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Comments on the Paper "Non-Darcian Forced-Convection Flow of Viscous Dissipating Fluid over a Flat Plate Embedded in a Porous Medium" by O. Aydin and A. Kaya, Transport in Porous Media, doi:10.1007/s11242-007-9200-x, 2008

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Abstract In a very recent paper by Aydin and Kaya (Transp. Porous Media (to appear), 2008) the combined effects of viscous dissipation and surface mass flux on the forced-convection boundary-layer flow was considered. However, as the present Note shows, the thermal boundary condition imposed at the outer edge of the boundary-layer by Aydin and Kaya is incompatible with the energy equation, and thus the results of their paper are in error.

Keywords Forced-convection · Porous media · Boundary-layer · Viscous dissipation · Brinkman terms

In the last few years, there has been much interest in how the presence of viscous dissipation affects free, mixed and forced convective flows in porous media. The paper of Aydin and Kaya (2008) is the most recent, and these authors consider the combined effect of viscous dissipation and suction/injection at the heated surface on the streamwise evolution of a forced-convection boundary-layer flow in a porous medium. Unfortunately, the paper of Aydin and Kaya (2008) does not take into account some well-established results reported in the latter years concerning the effect of viscous dissipation on the far-field thermal boundary conditions of forced- and mixed-convection boundary-layer flows in porous media. As a consequence, their results are physically inconsistent.

Indeed, it has been emphasized by Magyari et al. (2003a,b, 2005), and Nield (2004), that a mixed or forced-convection boundary-layer flow generates heat *everywhere* when viscous dissipation is present, including the free stream region outside the boundary-layer. The rate $q'''_\infty = (\mu/K)\, u_\infty^2$ of this volumetric heat generation by viscous dissipation in the

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far-field $y \rightarrow \infty$ is non-vanishing, and thus the asymptotic thermal boundary condition $T(x, y \rightarrow \infty) = \text{const.} = T_\infty$ (see Fig. 1 and Eq. 4 of the paper) is not compatible with the energy balance equation. This fact may also be deduced by taking the $y \rightarrow \infty$ limit of the energy equation (3) of Aydin and Kaya (2008),

$$u\frac{\partial T}{\partial x} + \text{v}\frac{\partial T}{\partial y} = \frac{\nu}{Pr}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\nu}{Kc_p}u^2$$

Bearing in mind that $\partial T/\partial y \rightarrow 0$ as $y \rightarrow \infty$ (since heat cannot escape to infinity by conduction at any chosen value of x), and $u \rightarrow u_\infty$, one obtains

$$u_\infty \lim_{y\rightarrow\infty}\frac{\partial T}{\partial x} = \frac{\nu}{Kc_p}u_\infty^2. \tag{1}$$

On integrating this equation once and taking into account the "initial condition" at $x = 0$ (i.e. the first line of Eq. 4 of Aydin and Kaya), one gets

$$T(x, \infty) = T_\infty + \frac{\nu u_\infty}{Kc_p}x. \tag{2}$$

Therefore, a consistent description of the present mixed convection problem requires one to replace the (unsuitable) boundary condition $T(x, \infty) = T_\infty$ by the above condition (2) which specifies an asymptotic temperature which is not a constant, but is a linear function of the wall coordinate x. This result could also be predicted intuitively by considering the case where a cold fluid is forced along a uniform thickness porous channel with insulated impermeable sidewalls. The temperature of the fluid will rise uniformly with distance from the inlet due to viscous dissipation. The same phenomenon also happens in mixed convection in a semi-infinite domain with an adiabatic plane wall. The zero flux boundary condition in the far-field is satisfied here automatically.

When written in terms of the dimensionless variables of Aydin and Kaya (2008), the far-field condition (2) becomes,

$$\theta(\xi, \infty) = \text{Ec}\,\xi, \tag{3}$$

which differs from the condition $\theta(\xi, \infty) = 0$ given as Eq. 10 in Aydin and Kaya. Equation 3 also shows that, as soon as ξ exceeds Ec^{-1}, the far-field becomes even hotter than the heated surface at $y = 0$. This effect, which is a direct consequence of the energy balance equation, cannot be seen in Figs. 7 and 8 of Aydin and Kaya, simply because their computations extend over only a very short distance ξ from the leading edge. Given that their present paper is concerned primarily with *forced-convection*, for which $\text{Ec} \neq 0$, it is strange that only two of the 16 figures show plots corresponding to non-zero values of the Eckert number Ec. Similarly, all three tables of the paper report results only for $\text{Ec} = 0$.

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