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Boundary layer flow and heat transfer on a continuous moving wavy surface

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Summary. The effect of spatially stationary surface waves on the forced convection induced by a moving surface in an otherwise quiescent fluid is examined. We consider the boundary layer regime where the Reynolds number Re is very large, and assume that the surface waves have $O(1)$ amplitude and wavelength. The boundary layer approximation is valid and the resulting parabolic equations are solved using the Keller-box scheme. Detailed results for the local skin-friction coefficient are presented, as are results for the local Nusselt number for both the cases of a constant wall temperature and a constant wall heat flux.

Notation

a	amplitude of the wavy surface
C_f	skin friction coefficient
f	reduced streamfunction
g, h	reduced temperatures
k	thermal conductivity
l	lengthscale associated with the surface waves
$\boldsymbol{n}$	unit vector normal to the wavy surface
Nu	local Nusselt number
p	pressure
Pr	Prandtl number
q	rate of heat flux
Re	Reynolds number based on l
Re_x	local Reynolds number
s_t	surface profile
T	temperature
u, v	fluid velocities in the (x, y)-directions
x, y	streamwise and cross-stream cartesian coordinates

Greek symbols

ξ, η	pseudo-similarity variables
σ	notation; see equation (5)
θ	dimensionless temperature
μ	dynamic viscosity
ν	kinematic viscosity
ϱ	density
τ	skin friction
ψ	streamfunction
ϕ	wave phase

* Dedicated to Professor Dr.-Ing. Dr. techn. E. H. Jürgen Zierep on the occasion of his 65th birthday

Superscripts

$\bar{}$	dimensional variables
$\hat{}$	transformed variables ($\xi < 1$)
$\tilde{}$	boundary layer variables
$'$	differentiaition with respect to $\tilde{x}$ or ξ

Subscripts

w	condition at the wall
∞	condition at infinity

1 Introduction

The study of the flow and heat transfer created by a moving wall in an otherwise quiescent fluid is relevant to several applications in the fields of metallurgy and chemical engineering. A number of technical processes concerning polymers involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. In these cases the properties of the final product depend to a great extent on the rate of cooling which, in turn, is governed by the structure of the boundary layer near the moving strip. Due to the entrainment of the ambient fluid, this boundary layer is different from the Blasius boundary layer flow past a flat plate. Sakiadis [1] was probably the first to study the boundary layer flow due to a moving wall in fluid at rest. Subsequently, many investigators have studied various aspects of this important problem, but we mention here only the papers in references [2]–[15].

The present analysis aims to study the flow due to and the heat transfer from a moving wavy surface in a fluid which is at rest. The effects of the amplitude of the wavy surface is studied in the forced convection regime where the boundary layer approximation is valid. We consider two different cases, namely, (i) that of a prescribed constant wall temperature (CWT), and (ii) a prescribed constant heat flux (CHF). Such surfaces can been considered to be good approximations to many practical geometries for which flow and heat transfer characteristics are of interest. The resulting non-similar boundary equations form a set of parabolic partial differential equations which is solved using the Keller-box method. The distribution of the velocity and temperature fields, as well as the skin friction coefficient, Nusselt number and the wall temperature along the wavy surface are presented. We note, in passing, that the method used in this paper is similiar to that used by the authors [16], [17] for the problem of free convection flow from a vertical wavy surface embedded in a porous medium.

2 Governing equations

Consider a wavy surface moving tangentially to itself with a constant speed U_0 through a stagnant incompressible fluid of constant temperature T_∞ as shown in Fig. 1. We assume either that the surface temperature remains uniform at $T_w (T_w \neq T_\infty)$, or that the heat flux at the surface remains uniform at q_w. The basic nondimensional equations governing steady flow are

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x\,\partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\,\nabla^2\left(\frac{\partial \psi}{\partial y}\right), \tag{1}$$

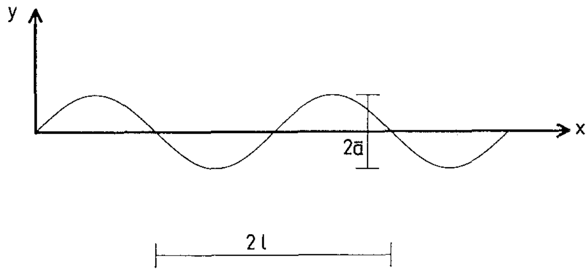

Fig. 1. Physical model and coordinate system depicting transverse surface waves

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x^2}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x\partial y}=\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\nabla^2\left(\frac{\partial\psi}{\partial x}\right), \tag{2}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\frac{1}{(\mathrm{Pr\,Re})}\nabla^2\theta. \tag{3}$$

The boundary conditions at $y = s_t(x) = a \sin(\pi x - \phi)$ are

$$\psi=0,\quad \frac{\partial\psi}{\partial y}=\sigma^{-1},\quad \theta=1\quad(\mathrm{CWT}),\quad \frac{\partial\theta}{\partial y}-s_t'\frac{\partial\theta}{\partial x}=-\sigma,\quad(\mathrm{CHF}) \tag{4.1}$$

whilst we have, as $y \to \infty$,

$$\frac{\partial\psi}{\partial x},\quad \frac{\partial\psi}{\partial y},\quad \theta,\quad (p-p_\infty)\to 0. \tag{4.2}$$

Here, σ is defined according to

$$\sigma=\sqrt{1+(s_t')^2}. \tag{5}$$

The nondimensional variables are obtained using the following scalings:

$$x=\frac{\bar{x}}{l},\quad y=\frac{\bar{y}}{l},\quad \psi=\frac{\bar{\psi}}{U_0 l},\quad p=\frac{\bar{p}}{\varrho U_0{}^2},\quad a=\frac{\bar{a}}{l},\quad \mathrm{Re}=\frac{U_0 l}{u},$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\ (\mathrm{CWT})\quad \theta=\frac{T-T_\infty}{q_w l}\ (\mathrm{CHF}). \tag{6}$$

Here ψ is a nondimensional streamfunction which is defined in the usual way: $(u, v) = (\psi_y, -\psi_x)$, and $\bar{a}$ is the amplitude of the wavy surface.

The effect of the wavy surface can be transferred from the boundary conditions (4) to the equations by means of the transformation given by

$$\hat{x}=x,\quad \hat{y}=y-s_t(x). \tag{7}$$

Equations (1) to (3) then become

$$\frac{\partial\psi}{\partial\hat{y}}\frac{\partial^2\psi}{\partial\hat{x}\,\partial\hat{y}}-\frac{\partial\psi}{\partial\hat{x}}\frac{\partial^2\psi}{\partial\hat{y}^2}=-\frac{\partial p}{\partial\hat{x}}+s_t'(\hat{x})\frac{\partial p}{\partial\hat{y}}+\mathrm{Re}^{-1}\mathscr{L}_1\psi, \tag{8}$$

$$\frac{\partial \psi}{\partial \hat{y}} \frac{\partial^2 \psi}{\partial \hat{x}^2} - \frac{\partial \psi}{\partial \hat{x}} \frac{\partial^2 \psi}{\partial \hat{x}\, \partial \hat{y}} + s_t'(\hat{x}) \left(\frac{\partial \psi}{\partial \hat{x}} \frac{\partial^2 \psi}{\partial \hat{y}^2} - \frac{\partial \psi}{\partial \hat{y}} \frac{\partial^2 \psi}{\partial \hat{x}\, \partial \hat{y}} \right) + s_t''(\hat{x}) \left(\frac{\partial \psi}{\partial \hat{y}} \right)^2 = \frac{\partial p}{\partial \hat{y}} + \frac{\mathscr{L}_2 \psi}{\mathrm{Re}}, \tag{9}$$

$$\frac{\partial \psi}{\partial \hat{y}} \frac{\partial \theta}{\partial \hat{x}} - \frac{\partial \psi}{\partial \hat{x}} \frac{\partial \theta}{\partial \hat{y}} = \frac{\mathscr{L}_3 \theta}{\mathrm{Pr\, Re}} \tag{10}$$

which are to be solved subject to

$$\psi = 0, \quad \frac{\partial \psi}{\partial \hat{y}} = \sigma^{-1}, \quad \theta = 1 \quad (\mathrm{CWT}), \quad \sigma^2 = \frac{\partial \theta}{\partial \hat{y}} - s_t'(\hat{x}) \frac{\partial \theta}{\partial \hat{x}} = -\sigma, \quad (\mathrm{CHF}) \tag{11.1}$$

on $\hat{y} = 0$, and

$$\frac{\partial \psi}{\partial \hat{x}}, \quad \frac{\partial \psi}{\partial \hat{y}}, \quad \theta, \quad (p - p_\infty) \to 0 \tag{11.2}$$

as $\hat{y} \to \infty$. In (10) the three operators, $\mathscr{L}_1$, $\mathscr{L}_2$ and $\mathscr{L}_3$ are defined as follows:

$$\mathscr{L}_1 = \sigma^2 \frac{\partial^3}{\partial \hat{y}^3} + \frac{\partial^3}{\partial \hat{y}\, \partial \hat{x}^2} - 2 s_t'(\hat{x}) \frac{\partial^3}{\partial \hat{y}^2\, \partial \hat{x}} - s_t''(\hat{x}) \frac{\partial^2}{\partial \hat{y}^2}, \tag{12.1}$$

$$\begin{aligned} \mathscr{L}_2 = & -s_t'(\hat{x})\, \sigma^2 \frac{\partial^3}{\partial \hat{y}^3} + (1 + 3 s_t''(\hat{x})) \frac{\partial^3}{\partial \hat{y}^2\, \partial \hat{x}} + \frac{\partial^3}{\partial \hat{x}^3} - 3 s_t'(\hat{x}) \frac{\partial^3}{\partial \hat{y}\, \partial \hat{x}^2} - 3 s_t''(\hat{x}) \frac{\partial^2}{\partial \hat{x}\, \partial \hat{y}} \\ & + 3 s_t'(\hat{x})\, s_t''(\hat{x}) \frac{\partial^2}{\partial \hat{y}^2} - s_t'''(\hat{x}) \frac{\partial}{\partial \hat{y}}, \end{aligned} \tag{12.2}$$

$$\mathscr{L}_3 = \sigma^2 \frac{\partial^2}{\partial \hat{y}^2} + \frac{\partial^2}{\partial \hat{x}^2} - 2 s_t'(\hat{x}) \frac{\partial^2}{\partial \hat{x}\, \partial \hat{y}} - s_t''(\hat{x}) \frac{\partial}{\partial \hat{y}}. \tag{12.3}$$

Next we introduce the boundary layer scalings,

$$\tilde{x} = \hat{x}, \quad \tilde{y} = \mathrm{Re}^{1/2} \hat{y}, \quad \tilde{\psi} = \mathrm{Re}^{1/2} \psi, \quad \tilde{p} = p - p_\infty, \tag{13.1}$$

$$\tilde{\theta} = \theta \quad (\mathrm{CWT}) \quad \tilde{\theta} = \mathrm{Re}^{1/2} \theta \quad (\mathrm{CHF}). \tag{13.2}$$

Thus, on introducing (13) into (8) to (11) and formally letting $\mathrm{Re} \to \infty$, we obtain the following *boundary layer* equations,

$$\frac{\partial \tilde{\psi}}{\partial \tilde{y}} \frac{\partial^2 \tilde{\psi}}{\partial \tilde{x}\, \partial \tilde{y}} - \frac{\partial \tilde{\psi}}{\partial \tilde{x}} \frac{\partial^2 \tilde{\psi}}{\partial \tilde{y}^2} = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + s_t'(\tilde{x}) \sqrt{\mathrm{Re}} \frac{\partial p}{\partial \tilde{y}} + \sigma^2 \frac{\partial^3 \tilde{\psi}}{\partial \tilde{y}^3}, \tag{14}$$

$$s_t'(\tilde{x}) \left(\frac{\partial \tilde{\psi}}{\partial \tilde{x}} \frac{\partial^2 \tilde{\psi}}{\partial \tilde{y}} - \frac{\partial \tilde{\psi}}{\partial \tilde{y}} \frac{\partial^2 \tilde{\psi}}{\partial \tilde{x}\, \partial \tilde{y}} \right) - s_t''(\tilde{x}) \left(\frac{\partial \tilde{\psi}}{\partial \tilde{y}} \right)^2 + s_t'(\tilde{x})\, \sigma^2 \frac{\partial^3 \tilde{\psi}}{\partial \tilde{y}^3} = \sqrt{\mathrm{Re}} \frac{\partial p}{\partial y}, \tag{15}$$

$$\frac{\partial \tilde{\psi}}{\partial \tilde{y}} \frac{\partial \tilde{\theta}}{\partial \tilde{x}} - \frac{\partial \tilde{\psi}}{\partial \tilde{x}} \frac{\partial \tilde{\theta}}{\partial \tilde{y}} = \frac{1}{\mathrm{Pr}} \sigma^2 \frac{\partial^2 \tilde{\theta}}{\partial \tilde{y}^2}, \tag{16}$$

together with the boundary conditions,

$$\tilde{\psi} = 0, \quad \frac{\partial \tilde{\psi}}{\partial \tilde{y}} = \sigma^{-1}, \quad \tilde{\theta} = 1 \quad (\mathrm{CWT}), \quad \sigma^2 \frac{\partial \tilde{\theta}}{\partial \tilde{y}} - s_t'(\tilde{x}) \frac{\partial \tilde{\theta}}{\partial \tilde{x}} = -\sigma, \quad (\mathrm{CHF}) \tag{17.1}$$

on $\tilde{y} = 0$, and

$$\frac{\partial \tilde{\psi}}{\partial \tilde{y}}, \quad \tilde{\theta}, \quad \tilde{p} \to 0 \quad \text{as} \quad \tilde{y} \to \infty. \tag{17.2}$$

Equation (15) indicates that the pressure gradient in the y-direction must be $O(\mathrm{Re}^{-1/2})$. This implies that the lowest order pressure gradient in the x-direction can be determined from the inviscid flow solution. For the present problem, the inviscid flow field is at rest and hence the pressure gradient is zero. For the present problem, Eq. (15) shows that $\mathrm{Re}^{1/2}\, \partial\tilde{p}/\partial\tilde{y}$ is $O(1)$ and is determined by the left-hand side of the equation. Elimination of this pressure gradient Eqs. (14) and (15) results in the following equations,

$$\frac{\partial \tilde{\psi}}{\partial \tilde{y}} \frac{\partial^2 \tilde{\psi}}{\partial \tilde{x}\, \partial \tilde{y}} - \frac{\partial \tilde{\psi}}{\partial \tilde{x}} \frac{\partial^2 \tilde{\psi}}{\partial \tilde{y}^2} + \frac{\sigma'}{\sigma}\left(\frac{\partial \tilde{\psi}}{\partial \tilde{y}}\right)^2 = \sigma^2 \frac{\partial^3 \tilde{\psi}}{\partial \tilde{y}^3}, \tag{18}$$

$$\frac{\partial \tilde{\psi}}{\partial \tilde{y}} \frac{\partial \tilde{\theta}}{\partial \tilde{x}} - \frac{\partial \tilde{\psi}}{\partial \tilde{x}} \frac{\partial \tilde{\theta}}{\partial \tilde{y}} = \frac{1}{\mathrm{Pr}} \sigma^2 \frac{\partial^2 \tilde{\theta}}{\partial \tilde{y}^2}, \tag{19}$$

where $\sigma' = d\sigma/d\tilde{x}$. These equations are subject to the boundary conditions (17).

To solve Eqs. (18) and (19), we introduce the further transformation,

$$\xi = \tilde{x}, \quad \tilde{\psi} = \sigma \sqrt{\xi}\, f(\xi, \eta), \tag{20.1}$$

$$\tilde{\theta} = g(\xi, \eta) \quad \text{(CWT)}, \quad \tilde{\theta} = \sqrt{\xi}\, h(\xi, \eta) \quad \text{(CHF)}, \tag{20.2}$$

where

$$\eta = \frac{\tilde{y}}{\sigma \sqrt{\tilde{x}}} \tag{20.3}$$

is the pseudo-similarity variable. Substituting (20) into Eqs. (18) and (19) gives

$$f_{\eta\eta\eta} + \frac{f f_{\eta\eta}}{2} + \frac{\sigma'}{\sigma}\, \xi (f f_{\eta\eta} - f_\eta^2) = \xi (f_\eta f_{\eta\xi} - f_\xi f_{\eta\eta}), \tag{21}$$

$$\frac{1}{\mathrm{Pr}}\, g_{\eta\eta} + \frac{f g_\eta}{2} + \frac{\sigma'}{\sigma}\, \xi f g_\eta = \xi (f_\eta g_\xi - f_\xi g_\eta), \tag{22}$$

for the (CWT) case and

$$\frac{1}{\mathrm{Pr}}\, h_{\eta\eta} + \frac{f h_\eta - f_\eta h}{2} + \frac{\sigma'}{\sigma}\, \xi f h_\eta = \xi (f_\eta h_\xi - f_\xi h_\eta), \tag{23}$$

for the (CHF) case. The corresponding boundary conditions are

$$f = 0, \quad f_\eta = \frac{1}{\sigma}, \quad g = 1 \quad \text{(CWT)}, \quad h_\eta = -1 \quad \text{(CHF)} \quad \text{at} \quad \eta = 0, \tag{24.1}$$

and

$$f_\eta, g, h \to 0 \quad \text{as} \quad \eta \to \infty. \tag{24.2}$$

Important physical quantities are the local skin friction coefficient and the local Nusselt number defined as

$$C_f = \frac{\tau_w}{\varrho U_0{}^2}, \qquad Nu = \frac{\bar{x} q_w}{k(T_w - T_\infty)} \tag{25}$$

where

$$\tau_w = \mu \boldsymbol{n} . \nabla \bar{\boldsymbol{u}}, \qquad q_w = -k \boldsymbol{n} . \nabla T \tag{26}$$

and $\boldsymbol{n} = \left(-s_t'(x), 1\right)/\sigma$ is the unit vector normal to the wavy surface. Using (6), (7) and (13), we obtain

$$\frac{C_f}{\sqrt{\mathrm{Re}_x}} = \sigma f_{\eta\eta}(\xi, 0), \tag{27.1}$$

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}_x}} = -g_\eta(\xi, 0), \quad \text{(CWT)} \tag{27.2}$$

where the local Reynolds number is $\mathrm{Re}_x = U_0 \bar{x}/\nu$. For the CHF case the wall temperature is given by $T_w - T_\infty = (q_w l/k)\, \mathrm{Re}_x{}^{-1/2}\, \xi h(s, 0)$ and hence

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}} = -\frac{1}{h(\xi, 0)}. \tag{27.3}$$

3 Results and discussion

Equations (21) to (23) subject to the boundary conditions (24) were solved numerically using the Keller-box method for different values of the wave amplitude, a and wave phase, ϕ. In all the computations presented here we used a step size of 0.01 in the ξ-direction and we took a nonuniform grid comprised of 80 points over the range $0 \leqq \eta \leqq 40$; we claim that our solutions are accurate to between the 3rd and the 4th significant figure. Using a Prandtl number of 0.7, to which we restrict ourselves in this paper, we find that the code reproduces the plane-wall ($a = 0$) solutions given by

$$f_{\eta\eta}(\xi, 0) = -0.4438 \quad \text{(see Ref. [11])}, \tag{28.1}$$

$$g_\eta(\xi, 0) = -0.3492 \quad \text{(see Ref. [2])}, \tag{28.2}$$

$$h(\xi, 0) = 2.8633, \tag{28.3}$$

where the final figure has been computed by ourselves.

In Figs. 2a, b, c we present the variations of $f_{\eta\eta}(\xi, 0)$, $g_\eta(0)$, and $h(\xi, 0)$, respectively, for various wave amplitudes when the wave phase is $\phi = \pi/2$; this phase corresponds to the surface being parallel to the x-axis at the origin. As the wave amplitude increases the skin-friction coefficient and the Nusselt number for both the (CWT) and (CHF) cases decrease. The graph of $f_{\eta\eta}(\xi, 0)$ seems to settle very quickly into a periodic state, whilst the graphs of $g_\eta(\xi, 0)$ and $h(\xi, 0)$ both have a periodic component which decays, the decay being very slow in the (CWT) case but quick in the (CHF) case. The corresponding graphs for the wave phase, $\phi = 0$, are given in Figs. 3a, b, c. For

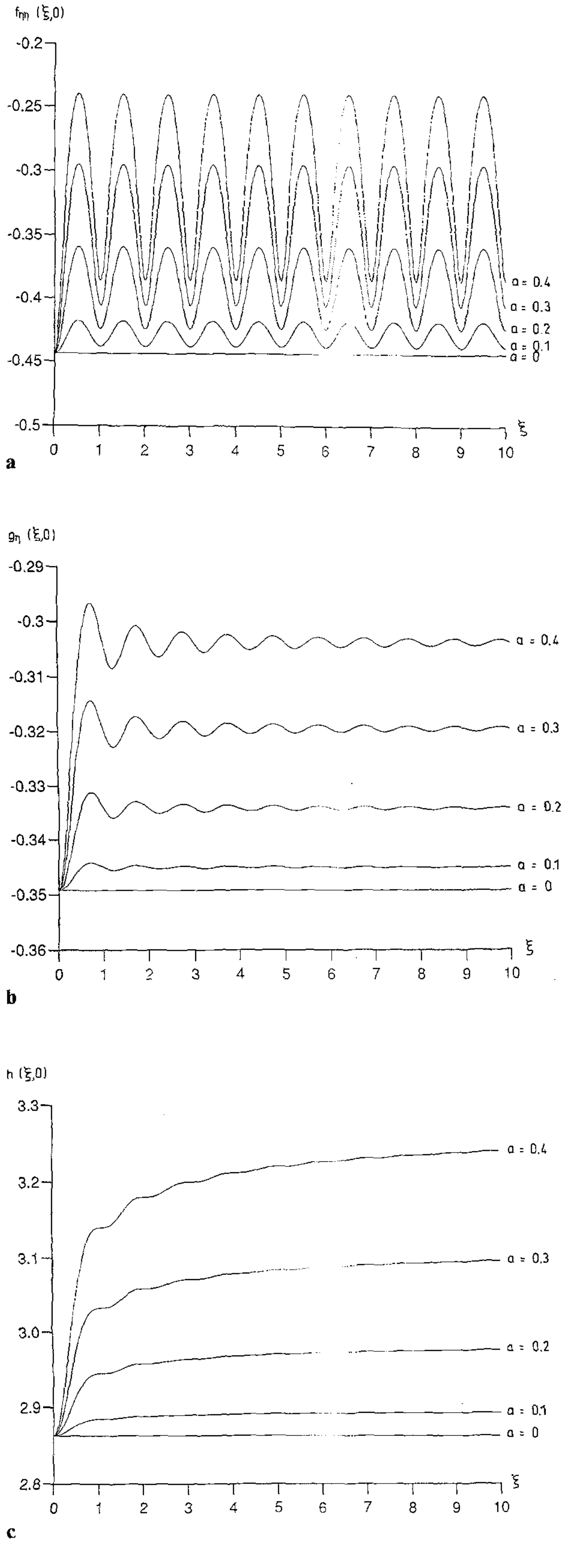

Fig. 2. Variation of **a** the skin-friction, $f_{\eta\eta}(\xi, 0)$, **b** the local rate of heat transfer, $g_\eta(\xi, 0)$ (CWT), and **c** the wall temperature, $h(\xi, 0)$ (CHF), for wave amplitudes, $a = 0$, 0.1, 0.2, 0.3 and 0.4 and phase, $\phi = 90°$

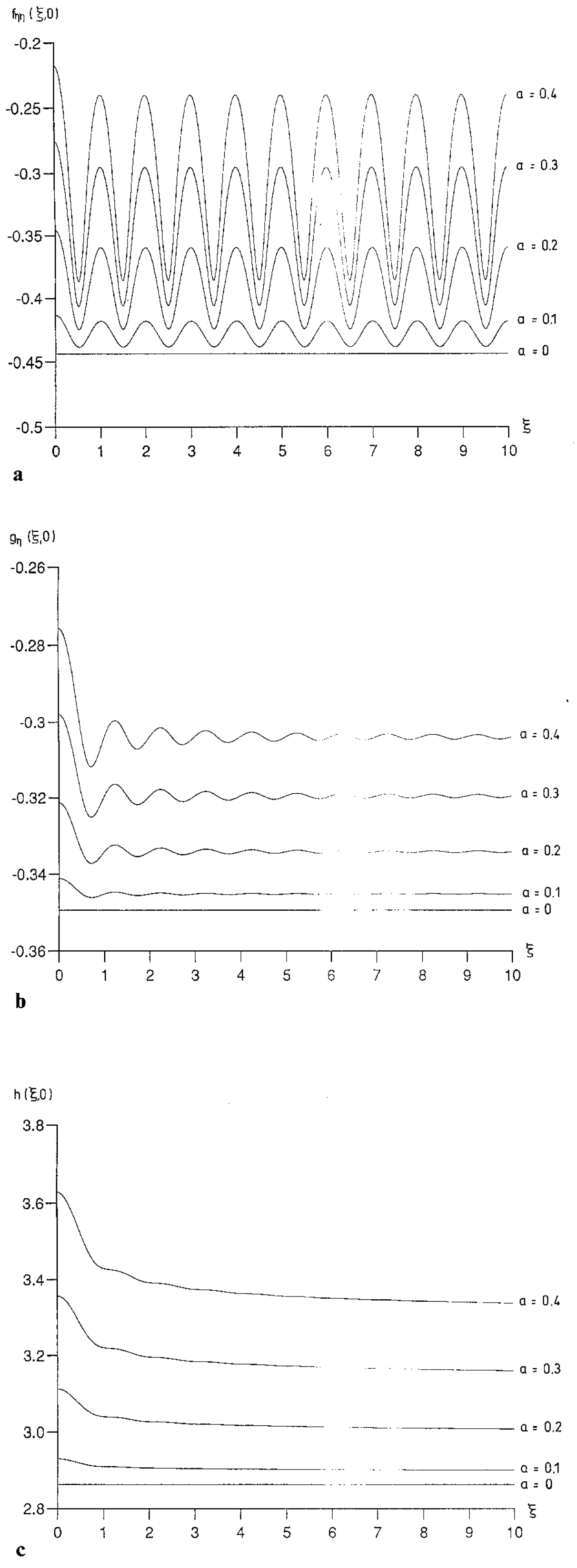

Fig. 3. Variation of **a** the skin-friction, $f_{\eta\eta}(\xi, 0)$, **b** the local rate of heat transfer, $g_\eta(\xi, 0)$ (CWT), and **c** the wall temperature, $h(\xi, 0)$ (CHF), for wave amplitudes, $a = 0$, 0.1, 0.2, 0.3 and 0.4 and phase, $\phi = 0°$

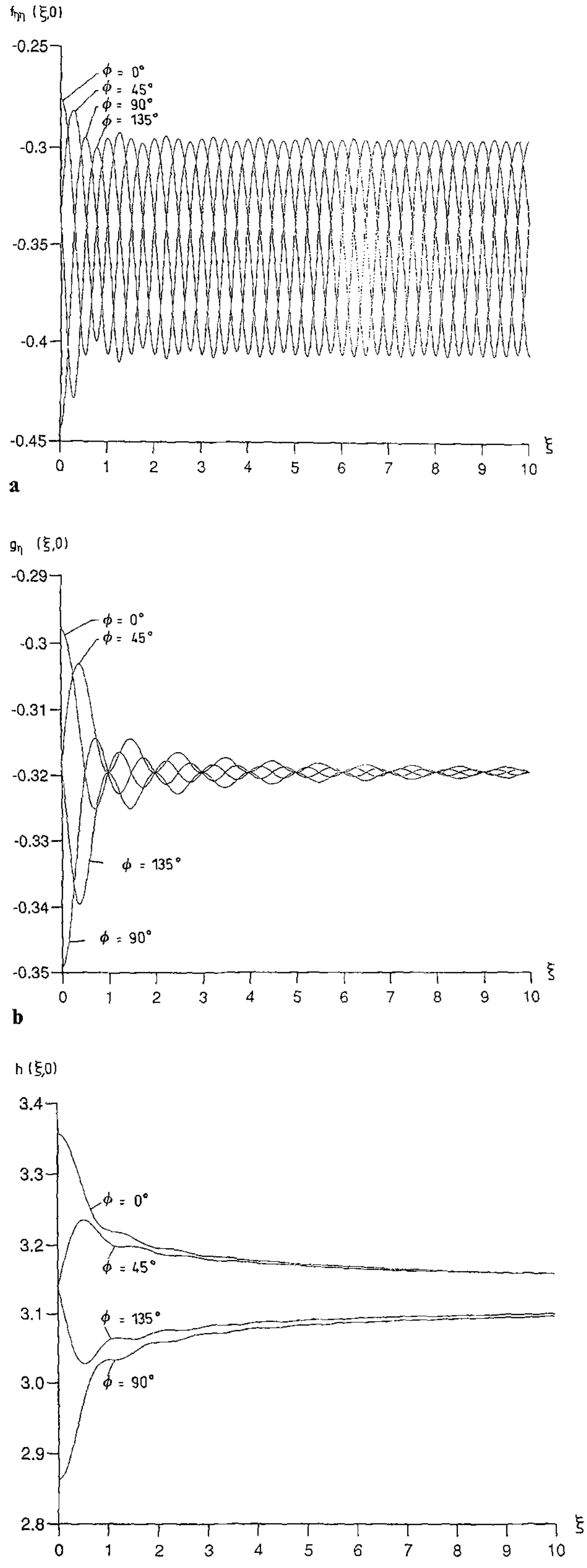

Fig. 4. Variation of **a** the skin-friction, $f_{\eta\eta}(\xi, 0)$, **b** the local rate of heat transfer, $g_\eta(\xi, 0)$ (CWT), and **c** the wall temperature, $h(\xi, 0)$ (CHF), for waves phases, $\phi = 0°$, $45°$, $90°$, $135°$ and amplitude, $a = 0.3$

this wave phase, and indeed for all others, the skin-friction coefficient and the Nusselt number decrease with increasing amplitude and exhibit correspondingly identical behaviour as ξ becomes large.

The variation of the solution with changing wave phase is given in Figs. 4 a, b, c for the wave amplitude $a = 0.3$. We note that solutions for wave phases ϕ and $\phi + \pi$ are identical. Again we see the approach to spatial periodicity for the skin-friction coefficient and the differing rates of decay of the sinusoidal components of the Nusselt numbers.

It is our intention to extend this work to considering the effect of longitudinal surface waves on the flow and heat transfer from a moving surface in a quiescent fluid.

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