

Fig. 2 Influence of thermal and flow distribution asymmetry for a two-tube model on the transient response characteristics of outlet vapor flow rate after a decrease in inlet flow rate

in its generalized form for an n -tube system, is simply a weighted average of the time constants for each of the individual tubes. The weighting factors are the corresponding flow distribution asymmetry parameters. Utilizing this concept, the ESTM can be used to approximate the transient response characteristics of a multitube evaporating system. For the case of a two-tube system, the equivalent single-tube time constant, $\tau_{s,2}$, becomes

$$\tau_{s,2} = \gamma\tau_1 + (1 - \gamma)\tau_2 \quad (8)$$

Therefore, the transient response characteristics of the outlet vapor flowrate for this two-tube evaporating flow system can be approximated by the following ESTM:

$$\frac{m_{t,o}(t) - m_{t,f}}{m_{t,i} - m_{t,f}} = e^{-t/\tau_m} + (1 - x_i) \times [1 - (\rho'/\rho)] \frac{(\tau_{s,2}/\tau_m)}{[(\tau_{s,2}/\tau_m) - 1]} \{e^{-t/\tau_{s,2}} - e^{-t/\tau_m}\} \quad (9)$$

A graph of the predictions of the ESTM, for various combinations of thermal and flow distribution asymmetry parameters, would plot almost on top of the curves depicted in Fig. 2, indicating that the ESTM is very accurate for the range of thermal and flow distribution parameters presented. Although no experimental verification has been presented in this paper, the predictive capability of the system mean void fraction model has been substantially verified with experimental measurements in previous papers, such as those listed in the references. Good agreement has always been found to exist, especially when consideration is given to the complexity of the physical mechanisms involved, and the relative simplicity of the model.

References

- Wedekind, G. L., Bhatt, B. L., and Beck, B. T., 1978, "A System Mean Void Fraction Model for Predicting Various Transient Phenomena Associated With Two-Phase Evaporating and Condensing Flows," *International Journal of Multiphase Flow*, Vol. 4, pp. 97-114.
- Wedekind, G. L., and Beck, B. T., 1981, "A Generalization of the System Mean Void Fraction Model for Transient Two-Phase Evaporating Flows," *ASME JOURNAL OF HEAT TRANSFER*, Vol. 103, pp. 81-85.
- Wedekind, G. L., Beck, B. T., Bhatt, B. L., and Roslund, G. L., 1984, "A Unified System Mean Void Fraction Model for Predicting Transient Charac-

teristics of Flow Quality, Void Fraction, Flowrate and Pressure Drop in Evaporating and Condensing Flows," *Two-Phase Flow and Heat Transfer; China-U.S. Progress*, Xue-jun Chen and T. N. Veziroglu, eds., Hemisphere Publishing Co., Washington, DC, pp. 633-646.

Wedekind, G. L., and Bhatt, B. L., 1989, "Modeling the Thermally Governed Transient Flow Surges in Multitube Condensing Flow Systems With Thermal and Flow Distribution Asymmetry," *ASME JOURNAL OF HEAT TRANSFER*, Vol. 111, pp. 786-791.

Zivi, S. M., 1964, "Estimation of Steady-State Stream Void Fraction by Means of the Principle of Minimum Entropy Production," *ASME JOURNAL OF HEAT TRANSFER*, Vol. 86, p. 247.

A Note on Free Convection Along a Vertical Wavy Surface in a Porous Medium

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Nomenclature

- a = amplitude of the wavy surface
 f = reduced streamfunction
 g = acceleration due to gravity
 k = thermal conductivity of the porous medium
 K = permeability of the porous medium
 l = wavelength of the surface undulations
 $Nu_x = -(\bar{x}/T_w - T_\infty)(\partial T/\partial \bar{y})_{y=a \sin x/l}$ = the local Nusselt number
 Q = integrated rate of heat transfer

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$Ra = g\beta K(T_w - T_\infty)l/\alpha\nu =$ the Darcy-Rayleigh number based on l

$Ra_x = g\beta K(T_w - T_\infty)\bar{x}/\alpha\nu =$ the local Darcy-Rayleigh number based on \bar{x}

$s =$ arc length of the surface

$T =$ temperature

$x, y =$ streamwise and cross-streamwise coordinates

$X =$ representative distance from the leading edge

$\alpha =$ thermal diffusivity of the saturated medium

$\beta =$ coefficient of thermal expansion

$\theta =$ dimensionless temperature

$\nu =$ kinematic viscosity

$\xi, \eta =$ transformed variables

$\psi =$ streamfunction

Superscripts

' = differentiation with respect to η

— = dimensional variables

^ = boundary layer variables

Subscripts

$w =$ condition at the wall

$\infty =$ condition at infinity

Introduction

The use of Darcy's law and the energy equation has met with considerable success in the modeling of flow and heat transfer within fluid-saturated porous media. This is reflected in the particularly large number of papers concerned with convection in porous media that have now been published. An excellent review of this topic to date can be found in the monograph by Nield and Bejan (1992).

Heat transfer by natural convection from vertical, horizontal, and inclined surfaces embedded in a porous medium are of fundamental importance for many practical applications. In general, it is not easy to determine the flow field and the associated heat transfer characteristics either in porous media flows, or, indeed, for flows of Newtonian fluids because of the complexity of practical geometries. The present note studies, for the first time, the free convection along a vertically oriented, isothermally heated, wavy surface placed within a porous medium. The configuration is analogous to that considered by Yao (1983) and Moulic and Yao (1989a, b) for a Newtonian fluid. Such surfaces can be considered as approximations to many practical geometries for which free convective heat transfer is of interest. It is shown that, with a generalized similarity transformation, the governing boundary layer equations and associated boundary conditions reduce to those of a vertical smooth isothermal surface first studied by Cheng and Minkowycz (1977).

Basic Equations and Analysis

Consider a vertical sinusoidally wavy surface embedded in a porous medium with constant ambient temperature, T_∞ , as shown in Fig. 1. The wavy surface is held at a constant temperature, T_w , where $T_w > T_\infty$, and the characteristic length scale associated with the sinusoidal waves is l .

The dimensional governing equations are Darcy's law and the energy equation framed in two-dimensional Cartesian coordinates, (\bar{x}, \bar{y}) . The dimensionless form of these equations, subject to the Boussinesq approximation, are

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = Ra \frac{\partial \theta}{\partial y}, \quad (1)$$

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y}, \quad (2)$$

with the boundary conditions

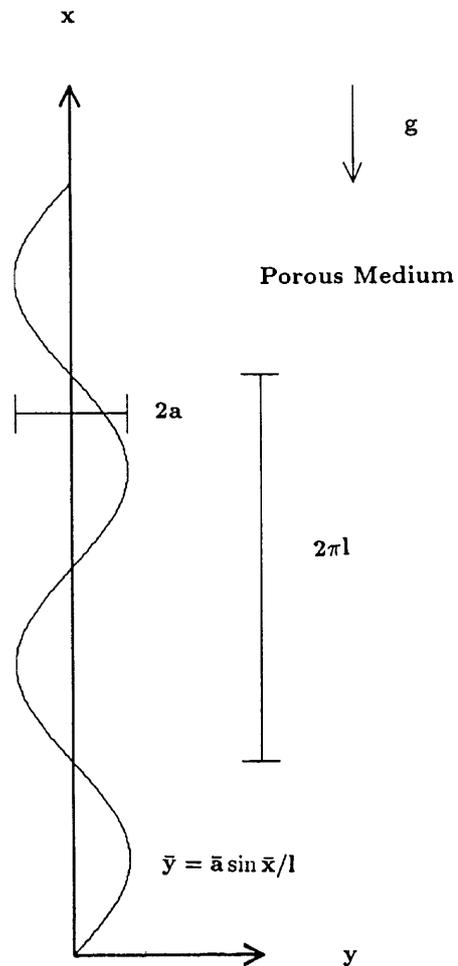


Fig. 1 Physical model and coordinate system

$$\psi = 0, \theta = 1 \text{ on } y = a \sin x, \quad (3a)$$

$$\psi_y \rightarrow 0, \theta \rightarrow 0 \text{ as } y \rightarrow \infty. \quad (3b)$$

The dimensionless variables are defined as

$$x = \bar{x}/l, \quad y = \bar{y}/l, \quad \psi = \bar{\psi}/\alpha, \quad (4a)$$

$$\theta = (T - T_\infty)/(T_w - T_\infty), \quad a = \bar{a}/l. \quad (4b)$$

Here $\bar{\psi}$ is the dimensional streamfunction, which is defined in the usual way, and \bar{a} is the amplitude of the wavy surface. In this fundamental study of the effect of surface undulations on free convection in porous media, we assume that boundary and inertia effects are negligible.

We now assume that the Darcy-Rayleigh number is large so that free convection takes place within a boundary layer whose cross-stream width is substantially smaller than the $O(1)$ amplitude of the surface waves. Accordingly we define new variables by subtracting out the effect of the surface waves and then introducing the usual boundary layer variables. Thus we transform according to

$$x = \xi, \quad y = \xi^{1/2} Ra^{-1/2} \eta + a \sin x, \quad (5a)$$

$$\psi = Ra^{1/2} \hat{\psi}, \quad (5b)$$

where $Ra = g\beta K(T_w - T_\infty)l/\alpha\nu$ is the Darcy-Rayleigh number. Equations (1) and (2) become

$$\nabla_1^2 \hat{\psi} = \xi^{1/2} \frac{\partial \theta}{\partial \eta}, \quad (6)$$

$$\nabla_1^2 \theta = \xi^{1/2} \left(\frac{\partial \hat{\psi}}{\partial \eta} \frac{\partial \theta}{\partial \xi} - \frac{\partial \hat{\psi}}{\partial \xi} \frac{\partial \theta}{\partial \eta} \right), \quad (7)$$

and the boundary conditions are now

$$\hat{\psi} = 0, \theta = 1 \text{ on } \eta = 0, \quad (8a)$$

$$\psi_\eta \rightarrow 0, \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty. \quad (8b)$$

Equations (6) and (7) are the full equations of motion and the operator, ∇_1^2 , is defined to be

$$\begin{aligned} \nabla_1^2 = & (1 + a^2 \cos^2 \xi) \frac{\partial^2}{\partial \eta^2} + a\xi^{1/2} Ra^{-1/2} \sin \xi \frac{\partial}{\partial \eta} \\ & - 2a\xi^{1/2} Ra^{-1/2} \cos \xi \left(\frac{\partial^2}{\partial \xi \partial \eta} - \frac{\eta}{2\xi} \frac{\partial^2}{\partial \eta^2} - \frac{1}{2\xi} \frac{\partial}{\partial \eta} \right) \\ & + Ra^{-1} \left(\xi \frac{\partial^2}{\partial \xi^2} - \eta \frac{\partial^2}{\partial \xi \partial \eta} + \frac{\eta^2}{4\xi} \frac{\partial^2}{\partial \eta^2} + \frac{3\eta}{4\xi} \frac{\partial}{\partial \eta} \right). \quad (9) \end{aligned}$$

If we formally let $Ra \rightarrow \infty$, then the leading order terms in Eqs. (6) and (7) become

$$(1 + a^2 \cos^2 \xi) \frac{\partial^2 \psi}{\partial \eta^2} = \xi^{1/2} \frac{\partial \theta}{\partial \eta}, \quad (10)$$

$$(1 + a^2 \cos^2 \xi) \frac{\partial^2 \theta}{\partial \eta^2} = \xi^{1/2} \left(\frac{\partial \psi}{\partial \xi} \frac{\partial \theta}{\partial \xi} - \frac{\partial \psi}{\partial \xi} \frac{\partial \theta}{\partial \eta} \right); \quad (11)$$

these equations are to be solved subject to the boundary conditions (8).

A similarity solution of Eqs. (10) and (11) is possible, and the streamfunction and temperature take the form

$$\psi = \xi^{1/2} f(\hat{\eta}), \quad \theta = \theta(\hat{\eta}), \quad (12a)$$

where the similarity variable, $\hat{\eta}$, is defined as

$$\hat{\eta} = \frac{\eta}{1 + a^2 \cos^2 \xi}. \quad (12b)$$

Using Eq. (12), Eqs. (10) and (11) reduce to

$$f''' + \frac{1}{2} f f'' = 0 \quad (13a)$$

and the boundary conditions are

$$f(0) = 0, f'(0) = 1, \text{ and } f'(\hat{\eta}) \rightarrow 0 \text{ as } \hat{\eta} \rightarrow \infty, \quad (13b)$$

where primes denote differentiation with respect to $\hat{\eta}$. It should be noted that the temperature field is given by $\theta = f'(\hat{\eta})$. Equation (13) for f is precisely that obtained by Cheng and Minkowycz (1977) in their study of thermal boundary layer flow induced by an isothermally heated plane vertical surface embedded in a porous medium.

The local Nusselt number, defined in terms of the temperature drop, $T_w - T_\infty$, the thermal conductivity of the saturated medium, k , and the local Rayleigh number based on the downstream distance, \bar{x} , Ra_x , can be expressed as

$$\begin{aligned} Nu_x &= \frac{-\theta'(0) Ra_x^{1/2}}{(1 + a^2 \cos^2 x)^{1/2}} \\ &\approx \frac{0.44375 Ra_x^{1/2}}{(1 + a^2 \cos^2 x)^{1/2}}, \quad (14) \end{aligned}$$

where the numerical value for $-\theta'(0)$ has been taken from Rees and Bassom (1991). The variation of Nu_x with x for various values of the dimensionless wave amplitude is shown in Fig. 2. In this figure it may be seen that the local Nusselt number is less than or equal to that corresponding to a plane heated surface; this may be explained as follows. When the heated surface is not vertical the component of the buoyancy force along the surface is reduced from its maximum value when the surface is vertical—this is true for both upward and downward facing surfaces. Consequently, the boundary layer thickness is locally thicker, and hence local rates of heat transfer at the surface are reduced.

A quantity of greater physical significance than the local Nusselt number is the integrated rate of heat transfer:

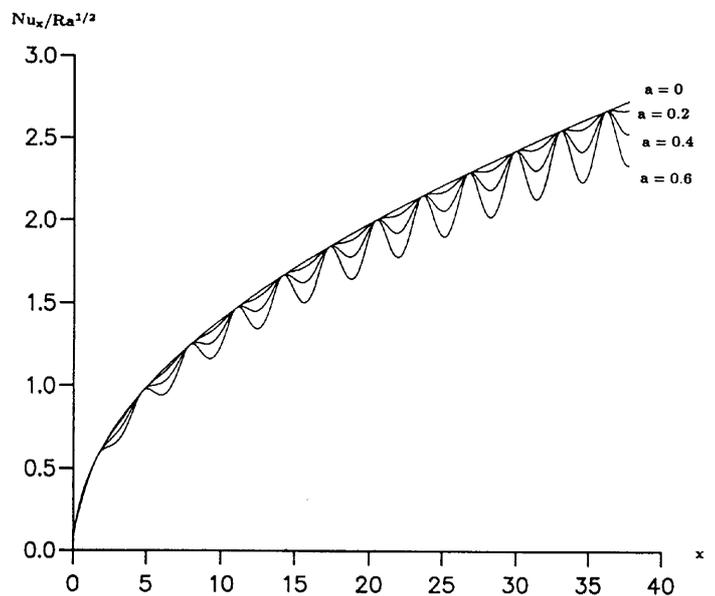


Fig. 2 Variation of the local Nusselt number, Nu_x , as a function of x for dimensionless wave amplitudes: $a = 0$, $a = 0.2$, $a = 0.4$, $a = 0.6$

$$Q = \int_0^X -k(\underline{n} \cdot \nabla T)_{y=a \sin(x/l)} \frac{d\bar{s}}{d\bar{x}} d\bar{x}, \quad (15)$$

where \bar{X} is the dimensional distance from the leading edge and \bar{s} is the dimensional arc length along the wavy surface. Equation (15) may be shown to reduce to

$$Q = 0.88750 k(T_w - T_\infty) Ra^{1/2} X^{1/2} / l \quad (16)$$

at leading order. Hence the total heat transfer between $x = 0$ and $x = X$ is independent of the amplitude and phase of the wavy surface (and even the shape of the surface providing that it is sufficiently smooth that boundary layer theory remains valid), and is, by implication, equal to that of a plane surface. Therefore we conclude that the presence of sufficiently smooth boundary deformations does not serve to change the rate of heat transfer into a porous medium at leading order.

Discussion

We have shown that the boundary layer flow induced by an isothermally heated surface exhibiting sinusoidal undulations is described by the solution of an ordinary differential equation identical to that which applies when no undulations are present. Therefore, the flow depends only on the local slope of the surface wave. This is in contrast to the identical configuration in a Newtonian fluid, which was considered by Yao (1983). There, the boundary layer equations formed a set of parabolic partial differential equations whose solution has to be obtained numerically and hence the flow also depends on conditions upstream. For the present problem we have seen that the total heat transfer rate in the medium is unchanged by the presence of surface waves, although the local heat transfer rate is less than or equal to that for a plane surface.

Our analysis is not confined to cases where the heated surface exhibits sinusoidal undulations but can be extended very easily to other shapes such as a plane surface with a $(1 - \cos)$ hump. More generally, a surface that has $O(1)$ variations and $O(1)$ gradients as $Ra \rightarrow \infty$ can also be incorporated into the analysis. Thus, if the boundary conditions (3a) are imposed at $y = F(x)$, and the transformation defined by Eq. (5a) has $a \sin x$ replaced by $F(x)$, where $F(x)$ and its first two derivatives are continuous, then the final solution, Eq. (12a), applies but with $\hat{\eta}$ defined according to

$$\hat{\eta} = \frac{\eta}{1 + (F'(x))^2}. \quad (17)$$

Even for this more general form of surface nonuniformity the total rate of heat transferred into the porous medium is unchanged to leading order.

Finally, it is necessary to point out that the solution presented here is valid for $O(1)$ distances from the leading edge; the corresponding boundary layer thickness is $O(Ra^{-1/2})$. An examination of definition (9) shows that when $x = O(Ra)$, streamwise diffusion terms can no longer be neglected. This situation corresponds to an $O(1)$ boundary layer thickness, which is the same order of magnitude as the amplitude of the undulations. For such large distances from the leading edge it may therefore be necessary to solve a set of elliptic partial differential equations.

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References

- Cheng, P., and Minkowycz, W. J., 1977, "Free Convection About a Vertical Flat Plate Embedded in a Porous Medium With Application to Heat Transfer from a Dike," *J. Geophys. Res.*, Vol. 82, pp. 2040-2044.
- Moulic, S. G., and Yao, L. S., 1989a, "Mixed Convection Along a Wavy Surface," *ASME JOURNAL OF HEAT TRANSFER*, Vol. 111, pp. 974-979.
- Moulic, S. G., and Yao, L. S., 1989b, "Natural Convection Along a Vertical Wavy Surface With Uniform Heat Flux," *ASME JOURNAL OF HEAT TRANSFER*, Vol. 111, pp. 1106-1108.
- Nield, D. A., and Bejan, A., 1992, *Convection in Porous Media*, Springer-Verlag, Berlin.
- Rees, D. A. S., and Bassom, A. P., 1991, "Some Exact Solutions for Free Convective Flows Over Heated Semi-infinite Surfaces in Porous Media," *Int. J. Heat Mass Transf.*, Vol. 34, pp. 1564-1567.
- Yao, L. S., 1983, "Natural Convection Along a Vertical Wavy Surface," *ASME JOURNAL OF HEAT TRANSFER*, Vol. 105, pp. 465-468.

A Unified Theory for Non-Darcy Free, Forced, and Mixed Convection Problems Associated With a Horizontal Line Heat Source in a Porous Medium

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Nomenclature

- C = Forchheimer constant
 C_p = specific heat of fluid at constant pressure
 g = acceleration due to gravity
 Gr_{Kc} = microscale Grashof number based on ΔT_c
 Gr_{Kq} = microscale Grashof number based on q^*
 k = equivalent thermal conductivity of the fluid-saturated porous medium
 K = permeability of the fluid-saturated porous media
 $m_l = d \ln \Delta T_c / d \ln x$

- Pe_x = local Peclet number based on u_e
 Pe_{xc} = modified local Peclet number based on u_c
 q^* = strength of line heat source
 Ra_{xq} = local Rayleigh number based on q^*
 Ra_{xq}^* = modified local Rayleigh number for Forchheimer free convection
 T = local temperature
 $\Delta T_c = T_c - T_e$ = centerline-ambient temperature difference
 $\Delta T_c^* = \Delta T_c / (q^* / k Pe_x^{1/2})$ = dimensionless temperature difference
 u, v = Darcian or superficial velocity components
 x, y = boundary layer coordinates
 $x_D^* = (Pe_x / Ra_{xq})^3$ = dimensionless coordinate for Darcy mixed convection
 $x_F^* = Pe_x^5 / Ra_{xq}^{5/2}$ = dimensionless coordinate for Forchheimer mixed convection
 α = equivalent thermal diffusivity of the fluid-saturated porous medium
 β = expansion coefficient of the fluid
 δ = plume width
 η = similarity variable
 μ = fluid viscosity
 ρ = fluid density

Subscripts

- c = plume centerline
 e = ambient

Introduction

Due to a wide range of practical applications in geophysics and energy-related problems such as geophysical flows, cooling of underground electric cables, and environmental impact of buried heat generating waste, numerous investigations have been carried out for the convection problems associated with concentrated heat sources in fluid-saturated porous media (Wooding, 1963; Yih, 1965; Hickox and Watts, 1980; Hickox, 1981; Nield and White, 1982; Cheng, 1978; Masuoka et al., 1986; Ingham, 1988; Lai, 1991a). All these previous investigations, however, are restricted to the case of pure free convection. Despite its importance in practical applications, the study of mixed convection in the plume in a porous medium (Cheng and Zheng, 1986; Lai, 1991b) is very much limited.

In this study, we propose a unified integral treatment for the non-Darcy mixed convection in the plume rising from a line heat source, along the lines of Nakayama and Pop (1991). From the proposed flow regime map, one can readily find out which convection mode is realized for given conditions, and then select the appropriate set of solutions to describe the corresponding velocity and temperature fields.

Governing Equations and Flow Regime Map

Consider the mixed convective flow in the thermal plume above a horizontal line heat source, as illustrated in Fig. 1. Under the boundary layer approximations, the governing equations are given as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\mu}{K} u + \frac{\rho C}{K^{1/2}} u^2 = \rho g \beta (T - T_e) + \frac{\mu}{K} u_e + \frac{\rho C}{K^{1/2}} u_e^2 \quad (2)$$

and

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

where the Boussinesq approximation is evoked for the buoy-

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