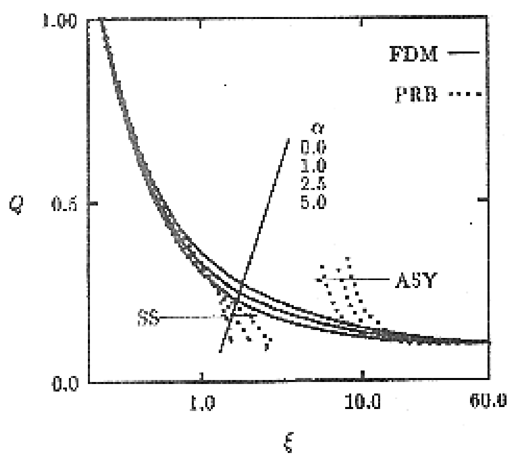

Fig. 1b. Local rate of heat transfer against ξ for different values of α with $M=1.0$ and $Pr=0.1$.

For M =1.0 and Pr =0.1 and different values of the buoyancy parameter α, the values of the shear stress and the rate of heat transfer are shown graphically in Figs. 1a and 1b, respectively. From Fig.1a it can be seen that the effect of varying α on the shear stress near the leading edge is less significant than in the downstream region. From the same figure we further observe that the shear stress increases with the increase of α. On the other hand, from Fig. 1b it can be seen that the effect of varying the buoyancy parameter in the region very close to the leading edge and in the downstream region is small compared with the intermediate region, which we can define as $\xi \in (0.5, 50.0)$. We observe that an increase in the buoyancy parameter leads to increase in the heat transfer.

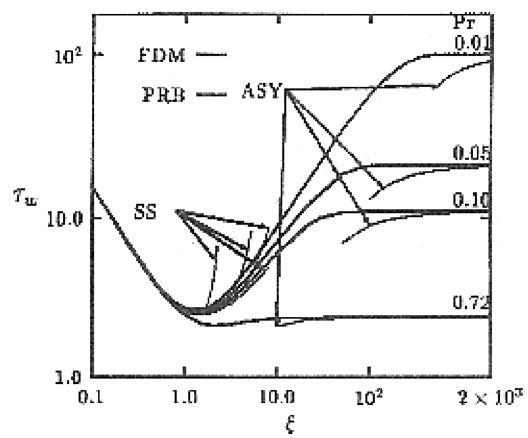

Fig. 2a. Local shear stress against ξ for different values of Pr with $M=1.0$ and $\alpha=1.0$

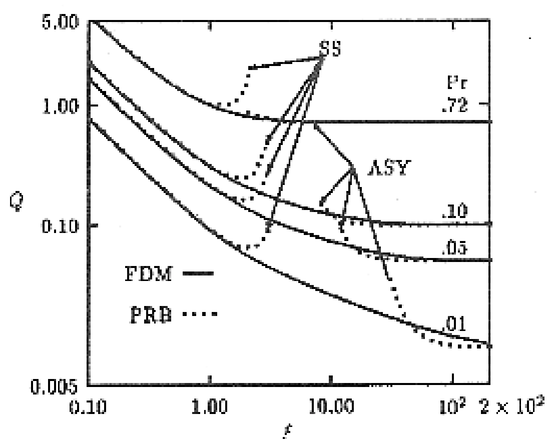

Fig. 2b. Local rate of heat transfer against ξ for different values of Pr with $M=1.0$ and $\alpha=1.0$

Figures 2a and 2b represent the shear stress and the rate of heat transfer against ξ for $\alpha=1.0$ and $M=1.0$ and for Pr taking the values, 0.72, 0.1, 0.05 and 0.01. We observe from Fig. 2a that the values of the local shear stress vary only slightly with changes in the Prandtl number when in the region $\xi \in (0.0, 0.8)$, but these differences increase with the increasing values of ξ. It may also be seen that the shear stress increases as *Pr* decreases. Here, it should be noted that a decreasing value of the Prandtl number delays the point where the FDM solution becomes almost indistinguishable from the asymptotic solution.

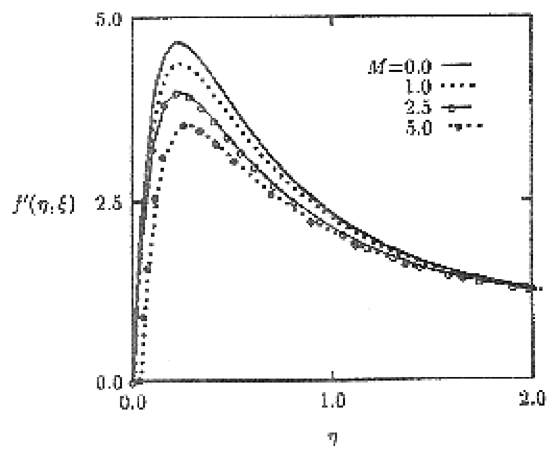

Fig. 3a. Velocity profiles against η for different values of *M* with *Pr=0.1* and $\alpha = 1.0$.

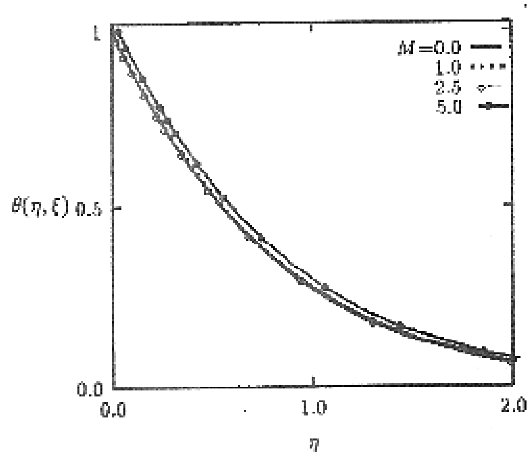

Fig. 3b. Temperature profiles against η for different values of *M* with *Pr=0.1* and $\alpha = 1.0$.

On the other hand, from Fig. 2b, one can see that the rate of heat transfer decreases with decreasing values of *Pr*.

Figures 3a, 4a and 5a represent the velocity distribution showing the separate effects of changing the parameters *M*, *Pr* and ξ, respectively. From Fig. 3a it can be concluded that the velocity decreases in magnitude as *M* increases. From

Fig. 4a, it is observed that the maximum fluid velocity increases as the Prandtl number decreases, and the point of maximum velocity recedes from the wall. Finally, from Fig. 5a we see that the point of maximum velocity gets closer to the wall as ξ increases. However, this conclusion is only valid in terms of η; in terms of y the boundary layer tends to a uniform thickness due to the action of suction, and therefore the velocity maximum asymptotes to a constant distance from the surface as ξ increases.

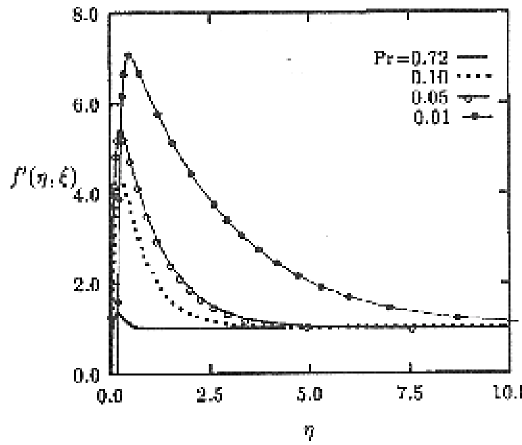

Fig. 4a. Velocity profiles against η for different values of Pr with $M=1.0$, $\alpha=1.0$ and $\xi=10.0$.

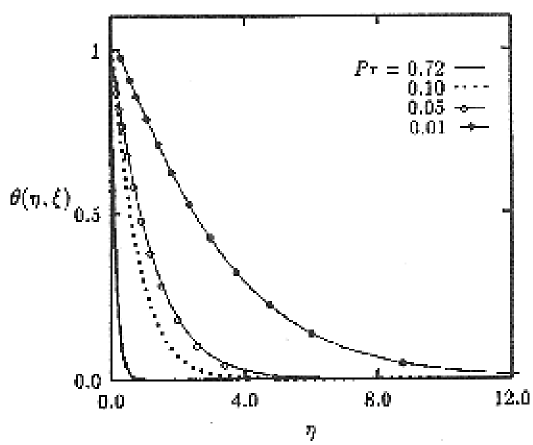

Fig. 4b. Temperature profiles against η for different values of Pr with $M=1.0$, $\alpha=1.0$ and $\xi=10.0$.

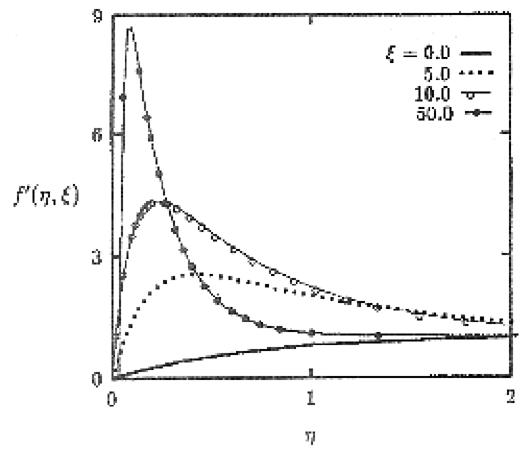

Fig. 5a. Velocity profiles against η for different ξ while $M=1.0$, $Pr=1.0$ and $\alpha=1.0$.

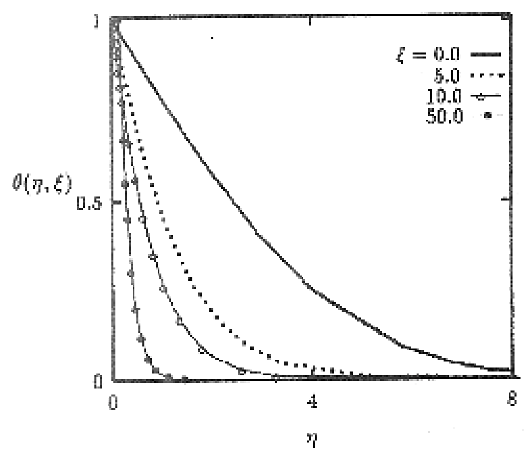

Fig. 5b. Temperature profiles against η for different values of ξ with $M=1.0$, $Pr=1.0$ and $\alpha=1.0$.

The separate effects of varying the parameters M, Pr and ξ on the temperature distribution are shown, respectively, in Figs. 3b, 4b and 5b. From Fig. 3b it is seen that the effect of magnetic field parameter on temperature distribution is negligible. From Figs. 4b and 5b, it can be see that the effect of varying the Prandtl number and transpiration parameter on temperature field is very strong near the surface. From these figures, it may, further, be seen that an increase in the Prandtl number or increasing values of ξ lead to a thinning boundary layer.

5. Conclusions

In the present study, the MHD forced and free convection flow along a vertical porous plate has been investigated theoretically by employing two distinct methodologies, namely, (i) the perturbation method for small and large values of ξ and (ii) the Keller box scheme.

It has been found that, near the leading edge, the effect of magnetic field parameter variations is to cause corresponding changes in both the local shear stress and the surface rate of heat transfer, but in the down-stream region its effects are absent. From the asymptotic analysis, it has been observed that the shear stress approaches $(\alpha + Pr)/Pr$ and the rate of heat transfer to Pr, but that they do so at different rates.

Acknowledgment

One of the authors (K.C.A. Alam) would like to acknowledge the gratitude to the *Ministry of National Science and Technology, Government of the Peoples' Republic of Bangladesh* for providing him a research fellowship during the period of doing this work.

Nomenclature

$B_0(x)$ – transverse magnetic field component
c_p – specific heat at constant pressure
f – dimensionless stream function
g – acceleration of gravity
Pr – Prandtl number
T_w – plate temperature
u – velocity component in the x-direction
v – velocity component in the y-direction
V_0 – uniform transpiration velocity
x – coordinate measuring distance along the plate
y – coordinate measuring distance normal to the plate
α – buoyancy parameter
β – coefficient of volume expansion
σ – electrical conductivity
η – kinematic coefficient of viscosity
ζ – scaled similarity variable
ξ – scaled streamwise coordinate
θ – scaled temperature
η – similarity variable
ψ – stream function
τ_w – nondimensional shear stress

Riley N. (1964): *Magnetohydrodynamic free convection.* - J. Fluid Mech., v.18, pp.267-247.

Singh K.R. and Cowling T.G. (1963): *Thermal convection in magnetohydrodynamic boundary layers.* - J. Mech. Appl. Math., v.16, pp.1-5.

Sparrow E.M., Eichorn R. and Gregg J.L. (1959): *Combined forced and free convection in a boundary layer flow.* - Physics Fluids, v.2, pp.319-328.

Sparrow E.M. and Gregg J.L. (1959): *Buoyancy effects in forced convention flow and heat transfer.* - ASME J. Appl. Mech., v.83, pp.133-134.

Sparrow E.M. and Cess R.D. (1961): *Effect of magnetic field on free convention heat transfer.* - Int. J. Heat Mass Transfer, v.3, pp.267-274.

Tingwei G., Bachrun P. and Daguent M. (1982): *Influence de la convection naturel le sur la convection force and dessus d'une surface plane vertical voumise a un flux de rayonnement.* - J. Heat Mass Transfer, v.25, pp.1061-1065.

Wilks G. (1973): *Combined forced and free convection flow on vertical surfaces.* - Int. J. Heat Mass Transfer, v.16, pp.1958-1964.

Wilks G. (1976): *Magnetohydrodynamic free convection about a semi-infinite vertical plate in a strong cross field.* - J. Appl. Math. Phys., v.27, pp.621-631.

Van Dyke M. (1975): *Perturbation Methods in Fluid Mechanics.* - California: Parabolic Press, pp.202-207.

Received: April 15, 1996