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Free Convection Induced by a Vertical Wavy Surface With Uniform Heat Flux in a Porous Medium

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Nomenclature

a = amplitude of the nonuniform surface = $\pi \bar{a}/l$
f = reduced streamfunction
g = acceleration due to gravity
h = reduced temperature
k = thermal conductivity
K = permeability of the porous medium
l = half-wavelength, or lengthscale, of the surface undulations
$\underline{n}$ = unit vector normal to the surface
$\bar{p}$ = pressure
q = heat flux
Ra = $g\beta K(q_w/k)l^2/\alpha\nu$, the Darcy–Rayleigh number based on q_w and l
s_t = shape function for the wavy surface
T = dimensional temperature: $T - T_\infty = (q_w l/k)\theta$
x, y = streamwise and cross-stream coordinates: $(x, y) = (\bar{x}/l, \bar{y}/l)$
α = thermal diffusivity of the saturated medium
β = coefficient of thermal expansion
η = pseudosimilarity variable
θ = dimensionless temperature
ν = kinematic viscosity
σ = function associated with the surface undulations
ψ = streamfunction = $\bar{\psi}/l$

Superscripts

$\bar{}$ = dimensional variables
$\hat{}$ = boundary layer variables
$'$ = differentiation with respect to $\hat{x}$

Subscripts

c = denoting center of hump
t = transverse variations
w = condition at the wall
∞ = condition at infinity

1 Introduction

The study of free convection heat transfer from uniform surfaces embedded in a saturated porous medium has attracted a great deal of interest for many investigators over the last two decades; see Nield and Bejan (1992) for a comprehensive review of this topic. Studies have centered on those cases where the thermal boundary conditions allow the use of similarity transformations to reduce the governing equations to a system of ordinary differential equations. In general, this means that the heated surface is plane. However, in practice, surfaces are sometimes roughened intentionally in order to enhance the heat transfer. Roughened surfaces are encountered in several heat transfer devices such as flat-plate solar collectors and flat-plate condensers in refrigerators. Larger scale surface nonuniformities are encountered in cavity wall insulating systems and grain storage containers. The only papers to date that study the effects of such nonuniformities on thermal boundary layer flow of a Newtonian fluid are those of Moulic and Yao (1989a, b) and Yao (1983). The present authors have very recently obtained similarity solutions for a vertical, sinusoidally wavy surface with a uniform *wall temperature* embedded in a porous medium (Rees and Pop, 1994a). However, the modeling of many practical configurations requires the imposition of a uniform *heat flux* from the bounding surface.

In this paper we consider the effects of transverse surface waves on the free convective boundary layer induced by a uniform heat flux vertical surface embedded in a porous medium. In this regard, the present study is akin to that of Moulic and Yao (1989b), who considered the Newtonian fluid analogue of this problem. Since the transformed boundary layer equations are nonsimilar, they are solved numerically using the Keller box method (see Keller and Cebeci, 1971). The distribution of the wall temperature along the wavy surface is presented and the accuracy is verified by comparing with previous results for a vertical flat plate embedded in a porous medium obtained by Kumari et al. (1990).

2 Analysis

Consider a vertical surface with transverse waves embedded in a porous medium with constant ambient temperature, T_∞, as shown in Fig. 1. In particular, we assume that the surface profile is given by

$$\bar{y} = \bar{s}_t(\bar{x}) = \bar{a} \sin(\pi \bar{x}/l), \qquad (1)$$

where $\bar{a}$ is the amplitude of the wavy surface and $2l$ is the wavelength. The local heat flux rate normal to the surface is maintained at a constant value, q_w. The flow is considered to be steady and two dimensional. All fluid and porous medium properties are considered to be constant except for the buoyancy term, and we

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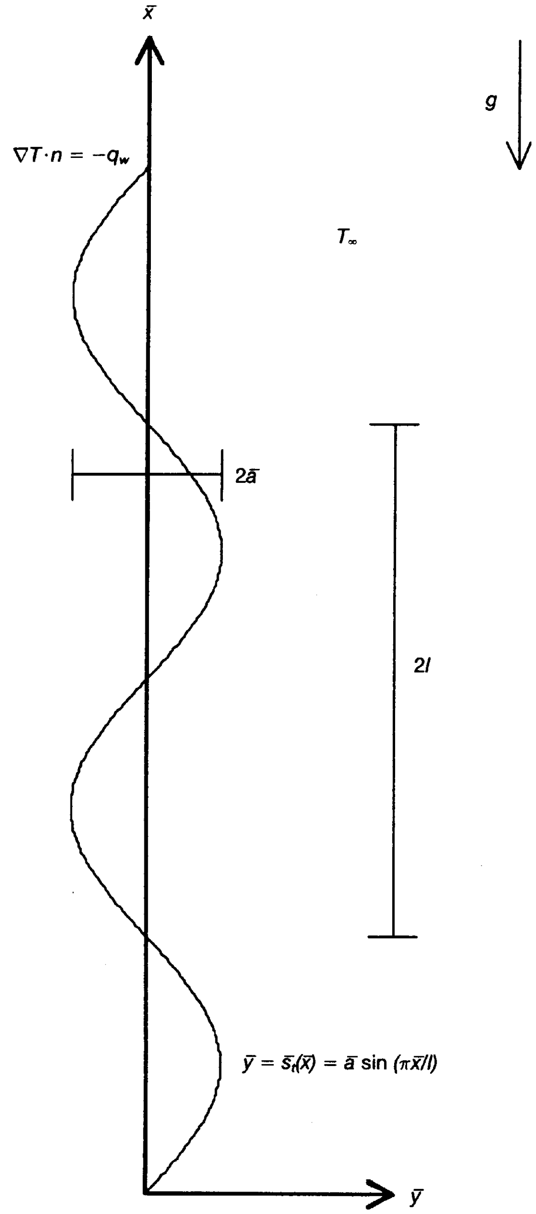

Fig. 1 Physical model and coordinate system depicting transverse surface waves

assume that the Boussinesq approximation is valid. In terms of dimensionless variables, the Darcy and energy equations can be written as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \mathrm{Ra}\,\frac{\partial \theta}{\partial y}, \tag{2}$$

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = \frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y}. \tag{3}$$

The boundary conditions that apply are the following:

$$\psi = 0, \quad \frac{\partial \theta}{\partial y} - a \cos \pi x \frac{\partial \theta}{\partial x} = -\sigma \quad \text{on}$$

$$y = s_t(x) = \frac{a}{\pi} \sin \pi x, \tag{4a}$$

$$\frac{\partial \psi}{\partial y} \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty, \tag{4b}$$

where σ, a function associated with the geometry of the surface, is given by

$$\sigma = \left(1 + \left(\frac{ds_t}{dx}\right)^2\right)^{1/2} = (1 + a^2 \cos^2 \pi x)^{1/2}, \tag{5}$$

and the dimensionless variables are defined in the nomenclature. In this paper we restrict attention to those cases for which the nondimensional wave amplitude remains $O(1)$ as $\mathrm{Ra} \rightarrow \infty$.

The effect of the wavy surface boundary conditions can be transferred to the governing equations by means of the transformation

$$\hat{x} = x, \quad \hat{y} = y - \frac{a}{\pi} \sin \pi x. \tag{6}$$

Equations (2) and (3) become

$$\nabla_1^2 \psi = \mathrm{Ra}\,\frac{\partial \theta}{\partial \hat{y}}, \tag{7}$$

$$\nabla_1^2 \theta = \frac{\partial \psi}{\partial \hat{y}}\frac{\partial \theta}{\partial \hat{x}} - \frac{\partial \psi}{\partial \hat{x}}\frac{\partial \theta}{\partial \hat{y}}, \tag{8}$$

and the boundary conditions are now

$$\psi = 0, \quad \frac{\partial \theta}{\partial \hat{y}} - a \cos \pi \hat{x} \frac{\partial \theta}{\partial \hat{x}} = -\sigma \quad \text{on} \quad \hat{y} = 0, \tag{9a}$$

$$\frac{\partial \psi}{\partial \hat{y}} \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as} \quad \hat{y} \rightarrow \infty, \tag{9b}$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial \hat{x}^2} + \sigma^2 \frac{\partial^2}{\partial \hat{y}^2} - 2a \cos \pi \hat{x} \frac{\partial^2}{\partial \hat{x} \partial \hat{y}} + a\pi \sin \pi \hat{x} \frac{\partial}{\partial \hat{y}}. \tag{10}$$

Next, we introduce the boundary layer variables

$$\psi = \mathrm{Ra}^{1/3} \hat{x}^{2/3} f(\hat{x}, \eta), \quad \theta = \mathrm{Ra}^{-1/3} \hat{x}^{1/3} h(\hat{x}, \eta), \tag{11a}$$

here

$$\eta = \mathrm{Ra}^{1/3} \hat{y} / \hat{x}^{1/3}. \tag{11b}$$

Thus, on introducing Eq. (11) into Eqs. (7) and (8), and formally letting $\mathrm{Ra} \rightarrow \infty$, we obtain the following boundary layer equations at leading order:

$$\sigma^2 f_\eta = h, \tag{12}$$

$$\sigma^2 h_{\eta\eta} + \tfrac{2}{3} f h_\eta - \tfrac{1}{3} f_\eta h = \hat{x}(f_\eta h_{\hat{x}} - h_\eta f_{\hat{x}}) \tag{13}$$

and the corresponding boundary conditions are

$$f = 0, \quad h_\eta = -1/\sigma \quad \text{on} \quad \eta = 0, \tag{14a}$$

$$h \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty. \tag{14b}$$

It is also convenient to introduce a third transformation to render the boundary conditions independent of $\hat{x}$:

$$f = \sigma^{1/3} \hat{f}(\hat{x}, \hat{\eta}), \quad h = \sigma^{2/3} \hat{h}(\hat{x}, \hat{\eta}), \tag{15a}$$

where

$$\eta = \sigma^{5/3} \hat{\eta}. \tag{15b}$$

Equations (12) and (13) become

$$\hat{f}_{\hat{\eta}} = \hat{h}, \tag{16}$$

$$\hat{h}_{\hat{\eta}\hat{\eta}} + \tfrac{1}{3}(2\hat{f}\hat{h}_{\hat{\eta}} - \hat{f}_{\hat{\eta}}\hat{h})$$

$$= \hat{x}(\hat{f}_{\hat{\eta}}\hat{h}_{\hat{x}} - \hat{h}_{\hat{\eta}}\hat{f}_{\hat{x}}) + \frac{\sigma'}{3\sigma}\hat{x}(2\hat{f}_{\hat{\eta}}\hat{h} - \hat{f}\hat{h}_{\hat{\eta}}), \tag{17}$$

and the boundary conditions transform to

$$\hat{f} = 0, \quad \hat{h}_{\hat{\eta}} = -1 \quad \text{on} \quad \hat{\eta} = 0, \tag{18a}$$

$$\hat{h} \rightarrow 0 \quad \text{as} \quad \hat{\eta} \rightarrow \infty, \tag{18b}$$

where the prime denotes differentiation with respect to $\hat{x}$.

3 Results and Discussion

Both the above-described systems of equations [i.e. (12, 13) and (16, 17)] were solved numerically by means of the Keller box method for different values of the wave amplitude, a. Integration in the η or $\hat{\eta}$ direction was over a nonuniform grid from 0 to 30, a value well outside the boundary layer. The grid chosen gave a value of $h_w = 1.29612$ for the surface temperature for the boundary layer flow over a flat plate (i.e., $\sigma = 1$ or $a = 0$ in Eqs. (12, 13)); this compares very well with the value, $h_w = 1.29618$, which is correct to five decimal places and which we obtained by solving the resulting similarity equations using a combination of the fourth-order Runge–Kutta method and Richardson extrapolation. It is worth mentioning that this value of h_w is in good agreement with the value $h_w = 1.29532$ obtained by Kumari et al. (1990) for a vertical plane surface with uniform surface heat flux embedded in a fluid-saturated porous medium.

Integration in $\hat{x}$ involved uniform steps of length 0.01, which generally gives very accurate results. At each step, convergence was assumed to have taken place when the maximum absolute change in any quantity was less than 10^{-10} and double precision arithmetic was used throughout. In general, convergence was increasingly difficult to achieve as $\hat{x}$ increased and as a increased. We presume that this is caused by increasing ill-conditioning of the Jacobian matrix used in the Newton–Raphson scheme. However, values of $\hat{x}$ up to 40 and wave amplitudes up to 0.5 gave no numerical difficulties.

In our discussion of the results we will confine our comments primarily to the surface temperature distribution. Figure 2 shows the distribution of the surface temperature, $h_w = h(\hat{x}, 0)$, for different values of a. The main effect of the presence of surface undulations is to raise the temperature of the bounding surface above the value 1.29612, corresponding to that of a plane surface. This is a consequence of the fact that the component of the buoyancy force parallel to the wavy surface is less than or equal to that of a plane surface, as can be seen by the factor, $1/\sigma^2$, in Eq. (12). We find that the positions of maximum and minimum surface temperature correspond almost exactly to where $d\sigma/d\hat{x} = 0$, that is, at integer values of $\hat{x}$. A similar observation was made by Moulic and Yao (1989b).

The corresponding distributions of the scaled temperature, $\hat{h}(\hat{x}, 0)$, are not shown for the sake of brevity. However, the effect of the scalings, Eq. (15), is to shift the "mean" value of the temperature close to 1.29612. Whereas $h(\hat{x}, 0)$, shown in Fig. 2, exhibits decaying oscillations, the corresponding behavior of $\hat{h}(\hat{x}, 0)$ tends fairly quickly to what looks like a periodic function. This was investigated numerically by solving Eqs. (15) and (16) for $a = 0.5$, a particularly large amplitude wave. Figure 3(a) shows the evolution of $\hat{h}_w(\hat{x}, 0) - \hat{h}_w(\hat{x} - 2, 0)$ as $\hat{x}$ increases, where 2 is the nondimensional wavelength of the waves. The envelope of this function decays as $\hat{x}^{-3/2}$, as is demonstrated by the graph of $(\hat{h}_w(\hat{x}) - (\hat{h}_w(\hat{x} - 2))\hat{x}^{3/2}$ shown in Fig. 3(b). From our numerical evidence we conclude that the large-$\hat{x}$ behavior of the flow field is periodic, although we have been unable to prove this analytically. It is important to state that no numerical evidence was found that an inner boundary layer structure exists within the basic boundary layer as $\hat{x}$ increases; it was originally thought that such a structure might occur since the width of the surface wave decreases relative to the boundary layer thickness as $\hat{x}$ increases. Indeed, since the submission of the original draft of this manuscript we have found that such a structure does exist when the heated surface is horizontal (Rees and Pop, 1994b).

Finally, we considered the effect of a quite different form of boundary nonuniformity, namely that of a single smooth hump on an otherwise plane surface. In particular, the hump was of

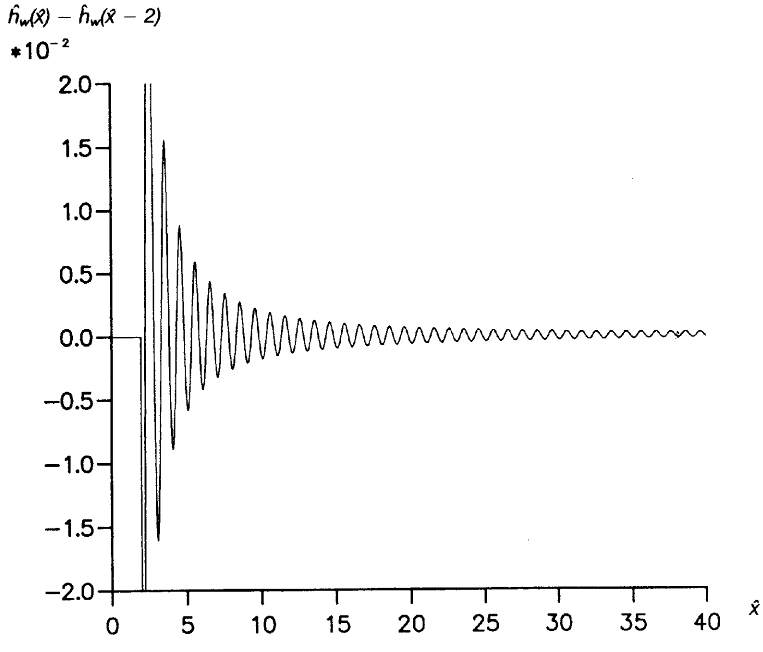

Fig. 3(a) Variation of $\hat{h}_w(\hat{x}) - \hat{h}_w(\hat{x} - 2)$ for $a = 0.5$

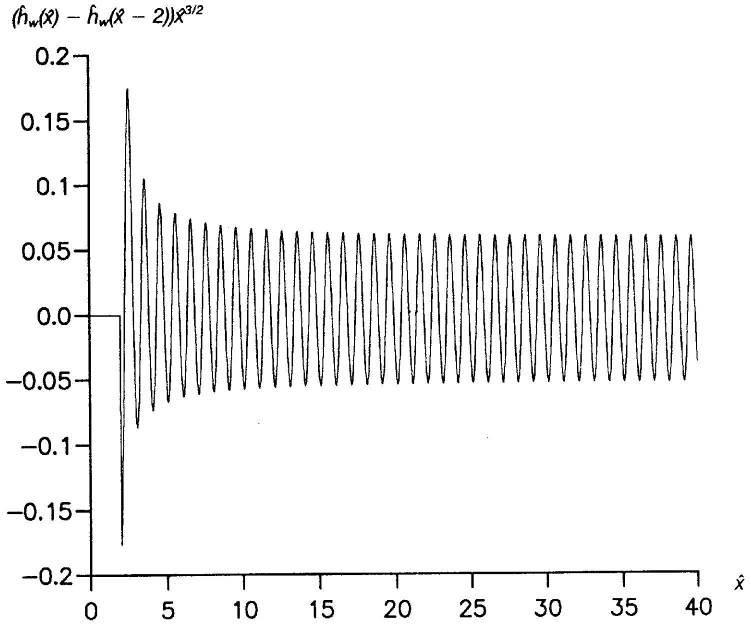

Fig. 3(b) Variation of $(\hat{h}_w(\hat{x}) - \hat{h}_w(\hat{x} - 2))\hat{x}^{3/2}$ for $a = 0.5$

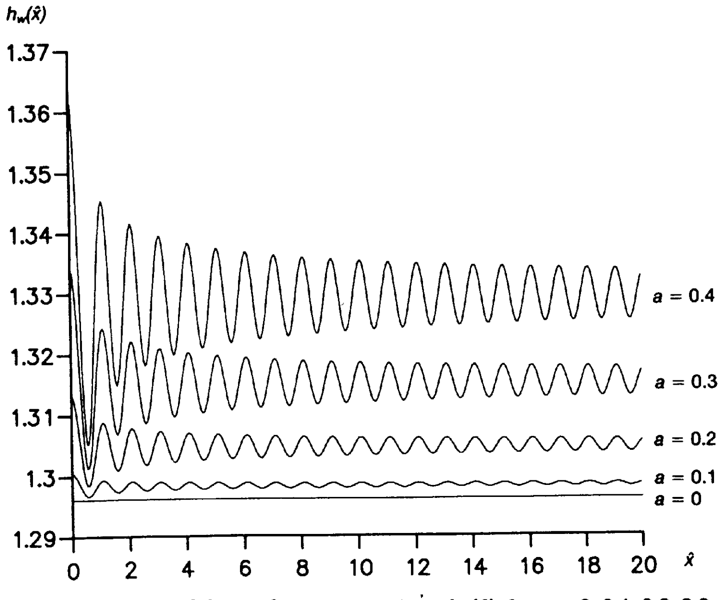

Fig. 2 Variation of the surface temperature, $h_w(\hat{x})$, for a = 0, 0.1, 0.2, 0.3, and 0.4

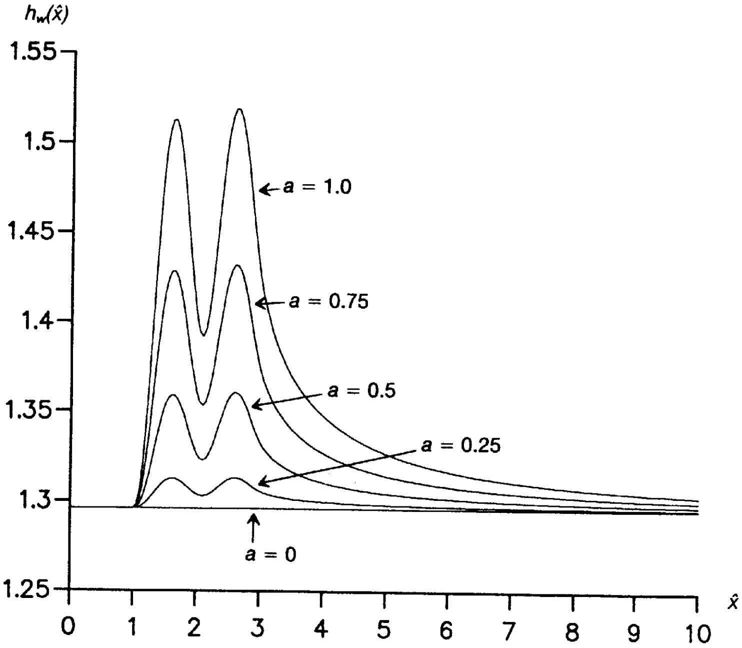

Fig. 4 Variation in the surface temperature, $h_w(\hat{x})$, for a hump of length 2, centered at $\hat{x}_c = 2$, for $a = 0, 0.25, 0.5, 0.75$, and 1

nondimensional length 2 and centered at $\hat{x} = \hat{x}_c = 2$. A surface profile having the form

$$s_t(\hat{x}) = 0, \quad 0 \le \hat{x} < 1$$

$$s_t(\hat{x}) = \frac{a}{\pi}(1 + \cos\pi\hat{x}), \quad 1 \le \hat{x} \le 3$$

$$s_t(\hat{x}) = 0, \quad 3 < \hat{x} \le 20. \tag{19}$$

was assumed and hence Eqs. (12) and (13) were solved with σ given by

$$\sigma = 1, \quad 0 \le \hat{x} < 1$$

$$\sigma = (1 + a^2 \sin^2 \pi\hat{x})^{1/2}, \quad 1 \le \hat{x} \le 3$$

$$\sigma = 1, \quad 3 < \hat{x} \le 20. \tag{20}$$

The wall temperature distribution, $h_w(\hat{x})$, as a function of both $\hat{x}$ and a, is displayed in Fig. 4. Although the temperature is found to increase (decrease) whenever the slope of σ increases (decreases), the presence of the boundary layer flow ensures that the positions of maximum temperature are shifted downstream of the positions of maximum slope. The two maxima in the surface temperature are caused by σ having two maxima. The effect of having such a nonuniformity on the surface temperature can be seen to persist a substantial distance downstream of the hump.

4 Conclusions

We have investigated the effects of large-scale surface nonuniformities on the boundary layer flow induced by a constant heat flux, vertically aligned, semi-infinite surface embedded in a porous medium. The amplitude of the waves of the surface were assumed to have the same order of magnitude as their wavelength as the Rayleigh number becomes large. Assuming the validity of the boundary layer approximation, it was found that the flow could only be determined by solving a pair of parabolic partial differential equations. Local hot spots were found to correspond almost exactly to points at which the slope of the function σ takes its maximum values. It was also found that the effect of an isolated hump on the surface persists a substantial distance downstream of the hump.

The analysis has been restricted to values of x that take $O(1)$ values as $\mathrm{Ra} \to \infty$. In this range of values of x, the boundary layer thickness is $O(\mathrm{Ra}^{-1/3})$, which is much smaller than the $O(1)$ length scale associated with the waves of the surface. It is now of some interest to investigate the regime where $x = O(\mathrm{Ra}^{1/3})$ for within this regime the width of the boundary layer is $O(1)$, which is comparable with the surface wavelength and amplitude; we hope to address this problem in the near future.

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Simulation of the Cyclic Injection Mold-Cooling Process Using Dual Reciprocity Boundary Element Method

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Nomenclature

C_p = specific heat of polymer melt
$[C]$ = square matrix for coefficients associated with $\dot{T}$
f_j = coordinate function at point j to approximate $\dot{T}$
$[G]$ = square matrix for coefficients associated with q
$[H]$ = square matrix for coefficients associated with T
h_{air} = heat transfer coefficient of the ambient air
h_c = heat transfer coefficient of the coolant
$\bar{J}_{avg}$ = cycle-averaged heat flux at cavity surface
K_m = thermal conductivity of mold
K_p = thermal conductivity of polymer melt
n = normal direction of mold boundary
$\{q^{n+1}\}$ = column matrix for heat flux at time $t = t_{n+1}$
r = distance
T = temperature field
$\dot{T}$ = time derivative of temperature

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