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Curved free convection plume paths in porous media

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Abstract—This short paper presents some numerical simulations of how free convection plumes in a porous medium are affected by the presence of a neighboring boundary or a neighboring plume. It is found that they are drawn towards a vertical boundary with the 'centreline' following a curved path from the source to the boundary. Thus the boundary entrains the plume in a manner which is reminiscent of the well-known Coandă effect in aerodynamics where a fluid jet is drawn towards a solid surface. When two plumes are present in a horizontally unbounded porous medium, the plumes are drawn towards one another before rising vertically. In many cases where one plume is weaker or lower than the other, the former is affected greatly by the latter, but not vice versa.

Key-words: porous media, free convection, plume, entrainment

1. Introduction

Free convection plumes usually rise vertically. However, this is true only in certain circumstances, namely, when the flow domain is symmetric about a vertical line through the heat source. This situation was assumed by *Afzal* (1985), where a line source of heat was placed at the intersection of two plane surfaces bounding a wedge-shaped region of porous medium, but where the boundaries are at equal but opposite inclinations away from the vertical. *Afzal* (1985) provided a detailed high order boundary layer theory to determine the manner in which such boundaries affect the strength of the plume, thereby extending the analysis of *Wooding* (1963). A more general situation was considered by *Bassom et al.* (2000), where a porous wedge was allowed to have a centreline which is no longer vertical. In this case the centreline of the plume remains straight, at least according to boundary layer theory, but it no longer remains vertical. In general, the direction of the plume is somewhere between the vertical and the direction corresponding to the centreline of the wedge, and it is, therefore, a compromise between the effect of buoyancy (which induces vertical forces) and the need to entrain equal amounts of fluid from either side of the plume (which draws the plume towards the centreline of the wedge). *Bassom et al.* (2000) used boundary layer theory and presented an analytical expression for the direction taken by the plume in the terms of the inclinations of the bounding surfaces. Similar situations arise for anisotropic porous media (*Rees et al.,* 2002) and for clear fluids (*Rees* and *Storesletten*, 2002; *Kurdyumov*, 2006).

In the context of groundwater studies the presence of groundwater flow, together with other effects, such as heterogeneities and variable saturation, also serve to modify the path taken by contaminant plume in porous medium; see *Harter* and *Yeh* (1996a, b) for example. It is also a matter of common experience that a crosswind will modify the direction of chimney plumes. Deviations from straight path were also found by *Shaw* (1985) when considering plume flow in a cavity with an inlet and outlet at different horizontal locations.

In the present paper we consider (i) how a plume path is modified by presence of an adjacent insulated vertical surface and (ii) the merging of two plumes. This is a purely numerical study of the fully elliptic equations of motion, and it is, therefore, not a boundary layer study. Indeed, we regard this as an exploratory investigation into the behaviour which may be displayed by free convection plumes in porous media, and we intend to follow this work by a more detailed quantitative study in the near future.

As we have used a time-dependent solver to determine the eventual steady state solutions, we conclude that such plume flows are stable, at least to two-dimensional perturbations. We find that, when a localized heat source is placed away from a vertical surface, the plume curves towards the surface, and the point of attachment changes its location as the Rayleigh number varies. We also consider the interaction of two plumes, a situation which has been reviewed by *Gebhart* (1979) for plumes in clear fluids.

2. *Equations of motion and numerical scheme*

The equations governing two-dimensional convection in a porous medium are,

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\bar{u} = -\frac{K}{\mu}\frac{\partial \bar{p}}{\partial \bar{x}}, \tag{2}$$

$$\bar{v} = -\frac{K}{\mu}\left[\frac{\partial \vec{p}}{\partial \bar{y}} - \rho g \beta (T - T_0)\right], \tag{3}$$

$$\sigma\frac{\partial T}{\partial \bar{t}}+\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\frac{k}{\rho C}\left(\frac{\partial^2 T}{\partial \bar{x}^2}+\frac{\partial^2 T}{\partial \bar{y}^2}\right)+\frac{q'''}{\rho C}, \tag{4}$$

where Darcy's law has been assumed to be valid for the momentum equation and the Boussinesq approximation holds. Here $\bar{x}$ and $\bar{y}$ are the horizontal and vertical coordinates, respectively, and the corresponding flux velocities are $\bar{u}$ and $\bar{v}$. In addition, $\bar{p}$ is the pressure and T is the temperature. Heat generation takes place within the porous medium with the rate q''', which represents a local source centreed at the horizontal distance L, from an insulated vertical surface. The other quantities, namely K, μ, ρ, g, β, k, C, and σ, take their usual meanings: permeability, dynamic viscosity, reference fluid density, gravity, coefficient of thermal expansion, thermal conductivity of the porous medium, specific heat of the fluid, and the ratio of thermal capacities of the porous medium and the fluid. Finally, T_0 is the ambient temperature of the porous medium.

Nondimensionalization takes place using the following transformations

$$(\bar{x},\bar{y})=L(x,y),\quad (\bar{u},\bar{v})=\frac{k}{L(\rho C)}(u,v),\quad \bar{p}=\frac{k\mu}{(\rho C)K}p,\quad T=T_0+\frac{QL^2}{k}, \tag{5}$$

where

$$Q=L^{-2}\int_0^\infty\int_0^\infty q'''\,d\bar{x}\,d\bar{y}. \tag{6}$$

On introduction of the streamfunction ψ, using

$$u=-\frac{\partial\psi}{\partial y},\qquad v=\frac{\partial\psi}{\partial x}, \tag{7}$$

the governing equations become

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=\mathrm{Ra}\frac{\partial\theta}{\partial y}, \tag{8}$$

$$\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}-\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}+S(x,y), \tag{9}$$

where the Darcy-Rayleigh number is given by

$$\mathrm{Ra}=\frac{\rho_0(\rho C)g\beta KL^3Q}{k^2\mu}, \tag{10}$$

and where $S(x,y)$ is a local heat source centred at $(x,y) = (x_c, y_c)$ which, given that $S = q'''/Q$ must satisfy

$$\int_0^\infty \int_0^\infty S(x,y)\,dx\,dy = 1. \tag{11}$$

When the plume is situated on the horizontal bounding surface at y=0, we use

$$S = \frac{c}{2\pi} e^{-c((x-x_c)^2+y^2)}, \tag{12a}$$

but when it is well above this surface we use

$$S = \frac{c}{\pi} e^{-c((x-x_c)^2+(y-y_c)^2)}. \tag{12b}$$

We use the value c=2 here.

The statement of the problem is completed by the boundary conditions. For the case of a single plume, the flow is bounded by surfaces at $x = 0$ and $y = 0$ with the porous medium contained in the quarter plane, $x, y \geq 0$. Each surface is a streamline and both are assumed to be insulated. Therefore, we set

$$\psi = 0, \quad \frac{\partial \theta}{\partial n} = 0 \quad \text{on } x = 0 \text{ and } y = 0. \tag{13}$$

Inflow occurs at $x = x_{\max}$, and we set

$$\frac{\partial^2 \psi}{\partial x^2} = \theta = 0 \quad \text{on} \quad x = x_{\max}. \tag{14}$$

Outflow occurs at the upper surface, and the conditions used here are

$$\frac{\partial \psi}{\partial y} = \frac{\partial \theta}{\partial y} = 0 \quad \text{on } y = y_{\max}. \tag{15}$$

Outflow conditions are not as destructive for convective flows in porous media as they are for the flows of clear fluids. For the present problem it was found that the outflow conditions given by Eq. (15) cause small streamwise oscillations for only a few grid points upstream of the upper boundary which, given that the upper boundary is very far from where plume attachment takes place due to the use of a coordinate transformation, means that the results presented are essentially independent of the outflow conditions given by Eq. (15).

Eqs. (8) and (9) were solved using second order finite differences in space on a nonuniform grid and first order backward differences in time. The accuracy with respect to time is not of importance here since every simulation eventually yielded a steady state solution. The use of backward differences means that the system being solved is fully implicit and, therefore, we employed a Full Approximation Scheme multigrid methodology to the problem, where iterations on each grid were undertaken using the line Gauss-Seidel method. While this complicates the numerical coding, the fact that the method is implicit means that it is possible and indeed desirable to increase substantially the time steps towards the end of the calculation to enhance convergence to the steady state. Therefore, a crude timestep-changing methodology was employed. In many cases we determined steady state solutions on relatively coarse grids, interpolated these solutions onto finer grids and used this as an initial condition – this increased further the rapidity with which highly accurate solutions were obtained. The code used is a modified version of the one described in detail in *Rees* and *Bassom* (1993).

3. Results

The present paper is an exploratory work where, despite the complexity of the numerical code, we are solely interested in determining the qualitative nature of plume entrainment. Comments will be made later about planned improvements of the methodology.

In the numerical code we choose not to vary the value of the Rayleigh number from the chosen value of Ra = 200, rather we alter the location of the source. When the source is located at $x_c = d$, then the transformations

$$(x,y) = d(\hat{x},\hat{y}), \quad \psi = \hat{\psi}, \quad \theta = \hat{\theta}, \quad \mathrm{Ra} = \hat{\mathrm{Ra}}\, d, \quad S = \hat{S} d^{-2} \tag{16}$$

mean that Eqs. (8), (9), and (11) are reproduced precisely in the new variables with the source at $\hat{x} = 1$. Therefore, the setting of Ra = 200 and $x_c = d$ is equivalent to setting $\mathrm{Ra} = 200d$ and $x_c = 1$. The only difference between the two cases is the size of the spatial region over which the source is defined. The principle reason we followed this route rather than simply increasing the value of Ra is that solutions were obtained much more quickly.

Fig. 1 shows some typical streamlines and isotherms for cases where the source is centred on the axis. We have taken $x_c = 3$, 5, 7, and 9, and, therefore, the effective Rayleigh numbers are $\hat{\mathrm{Ra}}$ = 600, 1000, 1400, and 1800. The streamlines show clearly that fluid is entrained from the far right, turns near the corner, and travels upward in the expected manner. In addition there is a small region of weak recirculation which is bounded by a dividing streamline which may be regarded as ‘centreline’ of the plume from the point of view of fluid; this will be called the fluid centreline.

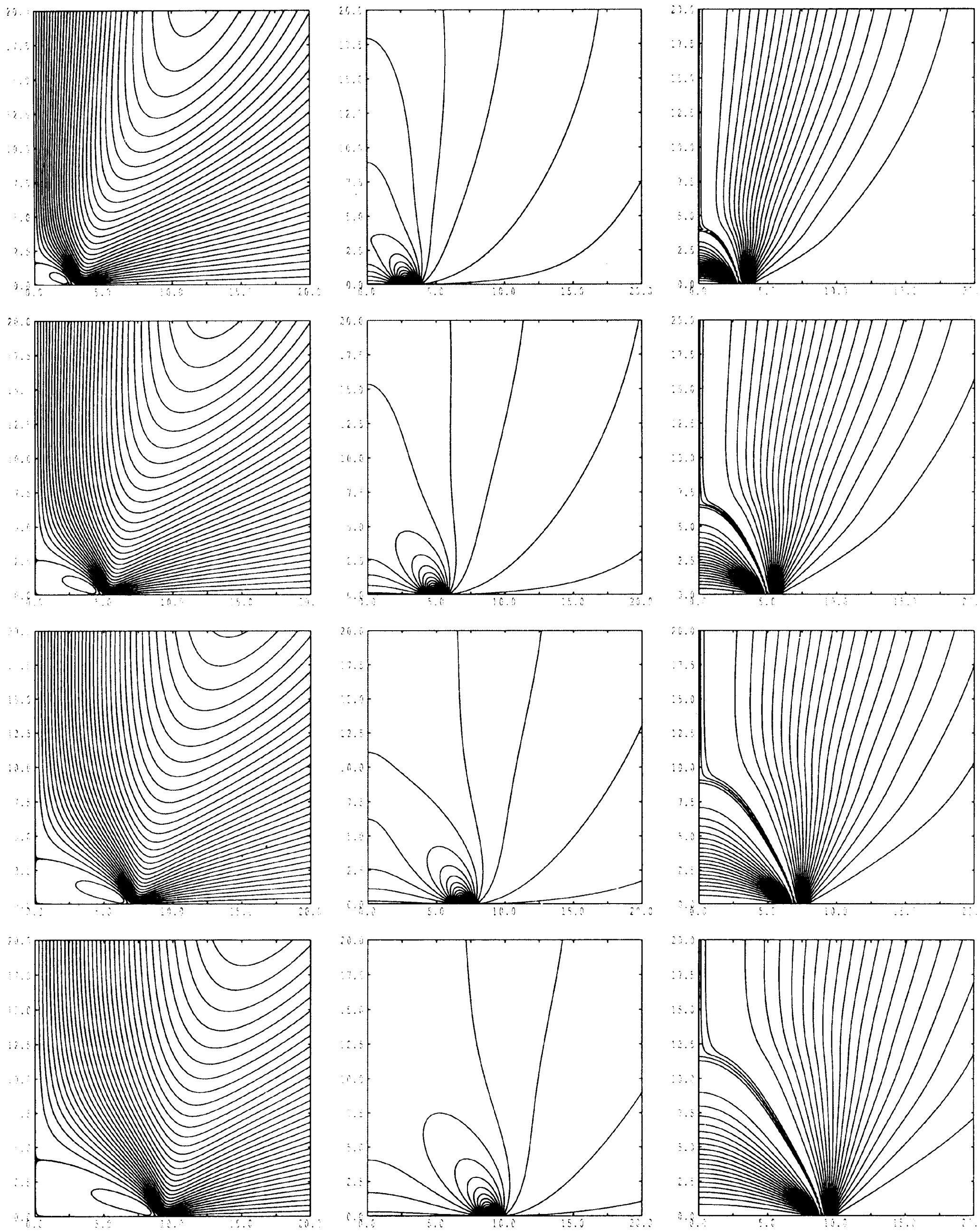


Fig. 1. Streamlines (left), isotherms (centre), and modified isotherms (right) for Ra = 200 with x_c=3, 5, 7, and 9 (from top to bottom) and y_c=0.

We have a situation where buoyancy forces cause the plume to rise, but the plume requires an equal amount of fluid to be entrained from each side for the plume to rise vertically. As the left side of the plume has only a finite amount of fluid (in terms of x) which may be entrained, the plume curves to the left to fulfil its need for fluid to entrain, and it attaches onto the vertical surface thereafter to rise as a wall plume (or, given the boundary conditions, as one half of a standard plume).

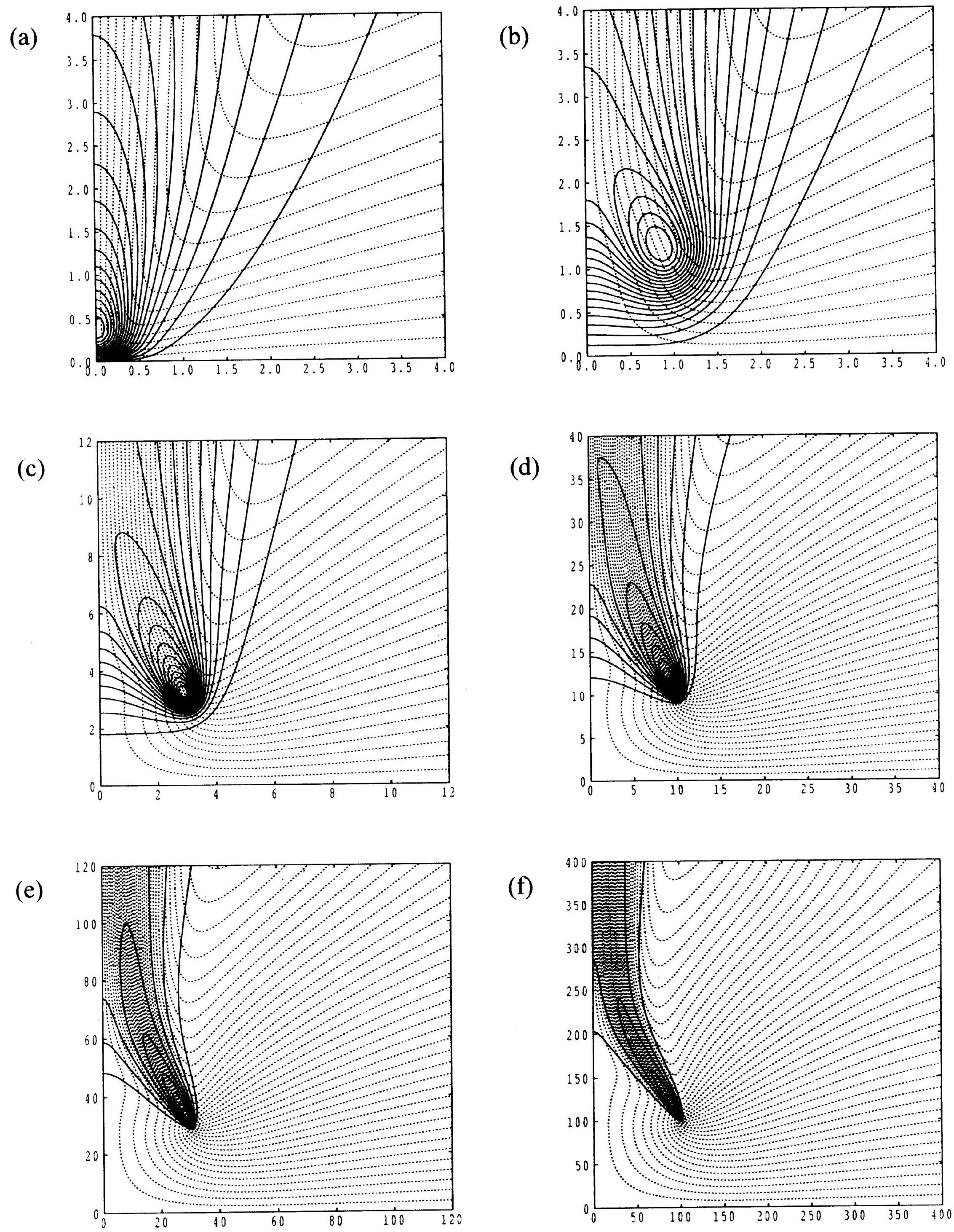
It is interesting to see that the point of attachment on the vertical surface (which should be labeled as y_c) is closer to the origin than is the horizontal distance of the source from the origin. *Table 1* shows that this attachment point gradually gets lower as Ra increases. Therefore, the need to entrain fluid appears to be a stronger effect than that due to buoyancy forces, which cause vertical motion.

Table 1. Attachment points for the plume; y_ψ and $\hat{y}_\psi$ correspond to where the dividing streamline joins onto the vertical surface, and y_θ and $\hat{y}_\theta$ to where thermal centreline joins

Râ	y_ψ	$\hat{y}_\psi$	y_θ	$\hat{y}_\theta$
600	1.60	0.53	2.43	0.81
1000	2.50	0.50	6.60	0.94
1400	3.35	0.48	9.10	1.30
1800	4.05	0.45	11.75	1.31

The second column of subfigures in *Fig. 1* shows the corresponding isotherms, but this is not particularly instructive, since the temperature of a free convection line plume in porous media decays roughly as $y^{-1/3}$ as y increases, and this masks the thermal behaviour of the plume that we wish to present. Therefore, the third column of subfigures has been prepared where the temperature at any point has been scaled with respect to the maximum temperature at that value of y. Thus, the contour plots show clearly where the maximum temperature is located and the path taken by this thermal centreline. *Table 1* gives the detailed values of where the thermal attachment point is as a function of Ra, and it is clear that this location (labeled as y_θ) increases slightly as Ra increases. However, we feel that further computation needs to be undertaken on this aspect as the position of the attachment point has not increased greatly between Ra = 1400 and Ra = 1800; it may be that this represents the large-Ra asymptotic limit, or it could presage a lowering of the attachment point following that of the dividing streamline. Further numerical work is needed to determine which of these scenarios is correct.

Fig. 2 shows how the plume reacts to changes in the location of its source, where $x_c = y_c$, or, equivalently, to changes in Ra. Here the source is above the horizontal surface and, therefore, fluid passes beneath the plume in order to feed the entrainment on the left side of the plume. It is important to note that the abscissa and ordinates of the subframes in *Fig. 2* have been scaled in such a way that each represents $0 \le \hat{x}, \hat{y} \le 4$. Given that computations were performed in terms of x and y with the source defined in Eq. (12), the increasing concentration of the isotherms around the source region as x_c increases is a direct consequence of the fact that the source has a diameter of roughly 1 in terms of x and y.



$$x_{\text{source}} = \begin{matrix} 0 & 1 \\ 3 & 10 \\ 30 & 100 \end{matrix} \quad \Rightarrow \quad \mathrm{Ra} = \begin{matrix} 100 & 100 \\ 900 & 10^4 \\ 9 \times 10^4 & 10^6 \end{matrix}$$

Fig. 2. Streamlines (dashed) and isotherms (continuous) for Ra = 200 with $x_c = y_c$, where x_c takes the values (a) 0, (b) 1, (c) 3, (d) 10, (e) 30, and (f) 100.

The chief effect of raising the source above $y = 0$ is to delay the attachment of the plume onto the vertical surface. The concept of an attachment point in terms of the streamfunction now no longer exists since there is no recirculation, and the only places where $\psi = 0$ are the two bounding surfaces. However, the concept of thermal attachment still applies, and the presence of a strong upward flow past the plume source means that thermal attachment is delayed substantially. This is seen most clearly in *Fig. 2c* which corresponds to $\hat{Ra} = 600$, which is the same as the case represented by the first row of *Table 1*. For the sake of comparison we shall define $\hat{y}_\theta$ to be based on the vertical distance between the attachment position and the location of the source: $\hat{y}_\theta = (y_\theta - y_c)/x_c$. As the maximum temperature at any value of y will correspond to that isotherm which has a turning point there, *Fig. 2c* shows clearly that y_θ is well above $y = 9$. Hence $\hat{y}_\theta$ is greater than 2, which is substantially further downstream than the value $\hat{y} = 0.81$ shown in *Table 1*. The attachment point in *Fig. 2d* is close to $\hat{y}_\theta = 3$, which is even greater.

Figs. 3, 4, and *5* show how two plumes interact, and each represents a different type of situation. The computational domain is now a half-plane. *Fig. 3* represents cases where two plumes have sources centred on $y_c = 0$, but where their horizontal locations are $x_c = \pm 4$; therefore, these correspond to cases where $Ra = 800$ and $\hat{x}_c = \pm 1$. The strength of the right and left hand plumes are given by the expressions given in Eq. (12), but where the right and left hand sides are multiplied by S_r and S_l, respectively. We take $S_r = 1$ and vary S_l between 0 and 1.

Fig. 3a depicts an isolated plume for comparison, while *Fig. 3b* shows that the presence of a weak second source nearby has little effect on the overall flow field and isotherm pattern, except for close to the horizontal bounding surface. We see that the plume generated by the weaker source exists independently for only a small distance above the surface before being absorbed into the main plume. As S_l increases, the left hand plume becomes stronger, and it begins to deflect the main plume towards itself, until, when $S_l = 1$ we are left with a perfectly symmetrical flow pattern.

Fig. 4 represents the same situation as *Fig. 3* except that both sources are placed at $y_c = 4$. The scenario described for *Fig. 3* in the above paragraph also occurs here, apart from the fact that the increased upward flow due to the positioning of the sources allows the weaker plume to exist for longer before being captured by the stronger plume. In this regard the behaviour is similar to that represented by *Fig. 2*.

Finally, two plumes of equal strength, but whose source heights are different, are depicted in *Fig. 5*. In *Fig. 5a* the lower plume is affected very strongly by the flow induced by the upper plume, although the upper plume is hardly affected by the presence of the lower plume, at least in terms of the part it takes. Indeed it is only when the source of the lower plume is as large as $y_c = 0.5$, which is shown in *Fig. 5c*, that the thermal centreline of the combined plume changes from being close to $x = 4$. When $y_c = 0.75$ for the lower plume, the centreline of the combined plume is now very close to $x = 0$, and symmetry is obtained when $y_c = 1$.

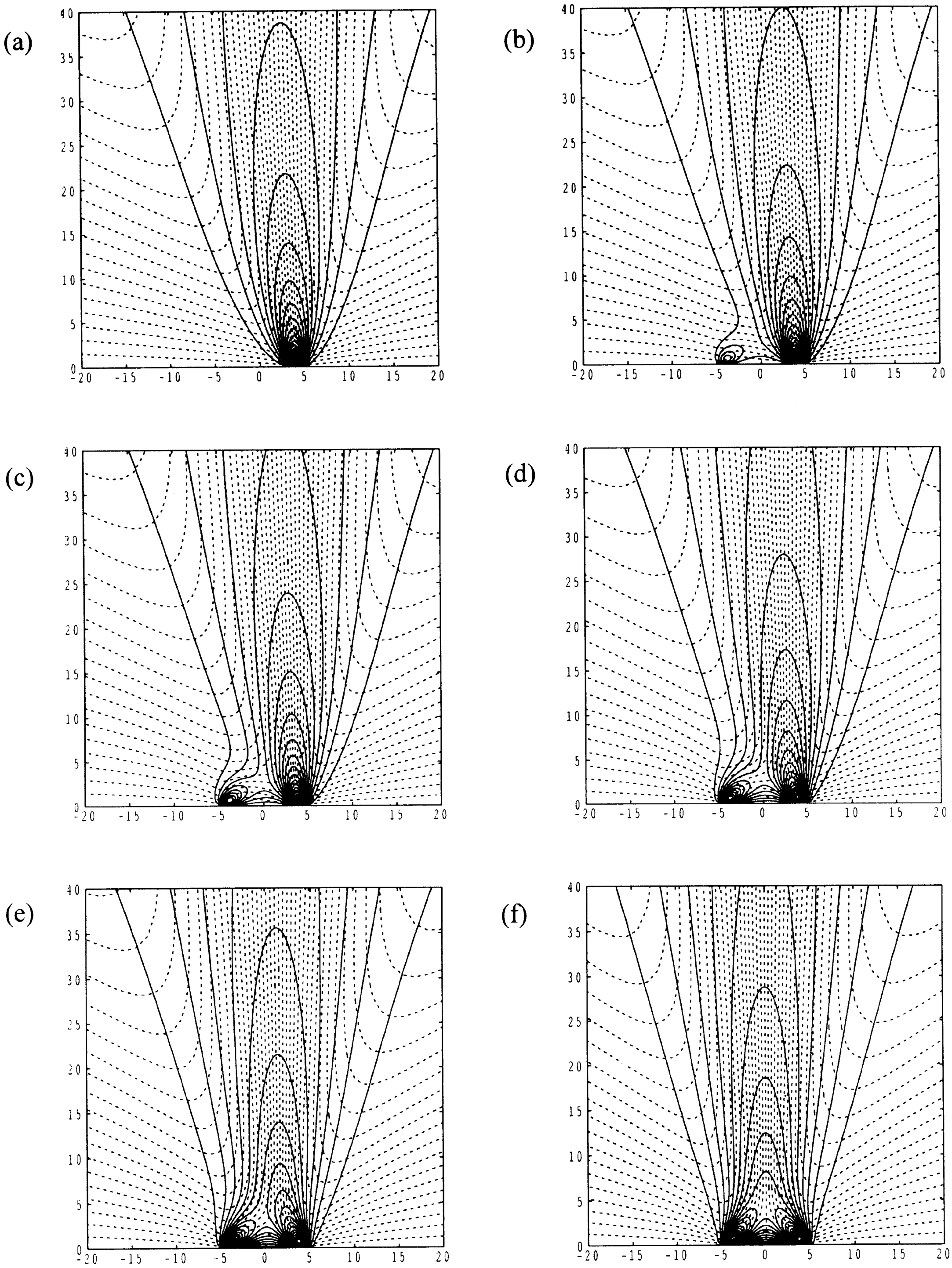


Fig. 3. Streamlines (dashed) and isotherms (continuous) for a situation with two plumes with Ra = 200 and $y_c = 0$. The strength of the right hand plume is S_r, and it is centred at $x_c = 4$. The strengths of the left hand plume are (a) $S_l = 0$, (b) 0.2, (c) 0.4, (d) 0.6, (e) 0.8, and (f) 1. The source of the left hand plume is centred at $x_c = -4$.

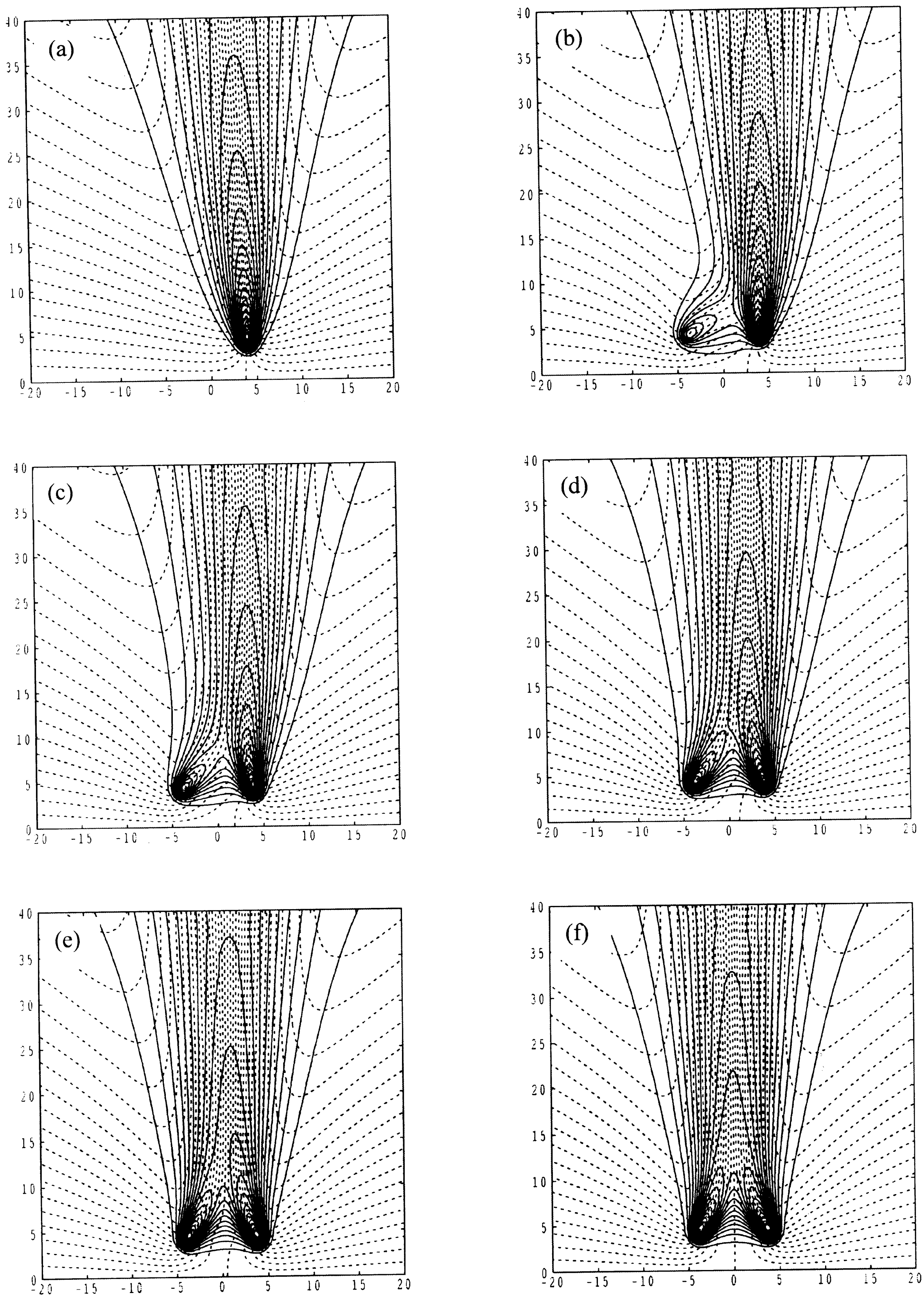


Fig. 4. As Fig. 3 but where the source of each plume is above the lower surface at $y_c = -4$.

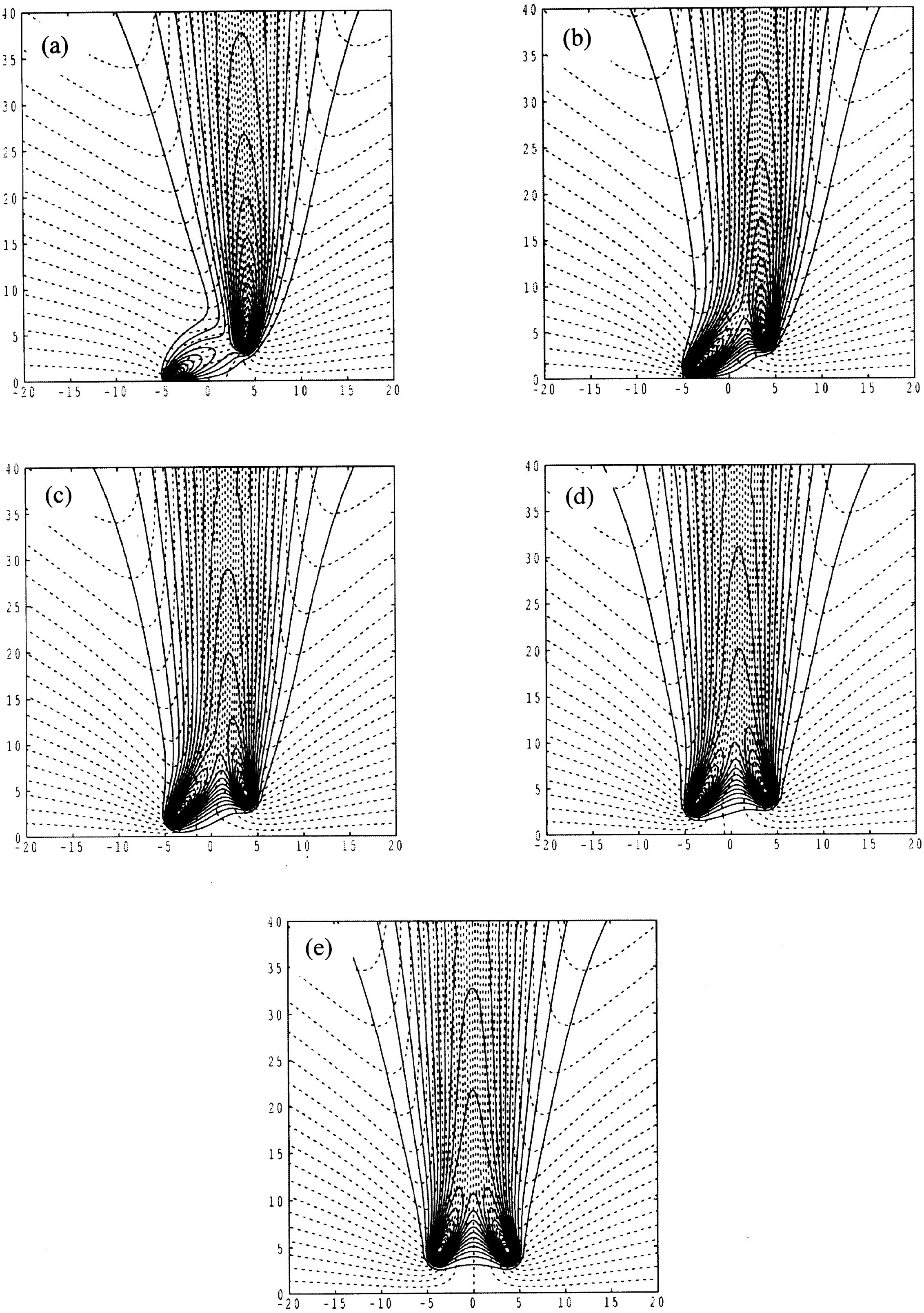


Fig. 5. Interaction of two plumes of equal strength ($S_r = S_l = 1$), where the source of the right hand plume is at $x_c = y_c = 4$, while the source of the left hand plume is at $x_c = -4$ with (a) $y_c = 0$, (b) 0.25, (c) 0.5, (d) 0.75, and (e) 1.

4. Conclusions

The main conclusion of this qualitative paper is that plumes are highly sensitive to their external environment. This was shown analytically by *Bassom et al.* (2000) where plumes in a wedge-shaped domain have a centreline which is straight, but not vertical, in general. In the present numerical study we have found that plumes will exhibit curved centrelines, although, in those cases where a fluid centreline may be defined, the fluid centreline does not coincide with the thermal centreline. In fact, within the parameters covered here, we find that the fluid attachment point descends as Ra increases, while the thermal centreline rises slightly. In addition, the restriction afforded by the placing of a source on a horizontal surface, means that there are substantial differences in both the flow field and the position of the thermal attachment point when the source is raised above the horizontal surface.

When two sources are present, the weaker one or the lower one is assimilated into the stronger one or the upper one, in some cases with little obvious qualitative effect on the latter.

It is our intention to perform further and more detailed computations, but given the sensitivity to domain shape and the need to keep the source region of constant size while the Rayleigh number is increased, it will be necessary to employ more sophisticated coordinate transformations than those employed here. In particular (i) the grid will need to be concentrated near the source region, as the whole flow field depends strongly on this, and (ii) the effective size of the computational domain in terms of the physical variables will need to be very large indeed in order to minimize the effects on the behaviour of the plume of the size and geometry of the computational domain and the inflow and outflow boundary conditions used.

Finally, we believe it to be true that the speed of migration of the plume towards the vertical wall, shown in *Figs. 1* and *2*, will be reduced substantially when point sources are considered. The flow that is induced in the y direction in these cases should provide some of the fluid for entrainment that is necessary to delay attachment.

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