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Influence of Darcy number on the onset of convection in a porous layer with a uniform heat source

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Abstract

This note considers the effect of different Darcy numbers on the onset of natural convection in a horizontal, fluid-saturated porous layer with uniform internal heating. It is assumed that the two bounding surfaces are maintained at constant but equal temperatures and that the fluid and porous matrix are in local thermal equilibrium. Linear stability theory is applied to the problem, and numerical solutions obtained using compact fourth order finite differences are presented for all Darcy numbers between $Da = 0$ (Darcian porous medium) and $Da \to \infty$ (the clear fluid limit). The numerical work is supplemented by an asymptotic analysis for small values Da.

1. Introduction

Natural convection has been a subject of intensive research in porous media in view of its wide range of application in many engineering and technological areas. Applications include high performance insulation for buildings and cold storage, the insulation of high temperature gas-cooled reactor vessels, the burying of drums containing heat-generating chemicals in the earth, regenerative heat exchangers containing porous materials and exothermic chemical reactions in packed-bed reactors.

Many authors have considered the conditions for instability in a porous layer heated either from below or by means of internal volumetric heat generation. Horton and Rogers [1] and Lapwood [2] were the first to establish analytically the critical Rayleigh number for onset of convection in a fluid-saturated porous layer heated from below without internal heat generation. Their analysis has since been extended substantially to include other types of modeling of porous media, and to moderately and strongly nonlinear situations. The reviews by Rees [3] and Tyvand [4] may be consulted for further details.

Gasser and Kazimi [5] conducted a comprehensive study of the onset of thermal convection in a horizontal porous layer using a linear stability analysis of the basic nonlinear temperature distribution which is caused by both internal heat generation and heating from below. Therefore two Rayleigh numbers appear, one corresponding to internal heating, the other to the external temperature gradient. They determined how the critical internal Rayleigh number varies with the size of the external Rayleigh number and vice versa. When the external Rayleigh number is zero, then the internal Rayleigh number is approximately 470. Rudraiah et al. [6] studied the same problem subsequently using trial functions to solve the linearised stability equations. In a short work, Selimos and Poulikakos [7] extended the analyses of [5] and [6] by including a second diffusing component and by adopting the Darcy–Brinkman momentum equations. A comprehensive set of results are given in [7] for a small selection of values of the Darcy number. Vasseur and Robillard [8] considered convection in a layer with unequal but constant heat fluxes imposed on the boundaries. On subtracting out the mean temperature rise, they obtained a stability problem similar to that of Gasser and Kazimi [5] except that the disturbance temperatures satisfy Neumann rather than Dirichlet boundary conditions. Further work has appeared in the literature detailing nonlinear effects, the effect of different boundary

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Nomenclature

C	specific heat
Da	Darcy number (Eq. (6))
g	gravity
k	wavenumber of disturbance
K	permeability
L	depth of the convection layer
P	pressure
q'''	rate of heat generation
Ra	Darcy–Rayleigh number (Eq. (6))
t	time
T	temperature
u, v	horizontal and vertical velocity
x, y	horizontal and vertical Cartesian coordinate
Greek symbols	
α	diffusivity
β	coefficient of cubical expansion
ρ	density
σ	heat capacity ratio
μ	dynamic viscosity
μ_e	effective viscosity
ν	kinetic viscosity
ε	porosity
ψ	streamfunction
Ψ	streamfunction disturbance
θ	scaled temperature
Θ	temperature disturbance
λ	amplification rate of disturbance
Superscripts and subscripts	
$\hat{}$	dimensional
basic	basic state
PM	porous media
f	fluid
s	solid
0	wall temperature
$\cdot$	k-derivative
$'$	y-derivative

conditions and flow within finite cavities; the reader is referred to Nield and Bejan [9] for further discussion.

On the other hand, for layers filled with a clear fluid, Sparrow et al. [10] performed an analytical study of the thermal instability of an internally heated fluid layer both with and without heating from below. They showed that with increasing heat generation rate the fluid becomes more prone to instability, that is, the critical Rayleigh number decreases. Takashima [11] applied linear stability theory to the problem of the stability of natural convection that occurs in an inclined fluid layer with uniformly distributed internal heat sources and with constant and equal boundary temperatures. The marginal stability criteria for different ranges of Prandtl number and angles of inclination are reported.

The purpose of this short note is to determine how the critical Rayleigh number and the corresponding wavenumber vary with Darcy number for an internally heated porous layer. As such it gives the full transition between convection when Darcy's law applies and when the full Navier–Stokes equations apply. Comparison is then made between our results and those for the Darcy-flow and clear fluid limits. We provide a highly accurate set of numerical results and supplement this with an asymptotic theory for small values of the Darcy number.

2. Governing equations and basic solution

We consider an infinite porous layer confined between two parallel rigid plates which are separated by a distance L, as depicted in Fig. 1. It is assumed that the fluid layer is heated internally by a uniform heat sources of strength q''' and the two bounding surfaces are each maintained at the constant temperature T_0. The governing equations of motion of fluid in a homogeneous and isotropic porous medium follow the Brinkman model, and, subject to the Boussinesq approximation, the full two-dimensional governing equations take the form,

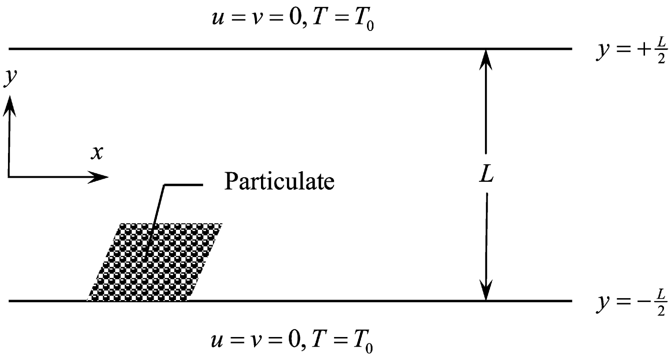

Fig. 1. Definition sketch of the horizontal porous layer with the coordinate system.

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0 \tag{1a}$$

$$\frac{\mu}{K}\hat{u} = -\frac{\partial \hat{P}}{\partial \hat{x}} + \mu_e\left(\frac{\partial^2 \hat{u}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{u}}{\partial \hat{y}^2}\right) \tag{1b}$$

$$\frac{\mu}{K}\hat{v} = -\frac{\partial \hat{P}}{\partial \hat{y}} + \mu_e\left(\frac{\partial^2 \hat{v}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{v}}{\partial \hat{y}^2}\right) + \rho g\beta(T - T_0) \tag{1c}$$

$$(\rho C)_{\mathrm{PM}}\frac{\partial T}{\partial \hat{t}} + (\rho C)_f\left(\hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{v}\frac{\partial T}{\partial \hat{y}}\right) = k_{\mathrm{PM}}\left(\frac{\partial^2 T}{\partial \hat{x}^2} + \frac{\partial^2 T}{\partial \hat{y}^2}\right) + q''' \tag{1d}$$

where x and y are the horizontal and vertical coordinates and u and v are the corresponding velocity components. All the other terms have their usual meaning for porous medium convection, and are given in the Nomenclature. The appropriate boundary conditions are,

$$\hat{u} = \hat{v} = T = 0 \quad \text{on } \hat{y} = \pm\frac{L}{2} \tag{2}$$

Here we have taken fixed temperature boundary and no-slip boundary conditions. Other choices of boundary condition have been made in the published literature, including having one surface cooled with the other insulated and/or stress free conditions. Our choice of thermal conditions means that the upper half of the layer is unstably stratified, and we have an example of penetrative convection, where disturbances may penetrate into the lower, stably stratified region below.

Eqs. (1a–d) may be nondimensionalised using the following substitutions,

$$\hat{t} = \frac{L^2\sigma}{\alpha_{\mathrm{PM}}}t, \qquad (\hat{x}, \hat{y}) = L(x, y), \qquad (\hat{u}, \hat{v}) = \frac{\alpha_{\mathrm{PM}}}{L}(u, v)$$
$$\hat{P} = \frac{\alpha_{\mathrm{PM}}\mu}{K}P, \qquad T = T_0 + \frac{q''' L^2}{k_{\mathrm{PM}}}\theta \tag{3}$$

where

$$\sigma = \frac{(\rho C)_{\mathrm{PM}}}{(\rho C)_f}, \qquad \alpha_{\mathrm{PM}} = \frac{k_{\mathrm{PM}}}{(\rho C)_f}$$
$$k_{\mathrm{PM}} = \varepsilon k_f + (1-\varepsilon)k_s, \qquad q''' = \varepsilon q_f''' + (1-\varepsilon)q_s''' \tag{4}$$

These transformations yield the following system of equations,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5a}$$
$$u = -\frac{\partial P}{\partial x} + Da\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{5b}$$
$$v = -\frac{\partial P}{\partial y} + Da\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + Ra\,\theta \tag{5c}$$
$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + 1 \tag{5d}$$

In above equations the nondimensional parameters, Da, and Ra are defined according to,

$$Da = \frac{\mu_e K}{\mu L^2} \quad \text{and} \quad Ra = \frac{g\beta K q''' L^3}{\nu \alpha_{\mathrm{PM}} k_{\mathrm{PM}}} \tag{6}$$

The boundary conditions are now,

$$u = v = \theta = 0 \quad \text{on } y = \pm\frac{1}{2} \tag{7}$$

From the continuity equation, (5a), a streamfunction ψ may be defined according to,

$$u = -\frac{\partial \psi}{\partial y} \quad \text{and} \quad v = \frac{\partial \psi}{\partial x} \tag{8}$$

After the elimination of the pressure P between Eqs. (5b) and (5c), Eqs. (5a–d) reduce to the system,

$$-Da\left(\frac{\partial^4 \psi}{\partial x^4} + 2\frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi}{\partial y^4}\right) + \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = Ra\frac{\partial \theta}{\partial x} \tag{9a}$$
$$\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + 1 \tag{9b}$$

which are to be solved subject to the boundary conditions,

$$\psi = \psi_y = \theta = 0 \quad \text{on } y = \pm\frac{1}{2} \tag{10}$$

The basic steady state now consists of no flow and the following parabolic temperature profile,

$$\theta_{\mathrm{basic}} = -\frac{1}{2}y^2 + \frac{1}{8} \tag{11}$$

3. Linear stability theory

We may assess the stability characteristics of the evolving basic state using a straightforward perturbation theory. Therefore we set,

$$\psi = \Psi(y)\mathrm{e}^{\lambda t}\cos kx \quad \text{and}$$
$$\theta = \theta_{\mathrm{basic}}(y) + \Theta(y)\mathrm{e}^{\lambda t}\sin kx \tag{12}$$

where Ψ and Θ are both of a sufficiently small amplitude that nonlinear terms may be neglected. The value, k, is the horizontal wavenumber of the disturbances. We assume that the principle of exchange of stabilities applies (see, for example, Drazin and Reid [12], and the discussion in Appendix A), indicating that the onset of convection is stationary, or, in other words, that neither travelling nor standing waves appear. Therefore the following system of linearised disturbance equations are obtained,

$$-Da(\Psi^{\mathrm{IV}} - 2k^2\Psi'' + k^4\Psi) + \Psi'' - k^2\Psi = Ra\,k\Theta \tag{13a}$$
$$\Theta'' - k^2\Theta + k\Psi\theta'_{\mathrm{basic}} = 0 \tag{13b}$$

and this system is subject to the boundary conditions,

$$\Psi = \Psi' = \Theta = 0 \quad \text{on } y = \pm\frac{1}{2}. \tag{14}$$

In Eqs. (13a,b) primes denote differentiation with respect to y. These equations cannot be solved analytically and therefore numerical methods must be employed.

4. Numerical simulations

Eqs. (13a,b) form an ordinary differential eigenvalue problem for Ra as a function of Da and the wavenumber, k. When $Da = 0$ we recover the Darcy-flow case considered by Gasser and Kazimi [5]. In this paper we solve the full system (13) using a direct method related closely to the one described in Rees [13]. Eqs. (13a,b) were reduced to a set of three second-order equations by introducing a vorticity-like variable, and then discretised using fourth order compact differences (Spotz [14]) on a uniform grid in the y-direction. The zero normal flow, tangential flow and temperature conditions provide a sufficient number of boundary conditions for these equations. However, the eigenvalue, Ra, also needs to be found, and this requires one more condition; this is provided by the following normalization condition,

$$\Theta' = 1 \quad \text{on } y = \frac{1}{2} \tag{15}$$

The resulting discretised system is then solved using a standard multidimensional Newton–Raphson iteration technique. The iteration matrix takes a block tridiagonal form where there is one further column and row of nonzero blocks, and therefore the block-Thomas algorithm was modified to account for these extra blocks; see Eq. (11) in Rees [13] where the full procedure is described in more detail.

Numerical experiments indicate that the neutral stability curves always have the same qualitative form, namely that there is one minimum value of Ra at a critical value of k with monotonic growth towards infinity as $k \to 0$ and as $k \to \infty$. Therefore we concentrate solely on these critical values, since such a minimum value of Ra signifies the point above which we may expect convection to take place in an infinite layer. This minimization was achieved by insisting that $\partial Ra/\partial k = 0$ and by supplementing Eqs. (13a,b) with their derivatives with respect to k. If we define the variables,

$$\dot{\Psi} = \frac{\partial \Psi}{\partial k} \quad \text{and} \quad \dot{\Theta} = \frac{\partial \Theta}{\partial k} \tag{16}$$

then differentiation of Eqs. (13a,b) with respect to k yields the following system,

$$\begin{aligned} -Da(\dot{\Psi}^{\mathrm{IV}} - 2k^2\dot{\Psi}'' + k^4\dot{\Psi}) + \dot{\Psi}'' - k^2\dot{\Psi} - Ra\,k\dot{\Theta} \\ = Da(-4k\Psi'' + 4k^3\Psi) + 2k\Psi + Ra\,\Theta \end{aligned} \tag{17a}$$

$$\dot{\Theta}'' - k^2\dot{\Theta} + k\dot{\Psi}\theta'_{\mathrm{basic}} = 2k\Theta - \Psi\theta'_{\mathrm{basic}} \tag{17b}$$

subject to the boundary conditions,

$$\dot{\Psi} = \dot{\Theta} = 0 \quad \text{on } y = \pm\frac{1}{2} \tag{18}$$

As the wave number is now a second eigenvalue, we need to impose a second normalization condition that $\dot{\Theta}'(\frac{1}{2}) = 1$, although any other value of this derivative yields precisely the same values of Ra and k. This new extended system now consists of six second order equations and two normalisation conditions. The block matrices which appear in the Newton–Raphson iteration matrix are now 6×6.

There is now only one parameter to vary, namely Da, and solutions are presented for the range, $10^{-6} \leqslant Da \leqslant 10^2$. Uniform grids were used in the computation and 200 intervals in the range $-0.5 \leqslant y \leqslant 0.5$ were used as a basic grid. Grid refinement was then used and the accuracy was improved further using Richardson's extrapolation technique to obtain over 10 significant figures of accuracy, even for values of the Darcy number as small as 10^{-6}.

Figs. (2a) and (2b) show the respective streamlines and isotherms of the disturbance shapes, $\Psi \cos kx$ and $\Theta \sin kx$, for $Da = 0.1$. Each frame displays contours corresponding to 20 equally-spaced subintervals between their respective maxima and minima. The streamlines and isotherms are displayed in (x, y) space. Given that it is only the top half of the layer that is unstably stratified, the disturbance temperature field is concentrated within this half with a pair of weak cells in the lower half. The streamlines also display a bias towards the upper half of the channel.

Fig. 3 shows normalized profiles of $-\Psi'$, which is related to the horizontal fluid velocity, in order to see how these profiles vary with the value of Da. When Da is relatively large, which corresponds to the clear fluid limit, the velocity profile varies relatively slowly across the cavity. At such large values of Da, the maximum absolute velocity occurs at $y = 0.35$,

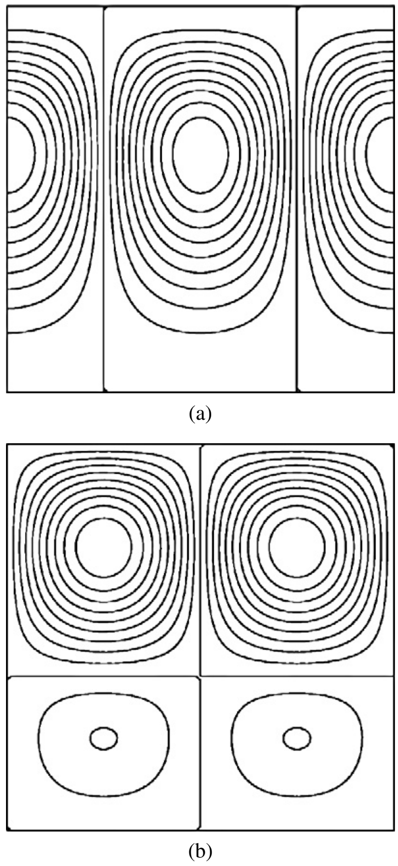

Fig. 2. Disturbance streamlines and isotherms corresponding to the $Da = 0.1$ in the (x, y)-plane: (a) streamlines, (b) isotherms.

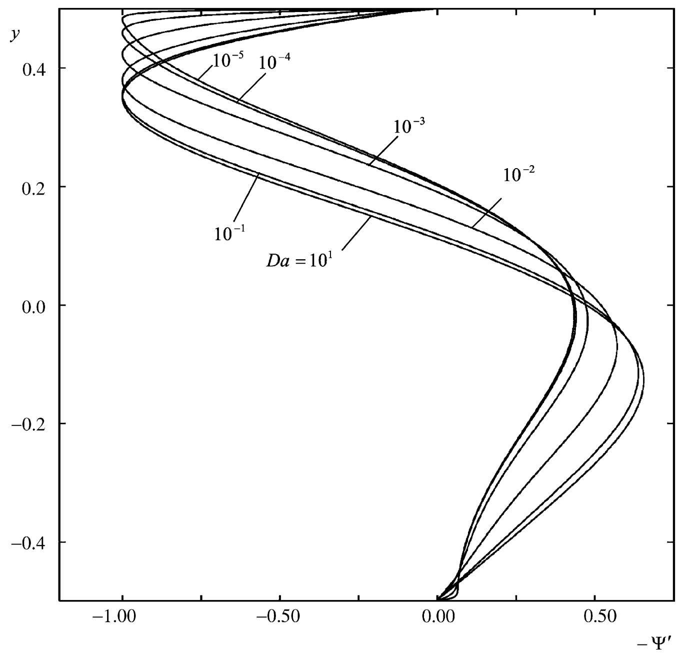

Fig. 3. Variation of a normalized horizontal velocity, $-\Psi'$ with y for different values of Da.

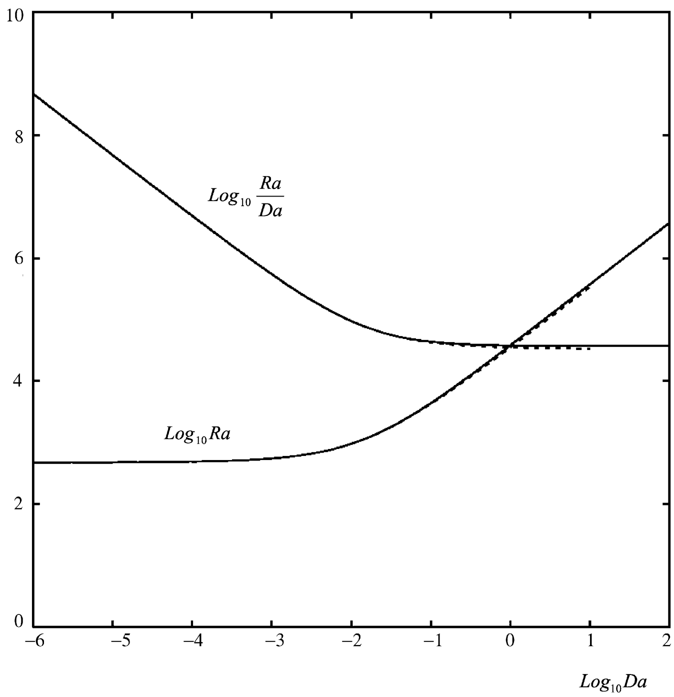

Fig. 4. Variation of critical Rayleigh number with $\mathrm{Log}_{10}Da$ for $10^{-6} \leqslant Da \leqslant 10^2$. Values are presented in terms of the porous Rayleigh number, Ra, and the clear fluid Rayleigh number Ra/Da. Dashed lines correspond to the small-Da solution given by Eq. (19a).

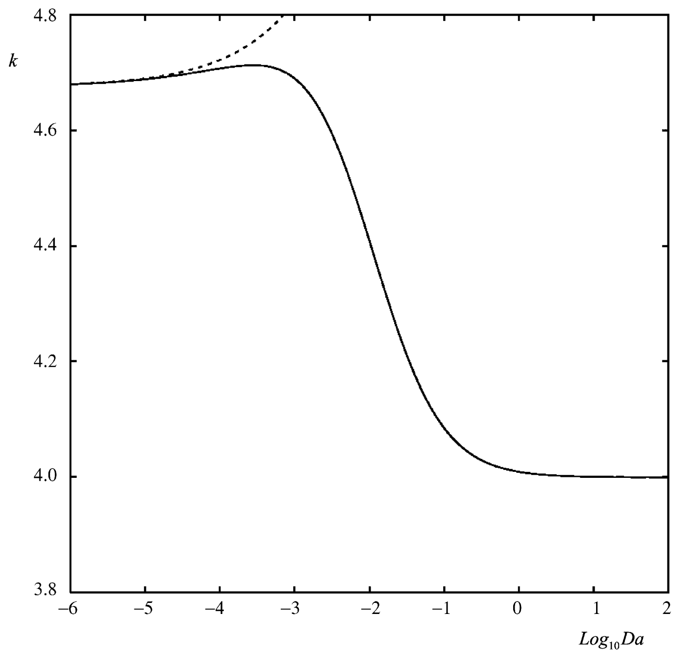

Fig. 5. Variation of critical wavenumber, k, with $\mathrm{Log}_{10}Da$ for $10^{-6} \leqslant Da \leqslant 10^2$. The dashed line corresponds to the small-Da solution given by Eq. (19b).

which is well within the upper half of the layer. However, as Da decreases towards zero, the position at which the maximum velocity occurs rises towards the upper surface. A very distinct boundary layer is formed at this upper surface within which the velocity changes rapidly to zero; a similar though weaker boundary layer is formed at the lower surface. It may be shown using a straightforward order-of-magnitude argument that the boundary layers are of thickness $\mathrm{O}(Da^{-1/2})$. A detailed asymptotic analysis of this phenomenon as it applies to Darcy–Bénard convection was given by Rees [13], and we shall present the results of a similar analysis below.

The respective variations in the critical values of Ra and k with Da are shown in Figs. 4 and 5. Fig. 4 shows the critical Rayleigh number in two forms, Ra, the porous medium Rayleigh number, and Ra/Da, the clear fluid Rayleigh number. The transition between the porous medium limit and the clear fluid limit is smooth with Ra increasing monotonically and Ra/Da decreasing monotonically. With regard to the former, it is to be expected that Ra should rise because viscous effects, as mediated by the Brinkman terms, increase in severity as Da increases. On the other hand, the variation of the critical wavenumber is not monotonic, a property it shares with the Darcy–Bénard problem (see [13]), but there is little overall variation.

The approach to the Darcy limit is seen clearly in these figures, where the dashed curves represent a small-Da asymptotic analysis. This analysis follows precisely the one described in detail in Rees [13] for the Darcy–Bénard problem, except that numerical solutions were required to solve the various ordinary differential equations which arise. We find that

$$Ra_c = 471.384663(1 + 3Da^{1/2} + 70.6226324Da + \cdots) \quad (19a)$$

$$k_c = 4.67518897(1 + Da^{1/2} + \cdots) \quad (19b)$$

where our computed data is correct to more than 10 significant figures. We note that the value, $Ra \sim 469$, which may be derived from the data presented in Gasser and Kazimi [5], is close to the leading term in (19a), but it was obtained using a small number of terms in a Galerkin expansion. We see from Fig. 5 that the above expression for k_c is accurate only for $Da < 10^{-4}$. However, Fig. 4 shows that Eq. (19a) is very accurate indeed for the whole range of values of Da, even though it is the result of a small-Da analysis. In particular, when Da is large, (19a) gives $Ra_c/Da \sim 33290.43$, which is very close indeed to the results of both Sparrow et al. [4] and Takashima [7], $Ra_c/Da = 37325.17$, an error of only 12%. Indeed, if we were to make an ad hoc modification to (19a) by taking the large-Da value of Ra into account:

$$Ra_c = 471.384663(1 + 3Da^{1/2} + 79.1820Da) \quad (20)$$

then this formula is in error by less than 1% over the whole range of Da values, with the largest error occurring at $Da \sim 0.1$. Eq. (20) may therefore be treated as a good correlation.

5. Conclusion

In this short paper we have determined how the presence of the Brinkman terms affects the onset criterion for the stability of natural convection in a horizontal porous layer with uniform heat generation and the standard no-slip boundary conditions. A smooth monotonic variation in the critical Rayleigh is found, with the porous and clear fluid limits being reproduced very accurately. The variation in the critical wavenumber, k, is not large, but it is not monotonic. In terms of the critical porous Rayleigh number, we may neglect the Brinkman terms and safely use Darcy's law when $Da < 10^{-3.5}$. On the other hand, $Da > 1$ reproduces the clear fluid limit with a high degree of accuracy. However, the formula given in (20) may be used over the whole range of Da with less than 1% error.

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Appendix A

If we were to replace ψ and θ in Eqs. (13) by $\Psi e^{\lambda t}$ and $\Theta e^{\lambda t}$, where λ is the exponential growth rate of disturbances, then Eqs. (13a) and (13b) become,

$$\Psi'' - k^2\Psi = Ra\, k\Theta \tag{A.1a}$$

$$\Theta'' - k^2\Theta = ky\Psi + \lambda\Theta \tag{A.1b}$$

When $Da = 0$ the appropriate boundary conditions are that

$$\Psi = \Theta = 0 \quad \text{on } y = \pm\frac{1}{2} \tag{A.1c}$$

We note that these boundary conditions, when combined with Eq. (A.1a), yield the fact that $\Psi'' = 0$ on the boundaries. Our intention here is to show that λ takes only real values, so that the Principle of Exchange of Stabilities applies.

We may eliminate Θ from Eqs. (A.1) to give,

$$\Psi'''' - 2k^2\Psi'' + (k^4 - Ra\, k^2)\Psi = \lambda(\Psi'' - k^2\Psi) \tag{A.2a}$$

which is subject to the boundary conditions,

$$\Psi = \Psi'' = 0 \quad \text{on } y = \pm\frac{1}{2} \tag{A.2b}$$

On taking Ψ to be complex in general, we may multiply Eq. (A.2a) by $\bar{\Psi}$ and perform a sufficient number of integrations by parts to obtain the following,

$$\int_{-1/2}^{1/2} \left[\Psi''\bar{\Psi}'' + 2k^2\Psi'\bar{\Psi}' + (k^4 - Ra\, k^2)\Psi\bar{\Psi}\right] dy$$

$$= -\lambda \int_{-1/2}^{1/2} \left[\Psi'\bar{\Psi}' + k^2\Psi\bar{\Psi}\right] dy \tag{A.3}$$

All the integrals in Eq. (A.3) are strictly real, and therefore λ must take real values. Therefore critical values of Ra correspond only to zero values of λ, and the Principle of Exchange of Stabilities applies to the Darcy case.

It is possible to extend this analysis easily to the more general Darcy–Brinkman case given by Eqs. (13), but only for stress-free boundary conditions (also see the discussion of Herron [15] and cited references). Therefore we shall assume that the Principle of Exchange of Stabilities is valid.

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