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The Effect of Anisotropic Permeability on Free Convective Boundary Layer Flow in Porous Media

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Abstract. The effect of an anisotropic permeability on thermal boundary layer flow in porous media is studied. The convective flow is induced by a vertical, uniformly heated surface embedded in a fluid-saturated medium. A leading-order boundary layer theory is presented. It is shown that the thickness of the resulting boundary layer flow is different from that obtained in an isotropic porous medium. In general, an anisotropic permeability induces a fluid drift in the spanwise direction, the strength of which depends on the precise nature of the anisotropy. Conditions are found which determine whether or not the boundary layer flow is three-dimensional.

Key words: Porous medium, convection, boundary layer, anisotropy, permeability.

1. Introduction

Thermally driven convection in porous media is of importance in a variety of geophysical and technological problems such as the modelling of geothermal reservoirs and thermal insulation systems, packed-bed catalytic reactors and heat storage devices. Work on the theory and modelling of porous media flows began nearly fifty years ago. So far, the substantial part of theoretical and experimental investigations have dealt with isotropic media. However, in many practical problems, the porous matrix is *anisotropic* in its mechanical and thermal properties. An example of such a medium is loft insulation which usually has a lower permeability across the insulating layer than it has in the perpendicular directions. Wooding (1978) also notes that, under certain circumstances, the horizontal permeability of a geothermal system can be up to ten times as large as the vertical component.

The first studies on natural convection in anisotropic porous media appeared in the middle of the seventies and have concentrated exclusively on the porous medium analogue of the Bénard problem, sometimes called the Lapwood problem. Castinel and Combarnous (1974) found the criterion for the onset of convection in a horizontal layer with anisotropic permeability. They also reported experimental results which agree fairly well with their theoretical predictions. Epherre (1975) extended the stability analysis by including anisotropy in the thermal diffusivity. It was shown that anisotropy in the mechanical and thermal properties affects the marginal stability criterion as well as the preferred width of the convection cells. On the other hand, Kvernvold and Tyvand (1979) showed that even a three-dimensioinal anisotropy does not lead to any essentially new flow patterns at onset compared with the isotropic case. But this is true only if one of the principal axes of the anisotropic medium is normal to the layer. Such a restriction has been maintained in almost all the former work in the field; see the review article by McKibbin (1984) and the monograph by Nield and Bejan (1992).

Tyvand and Storesletten (1991) seem to be the first to have studied natural convection in a horizontal porous layer where none of the principal axes is vertical. They considered the situation where the anisotropic permeability is transversely isotropic but the orientation of the longitudinal axis is arbitrary. This was sufficient to achieve qualitively new flow patterns with tilted planes of motion or tilted as well as curved cell walls; this depends, respectively, on whether the transverse permeability is larger or smaller than the longitudinal permeability. Storesletten (1993) has studied the analogous problem for a horizontal layer with anisotropy in the thermal diffusivity. There are again two different types of convection cells depending on whether the transverse diffusivity is smaller or larger than the longitudinal diffusivity. In the former case the convection cells have a rectangular cross-section with vertical lateral walls just like isotropic convection. In the latter case, however, the lateral cell walls are tilted as well as curved. Thus far, studies of natural convection in porous layers indicate that the effect of anisotropy in either the mechanical or the thermal properties of the medium has a much greater influence on the resulting convection pattern when none of the principal axes is normal to the layer.

The above-quoted papers all deal with Rayleigh–Bénard convection in anisotropic media. At present, the effect of anisotropy on thermal boundary layer flow in porous media is unknown; this is the subject of the present paper. The first papers to appear dealing with thermal boundary layer flow in porous media were by Cheng and Chang (1976) and Cheng and Minkowycz (1977). In these papers, certain geothermal formations are modelled by assuming that they are represented adequately by semi-infinite surfaces which are horizontal and vertical, respectively. Cheng and co-workers assumed further that the boundary layer approximation is valid and analysed the flow and heat transfer by determining the leading-order boundary layer flow. Later, this work was extended to higher-order by Chang and Cheng (1983), Daniels and Simpkins (1984), and Cheng and Hsu (1984) using the method of matched asymptotic expansions. These authors were able to obtain more accurate accounts of the rate of heat transfer into the porous medium.

In common with all the authors quoted in this paragraph we shall assume that the heated surface is maintained at a constant, steady temperature. However, we relax the assumption that the porous medium is mechanically isotropic. We consider a vertical heated surface and determine the resulting boundary layer flow. Our results can be shown to extend to generally inclined surfaces (except for small inclinations from the horizontal which results in a nonsimilar profile; see Rees and Riley (1985)) and to an upward-facing horizontal surface. Attention will be restricted to the leading-order boundary layer flow; preliminary results already completed indicate that a higher order analysis is quite involved and therefore different inclinations may have to be treated separately.

In Section 2 we derive the governing equations for anisotropic flow induced by a heated vertical surface. The leading-order boundary layer analysis is presented in Section 3 and the results are discussed briefly in Section 4.

2. Mathematical Formulation

We consider the free convective flow induced by heating uniformly a semi-infinite vertical surface embedded in an anisotropic fluid-saturated porous medium. The heated surface is maintained at a dimensional temperature T_1, whilst the ambient temperature is T_0, where $T_1 > T_0$. A Cartesian frame of reference is chosen, where the x-axis is aligned in the upward vertical direction, the y-axis is normal to the heated surface, and the z-axis is in the spanwise direction. The permeability tensor, $\mathbf{K}$, is given by

$$\mathbf{K} = K_1\mathbf{i}'\mathbf{i}' + K_2\mathbf{j}'\mathbf{j}' + K_3\mathbf{k}'\mathbf{k}', \tag{1}$$

where the right-handed set of unit vectors, $\mathbf{i}', \mathbf{j}'$ and $\mathbf{k}'$ are obtained by rotating the unit vectors in the x, y and z directions (respectively, $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$) by an angle α about the x-axis, followed by a rotation of an angle β about the y-axis and an angle γ about the z-axis, in that order. In other words we have,

$$\begin{aligned} \mathbf{i}' &= (\cos\beta\cos\gamma,\ \cos\beta\sin\gamma,\ -\sin\beta), \\ \mathbf{j}' &= (-\cos\alpha\sin\gamma + \sin\alpha\sin\beta\cos\gamma, \\ &\qquad \cos\alpha\cos\gamma + \sin\alpha\sin\beta\sin\gamma, \sin\alpha\cos\beta), \\ \mathbf{k}' &= (\sin\alpha\sin\gamma + \cos\alpha\sin\beta\cos\gamma, \\ &\qquad -\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma, \cos\alpha\cos\beta). \end{aligned} \tag{2}$$

The resulting permeability matrix in its most general form is rather complicated to present, but we can write it in the form

$$\mathbf{K} = \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{12} & K_{22} & K_{23} \\ K_{13} & K_{23} & K_{33} \end{pmatrix}, \tag{3}$$

where the K_{ij} values are given in the Appendix. We note that $\mathbf{K}$ is symmetric. Using K_1 as a reference permeability we can rewrite (3) as

$$\mathbf{K} = K_1\mathbf{L} = K_1 \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{pmatrix}, \tag{4}$$

where the various L_{ij} values $(i = 1, 2, 3, j = 1, 2, 3)$ are defined appropriately.

Steady flow in a porous medium for which Darcy's law and the Boussinesq approximation are both valid is governed by the equations

$$\operatorname{div} \mathbf{u} = 0, \tag{5a}$$

$$\mathbf{u} = -\frac{1}{\mu}\mathbf{K}\cdot(\nabla p + (T - T_0)\rho_0 b\mathbf{g}), \tag{5b}$$

$$\mathbf{u}\cdot\nabla T = \kappa\nabla^2 T, \tag{5c}$$

where μ is the fluid viscosity, κ, the thermal diffusivity of the saturated medium, ρ_0, the density of the fluid at temperature T_0, b, the coefficient of cubic expansion, $\mathbf{g}$, the gravity vector, $\mathbf{u}$, the velocity flux, T, the temperature, and p, the dynamic pressure. We nondimensionalise by setting

$$(x, y, z) = \frac{\mu\kappa}{\rho_0 g b K_1(T_1 - T_0)}(x^*, y^*, z^*), \tag{6a}$$

$$\mathbf{u} \equiv (u, v, w) = \frac{\rho_0 g b K_1(T_1 - T_0)}{\mu}(u^*, v^*, w^*), \tag{6b}$$

$$\theta = \frac{T - T_0}{T_1 - T_0} \quad \text{and} \quad p = \frac{\mu\kappa}{K_1}p^* \tag{6c,d}$$

into (5). On omitting the asterisks we obtain

$$\operatorname{div} \mathbf{u} = 0, \tag{7a}$$

$$\mathbf{u} = -\mathbf{L}\cdot\nabla p + \begin{pmatrix} L_{11} \\ L_{12} \\ L_{13} \end{pmatrix}\theta, \tag{7b}$$

$$\mathbf{u}\cdot\nabla\theta = \nabla^2\theta, \tag{7c}$$

We note that there is no nondimensional parameter in this problem. This is a consequence of the fact that there is no natural length scale, but rather that the material parameters of the fluid and the medium define a length scale according to (6a).

Although the medium is anisotropic and hence fluid motions will, in general, be induced in the z-direction, there is no reason to suppose that the resulting convective boundary layer exhibits z-variations. Hence Equations. (7) reduce to

$$u_x + v_y = 0 \tag{8a}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = -\mathbf{L} \cdot \begin{pmatrix} p_x \\ p_y \\ 0 \end{pmatrix} + \begin{pmatrix} L_{11} \\ L_{12} \\ L_{13} \end{pmatrix} \theta, \tag{8b}$$

$$u\theta_x + v\theta_y = \theta_{xx} + \theta_{yy} \tag{8c}$$

In view of the simplified continuity Equation, (8a), we define a stream function, ψ, according to

$$u = \psi_y, \qquad v = -\psi_x. \tag{9}$$

Finally, Equations (8) reduce to

$$L_{11}\psi_{xx} + 2L_{12}\psi_{xy} + L_{22}\psi_{yy} = (L_{11}L_{22} - L_{12}^2)\theta_y, \tag{10a}$$

$$\theta_{xx} + \theta_{yy} = \psi_y\theta_x - \psi_x\theta_y, \tag{10b}$$

$$w = \frac{(L_{13}L_{22} - L_{12}L_{23})\psi_y + (L_{12}L_{13} - L_{11}L_{23})\psi_x}{L_{11}L_{22} - L_{12}^2} \tag{10c}$$

The coefficients of ψ_y and ψ_x appearing in the numerator of (10c) are given in the Appendix.

3. Boundary Layer Analysis

In this section we develop the leading order boundary layer theory for convection induced in a porous medium with an anisotropic permeability tensor. We follow the same procedure as that of Riley and Rees (1985) who considered the flow at asymptotically large distances from the leading edge. It turns out that the boundary layer flow in an anisotropic medium can be expressed in terms of the isotropic solution, at least to leading order, and therefore we review the latter solution very briefly.

The isotropic case corresponds to when $L_{ij} = 1$ when $i = j$ and $L_{ij} = 0$ when $i \neq j$. On invoking the boundary layer approximation (that is, on taking $x \gg y$) Equations (10) reduce to

$$\psi_{yy} = \theta_y, \tag{11a}$$

$$\theta_{yy} = \psi_y\theta_x - \psi_x\theta_y, \tag{11b}$$

$$w = 0, \tag{11c}$$

which are to be solved subject to the boundary conditions, $\psi = 0, \theta = 1$ when $y = 0$, and $\psi_y, \theta \to 0$ as $y \to \infty$. Equations (11) admit the similarity solution

$$\psi = x^{1/2}F(\eta), \qquad \theta = G(\eta), \tag{12}$$

where $\eta = y/x^{1/2}$ is the similarity variable and where F and G satisfy

$$F'' = G'', \qquad G'' + \tfrac{1}{2}FG' = 0 \tag{13}$$

subject to the boundary conditions

$$F(0) = 0, \qquad F'(\eta) \to 0 \quad \text{as } \eta \to \infty, \tag{14a}$$

$$G(0) = 1, \qquad G(\eta) \to 0 \quad \text{as } \eta \to \infty. \tag{14b}$$

The solution to (13) and (14) is well-known and first appeared in the context of convection in porous media in Cheng and Minkowycz (1977).

We consider three special cases first before looking at the general case of arbitrary permeability ratios, K_2/K_1 and K_3/K_1, and rotations, α, β, and γ. Respectively, the four cases correspond to (i) rotation but the z-axis only, (ii) rotation about the x-axis only, (iii) rotation about the y-axis only, and (iv) arbitrary rotations. Case (i) corresponds to when the coefficients of both ψ_y and ψ_x in (10c) are both zero, case (ii) to when the coefficient of ψ_y is zero but that of ψ_x is nonzero, and cases (iii) and (in general) (iv) to when the coefficient of ψ_y is nonzero.

CASE 1. Rotation about the z-axis

Since both $\alpha = 0$ and $\beta = 0$, it follows from (A7) and (A8) that $w = 0$. Consequently there is no fluid drift in the z-direction and hence the flow is two-dimensional.

On invoking the boundary layer approximation, Equations (10) reduce to

$$\psi_{yy} = a^2\phi_y, \tag{15a}$$

$$\theta_{yy} = \psi_y\theta_x - \psi_x\theta_y, \tag{15b}$$

where

$$a^2 = \frac{K_2}{K_1\,\sin^2\gamma + K_2\,\cos^2\gamma}. \tag{16}$$

The boundary conditions are

$$\psi = 0, \qquad \theta = 1 \quad \text{when } y = 0, \tag{17a}$$

$$\psi_y, \theta \to 0 \quad \text{as } y \to \infty. \tag{17b}$$

Equations (15) admit the similarity solution

$$\psi = ax^{1/2}F(a\eta), \qquad \theta = G(a\eta), \tag{18}$$

where F and G satisfy Equations (13) and (14) corresponding to the isotropic case.

The primary effect of the anisotropic permeability, at leading order, is to change the boundary layer thickness from that of an isotropic medium. The boundary layer is thicker when $a < 1$, which is satisfied if and only if $K_1 > K_2$ and $\gamma \neq 0, \pi$. Conversely, the boundary layer is thinner when $K_1 < K_2$ and $\gamma \neq 0, \pi$. However, it is of the same thickness, and hence indistinguishable from the isotropic case, when $K_1 = K_2$ or when $\gamma = 0, \pi$.

As might have been anticipated, the magnitude of K_3 does not affect the flow at leading order.

CASE 2. Rotation about the x-axis

Since both $\beta = 0$ and $\gamma = 0$, it follows from (A7) that the coefficient of ψ_y in (10c) is zero, which implies that

$$w = -\frac{L_{23}}{L_{22}}\psi_x = \frac{1}{2}\frac{(K_2 - K_3)\sin(2\alpha)}{K_2\cos^2\alpha + K_3\sin^2\alpha}v. \tag{19}$$

On invoking the boundary layer approximation equations (10a) and (10b) reduce to (11a) and (11b), respectively, which corresponds to the isotropic case. Thus, a rotation of the principal axes about the x-axis yields (i) a flow in the x and y directions which is unchanged from the isotropic case, and (ii) an induced fluid drift in the z-direction which is positive when $(K_2 - K_3)\sin(2\alpha) < 0$ as the induced flow into the boundary layer, v, is negative. Hence w has the same sign as v if $(K_2 - K_3)\sin(2\alpha) > 0$. There is no drift in the z-direction if $K_2 = K_3$ or if $\alpha = \frac{1}{2}n\pi, n = 0, 1, 2$ or 3.

CASE 3: Rotation about the y-axis

Although both $\alpha = 0$ and $\gamma = 0$, the coefficient of ψ_y in (10c) is not necessarily zero; see (A7). We find that

$$w = \frac{L_{13}}{L_{11}}\psi_y = \frac{1}{2}\frac{(K_1 - K_3)\sin(2\beta)}{K_1\cos^2\beta + K_3\sin^2\beta}u. \tag{20}$$

On invoking the boundary layer approximation, Equations (10a) and (10b) reduce to a system of the same form as (15), where a is now given by

$$a^2 = L_{11} = \cos^2\beta + (K_3/K_1)\sin^2\beta. \tag{21}$$

As in Case 1, there exists a similarity solution given by (18), where F and G satisfy (13) and (14) which correspond to the isotropic case. Thus, a rotation of the principal axes about the y-axis yields a flow where the thickness of the boundary

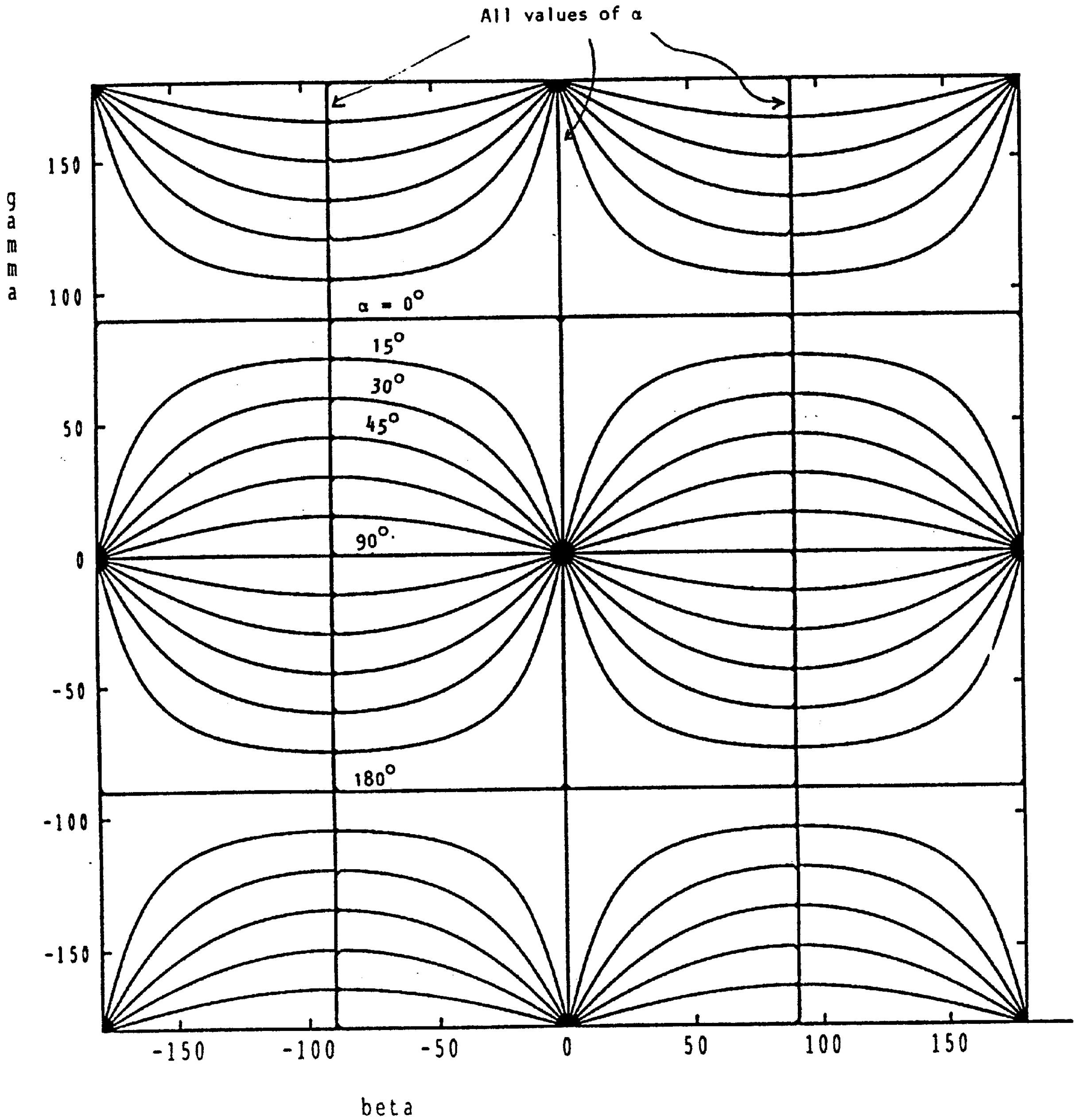

Fig. 1. Loci of values of β and γ for various values of α for which there is no O(1) spanwise fluid motion: $K_2 = K_1, K_3 = \frac{1}{2}K_1$.

layer is greater than in the isotropic case when $a < 1$, i.e. if and only if $K_1 > K_3$ and $\beta \neq 0, \pi$. Conversely, the boundary layer is thinner when $K_1 < K_3$ and $\beta \neq 0, \pi$. It is of the same thickness when either $K_1 = K_3, \beta = 0$ or $\beta = \pi$.

In addition, there is a fluid drift in the z-direction given by (20). There is no drift when $K_1 = K_3$ or $\beta = \frac{1}{2}n\pi (n = 0, 1, 2$ or $3)$. As $|\psi_y| \gg |\psi_x|$, the fluid drift is much stronger than in Case 2 (rotation about the x-axis). However, the leading-order drift is confined to the boundary layer in the present case as w is proportional to u, but in Case 2 it persists outside the boundary layer as w is proportional to v (however, this statement might need to be modified when higher-order effects are taken into account).

CASE 4. Arbitrary Rotations

The boundary layer approximation applied to Equations (10a) and (10b) yields equations of the same form as (15a,b) where

$$\begin{aligned} a^2 &= \frac{L_{11}L_{22} - L_{12}^2}{L_{22}} \\ &= \frac{\begin{array}{l} K_1K_2(\cos\alpha\cos\beta)^2 + K_1K_3(-\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma)^2 \\ \quad +K_2K_3(\sin\alpha\sin\gamma + \cos\alpha\sin\beta\cos\gamma)^2 \end{array}}{\begin{array}{l} K_1[K_1(\cos\beta\sin\gamma)^2 + K_2(\sin\alpha\sin\beta\sin\gamma - \cos\alpha\cos\gamma)^2 \\ \quad +K_3(\cos\alpha\sin\beta\sin\gamma + \sin\alpha\cos\gamma)^2] \end{array}} \end{aligned} \tag{22}$$

As in Cases 1 and 3 there exists a similarity solution given by (18) where F and G satisfy (13) and (14). Thus the boundary layer is thicker than, as thick as, or thinner than the isotropic boundary layer when $a < 1, a = 1$ or $a > 1$, respectively.

Since $x \gg y$ has been assumed as part of the boundary layer approximation, it follows that $|\psi_y| \gg |\psi_x|$, in general, and therefore the ψ_x term in (10c) can be neglected. To leading order, then, the fluid drift in the z-direction is given by

$$\begin{aligned} w &= \frac{L_{13}L_{22} - L_{12}L_{23}}{L_{11}L_{22} - L_{12}^2}\psi_y \\ &= \frac{\begin{array}{l} \cos\beta[K_1(K_3 - K_2)\sin\alpha\cos\alpha\sin\gamma \\ \quad +K_2(K_3 - K_1)\cos^2\alpha\sin\beta\cos\gamma \\ \quad +K_3(K_2 - K_1)\sin^2\alpha\sin\beta\cos\gamma] \end{array}}{\begin{array}{l} K_1K_2(\cos\alpha\cos\beta)^2 \\ \quad +K_1K_3(-\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma)^2 \\ \quad +K_2K_3(\sin\alpha\sin\gamma + \cos\alpha\sin\beta\cos\gamma)^2 \end{array}} u. \end{aligned} \tag{23}$$

In general, then, arbitrary rotations of the principal axes of the permeability tensor lead to an induced drift in the spanwise direction with a magnitude which is of $O(1)$ as $x \to \infty$. Although we have already dealt with two cases above (Cases 1 and 2) for which this $O(1)$ drift is absent, there are other configurations for which this is also true. Given the number of free parameters (three angles and two permeability ratios) we can not give a general statement about when this drift is absent except to make the obvious point that the numerator of (23) has to be zero. However, in Figures 1 and 2 we present some examples of configurations where the $O(1)$ drift is absent. In Figure 1 we set $K_2 = K_1$ and $K_3 = \frac{1}{2}K_1$ and plot loci in the β-γ plane of where the drift is absent, for various values of α. We note that this drift is absent for all values of α when β is an odd integer multiple of a right angle. Figure 2 is more complicated and represents a case where $K_2 = \frac{1}{2}K_1$ and $K_3 = \frac{1}{4}K_1$. In both cases the figures are identical modulo 180 degrees in α.

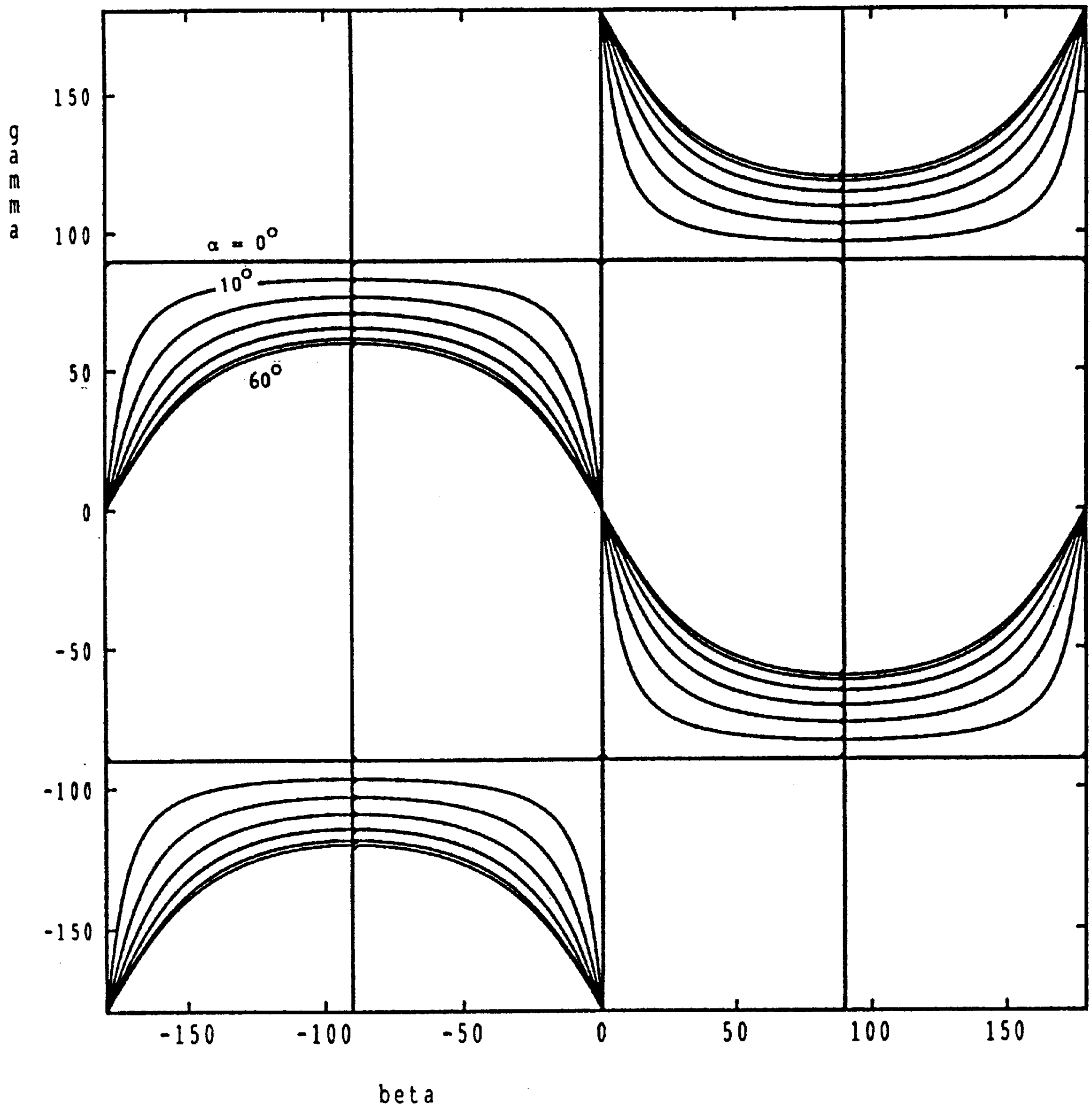

Fig. 2a. Loci of values of β and γ for various values of α for which there is no O(1) spanwise fluid motion: $K_2 = \frac{1}{2}K_1, K_3 = \frac{1}{4}K_1$. (a) $0° \leqslant \alpha \leqslant 60°$, (b) $60° \leqslant \alpha \leqslant 120°$, (c) $120° \leqslant \alpha \leqslant 180°$.

When the O(1) drift is absent we have to consider the ψ_x term in (10c). In such cases the induced drift is proportional to $x^{-1/2}$ as $x \to \infty$ and is given by

$$\begin{aligned} w &= \frac{L_{12}L_{13} - L_{11}L_{23}}{L_{11}L_{22} - L_{12}^2}\psi_x \\ &= -\frac{\begin{array}{c}\cos\beta[K_1(K_3 - K_2)\sin\alpha\cos\alpha\cos\gamma \\ -K_2(K_3 - K_1)\cos^2\alpha\sin\beta\sin\gamma \\ -K_3(K_2 - K_1)\sin^2\alpha\sin\beta\sin\gamma]\end{array}}{\begin{array}{c}K_1K_2(\cos\alpha\cos\beta)^2 \\ +K_1K_3(-\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma)^2 \\ +K_2K_3(\sin\alpha\sin\gamma + \cos\alpha\sin\beta\cos\gamma)^2\end{array}}v. \end{aligned} \quad (24)$$

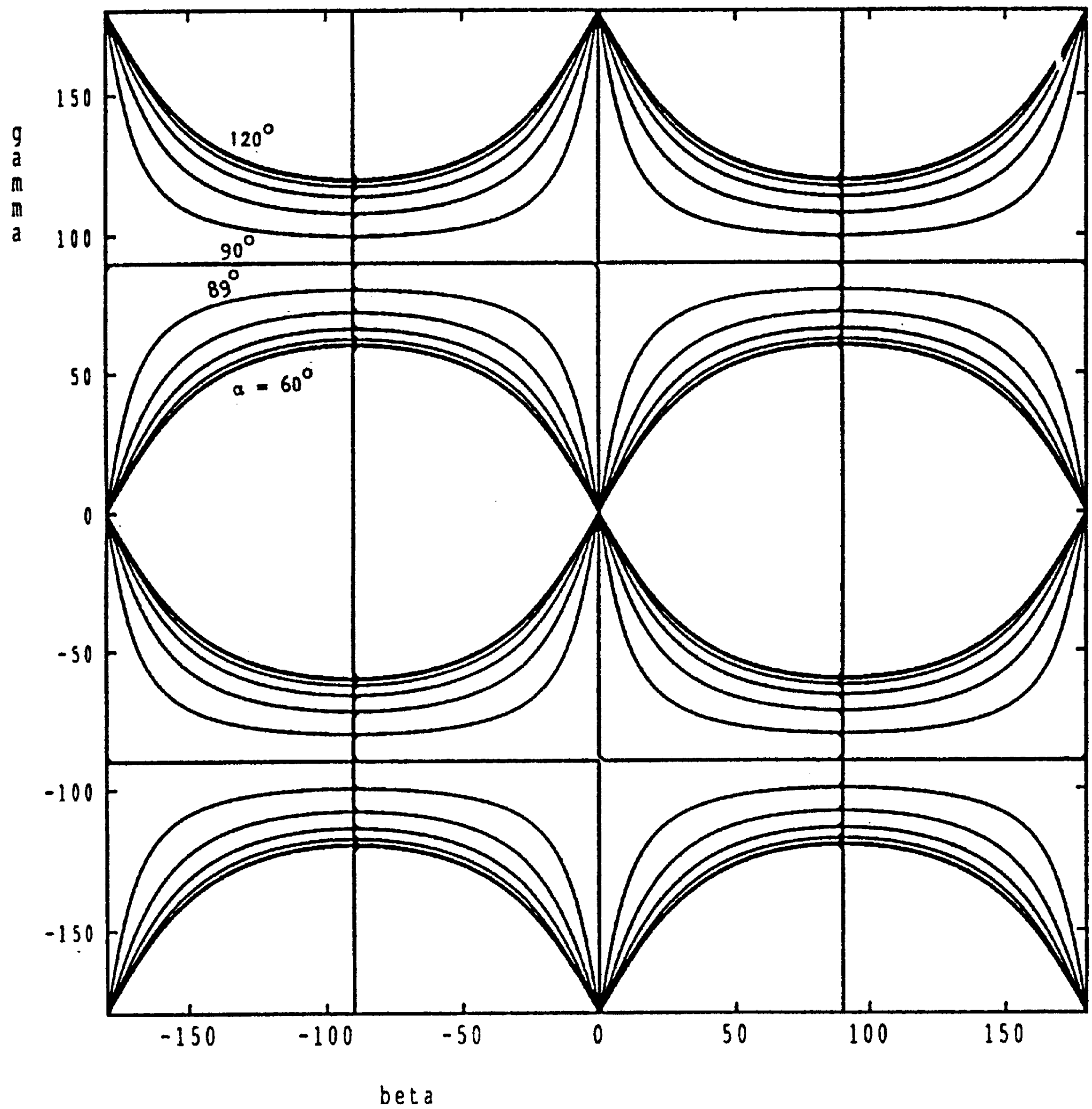

Fig. 2b.

In general the coefficient of ψ_x will be nonzero and therefore there will be a spanwise fluid motion.

Despite the complexity of the formulae (23) and (24) it is quite straightforward to derive the conditions for which there is no spanwise fluid drift. These conditions are as follows:

(i)	$K_1 = K_2 = K_3$	(isotropic medium)
(ii)	$K_2 = K_3,\ \beta = n\pi$	($\mathbf{i}'$ in the x–y plane)
(iii)	$K_1 = K_3,\ \alpha = n\pi$	($\mathbf{j}'$ in the x–y plane)
(iv)	$K_1 = K_2,\ \alpha = (n + \frac{1}{2})\pi$	($\mathbf{k}'$ in the x–y plane)
(v)	$\alpha = n\pi,\ \beta = m\pi$	(both $\mathbf{i}'$ and $\mathbf{j}'$ in the x–y plane)
(vi)	$\alpha = (n + \frac{1}{2})\pi,\ \beta = m\pi$	(both $\mathbf{i}'$ and $\mathbf{k}'$ in the x–y plane)
(vii)	$\beta = (m + \frac{1}{2})\pi$	(both $\mathbf{j}'$ and $\mathbf{k}'$ in the x–y plane)

where both n and m are integers.

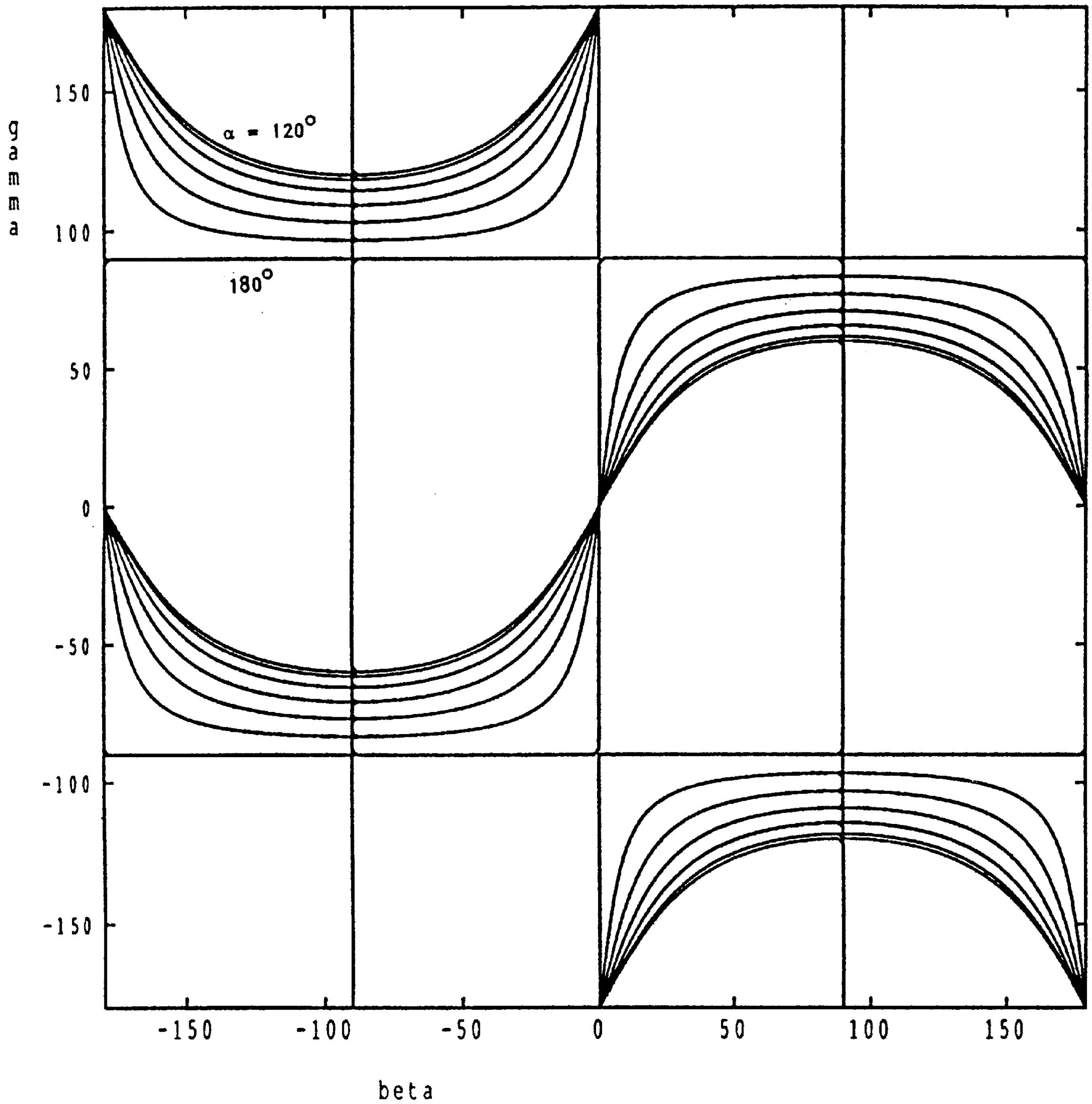

Fig. 2c.

4. Discussion

In the main body of the paper we have considered the effects of anisotropic permeability on convection induced by a vertical, uniformly heated surface embedded in a fluid-saturated porous medium. In particular we have focused on the effects of anisotropy on the leading order boundary layer flow. There are two primary effects which are evident when considering this flow, namely, (i) a changed boundary layer thickness (which has important ramifications for the rate of heat transferred into the porous medium), and (ii) an anisotropically induced fluid drift in the z-direction. It has been shown that the presence or absence of either or both of these effects is dependent on the precise nature of the anisotropy. In the case of the induced spanwise drift the asymptotic strength of the w-velocity is either of $O(1)$ as $x \to \infty$ or else it is of $O(x^{-1/2})$. Conditions have been presented which determine when drift is present or absent, and, if present, how strong it is.

An almost identical analysis can be presented for the equivalent horizontal thermal boundary layer flow in a porous medium even though the definition of the similarity variable is different for this case (see Cheng and Chang, 1976). When the heated surface is inclined from the horizontal with its leading edge below the rest of the surface an identical analysis to the above can be undertaken if account is taken of the reduced buoyancy force along the surface by means of a simple rescaling of the similarity variable.

It is our intention to extend the work of this paper to higher order in order to obtain a more accurate account of the effects of anisotropy on free convective boundary layers in porous media.

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Appendix

The definitions of the entries of the permeability tensor given in Equation (3) are the following,

$$\begin{aligned} K_{11} &= K_1 \cos^2\beta \cos^2\gamma + K_2(\sin\alpha\, \sin\beta\, \cos\gamma + \cos\alpha\, \sin\gamma)^2 \\ &\quad + K_3(\cos\alpha \sin\beta \cos\gamma + \sin\alpha \sin\gamma)^2, \end{aligned} \tag{A1}$$

$$\begin{aligned} K_{22} &= K_1 \cos^2\beta \sin^2\gamma + K_2(\sin\alpha\, \sin\beta\, \sin\gamma - \cos\alpha\, \cos\gamma)^2 \\ &\quad + K_3(\cos\alpha \sin\beta \sin\gamma + \sin\alpha \cos\gamma)^2, \end{aligned} \tag{A2}$$

$$K_{33} = K_1 \sin^2\beta + K_2 \sin^2\alpha \cos^2\beta + K_3 \cos^2\alpha\, \cos^2\beta, \tag{A3}$$

$$\begin{aligned} K_{12} &= K_1 \cos^2\beta\, \sin\gamma\, \cos\gamma + K_2(\sin\alpha\, \sin\beta \cos\gamma + \cos\alpha\, \sin\gamma) \\ &\quad \times(\sin\alpha\, \sin\beta\, \sin\gamma - \cos\alpha \cos\gamma) \\ &\quad + K_3(\cos\alpha\, \sin\beta\, \cos\gamma - \sin\alpha\, \sin\gamma) \\ &\quad \times(\cos\alpha\, \sin\beta\, \sin\gamma + \sin\alpha\, \cos\gamma), \end{aligned} \tag{A4}$$

$$\begin{aligned} K_{13} &= K_1 \sin\beta\, \cos\beta\, \cos\gamma - K_2(\sin\alpha\, \sin\beta\, \cos\gamma + \cos\alpha \sin\gamma) \sin\alpha\, \cos\beta \\ &\quad - K_3(\cos\alpha\, \sin\beta\, \cos\gamma - \sin\alpha\, \sin\gamma) \cos\alpha\, \cos\beta, \end{aligned} \tag{A5}$$

$$K_{23} = K_1 \sin\beta \cos\beta \sin\gamma - K_2(\sin\alpha \sin\beta \sin\gamma - \cos\alpha \cos\gamma) \sin\alpha \cos\beta \\ -K_3(\cos\alpha \sin\beta \sin\gamma + \sin\alpha \cos\gamma) \cos\alpha \cos\beta. \qquad (A6)$$

The coefficient of ψ_y appearing in the numerator of Equation (10c) is

$$\cos\beta[K_1(K_3 - K_2) \sin\alpha \cos\alpha \sin\gamma + K_2(K_3 - K_1) \cos^2\alpha \sin\beta \cos\gamma \\ +K_3(K_2 - K_1) \sin^2\alpha \sin\beta \cos\gamma]/K_1^2, \qquad (A7)$$

and the corresponding coefficient of ψ_x is

$$\cos\beta[K_1(K_3 - K_2) \sin\alpha \cos\alpha \cos\gamma - K_2(K_3 - K_1) \cos^2\alpha \sin\beta \sin\gamma \\ -K_3(K_2 - K_1) \sin^2\alpha \sin\beta \sin\gamma]/K_1^2. \qquad (A8)$$

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