Feedback on the May 2021 ME20021 (Modelling Techniques) Exam

General remarks

- Please do not scan your pages sideways! It's a pain to rotate the document in Inspera.
- I really don't like the use of *subbed* as a substitute for *substituted*. It certainly should never appear in formal documents.
- One student used a $\hat{}$ at the start of each line. Perhaps it was his/her version of \Rightarrow . Again, not in documents.
- In a large number of cases the u dv approach to Integration by Parts didn't work well. By contrast, there was only one instance where the Rees method didn't work, and that was because the student differentiated first rather than integrated.
- Congratulations to the two students who managed to provide a one-page complete set of solutions albeit in a two-column format. In one of the cases I would judge that slightly less than half a page was used!
- In general, the questions were answered very well indeed with the following comments being either neutral or representing the more common bits of dodgy practice.

Question 2.

- I am quite astonished by the fact that some people used the slow Separation of Variables technique shown on pages 28 to 30 of *https://people.bath.ac.uk/ensdasr/ME20021.bho/master.pdf* but which I didn't teach because it is so slow. A couple of students used up two pages doing this instead of the one-liner which I taught and which is based on the boundary conditions.
- Any of $\sin \frac{1}{2}n\pi$, $(-1)^{(n-1)/2}$ or $(-1)^{(n+3)/2}$ will do when n is odd. They are equivalent.
- Note that -1^n is not the same as $(-1)^n$. Likewise, $(-1)^{n-1/2}$ is not the same as $(-1)^{(n-1)/2}$; the former is purely imaginary when n is an integer while the latter is real.
- Note that $\cos \frac{1}{2}n\pi = 0$ when n is odd.
- In evaluating the Fourier coefficients a few were clearly so relieved to have done the integration by parts that they didn't substitute in the limits! However, this left an *x*-dependent Fourier coefficient which is definitely not correct.
- The solution to the ODE in part (a) should have given a sine and a cosine in time. This should be expected because it is the *wave* equation. Some used real exponentials not good. Some used complex exponentials and then declared that $e^{+jc\pi t/2}$ grows exponentially in time and removed it; it doesn't. The consequence was that only one initial condition could be applied, and the final answer to a real problem came out to be complex.

Question 3.

- There were a few students who decided to solve the question using the full Fourier Transform and not with the given Fourier Cosine Transform! Somehow they didn't notice that it was then impossible to apply the boundary condition at x = 0.
- Final sketch. Many problems here. This is a heat transfer problem on a semi-infinite domain, but often the sketches were confined just to $0 \le x \le 1$. The heat keeps diffusing outwards.
- Final sketch: Quite a few students used straight lines; they are curves!
- Final sketch: One student not only used a graphics package but uploaded the graph to Inspera! So I don't know which came first: the sketch or the graph.
- Final sketch: This was a triangular pulse rather than a square/rectangular pulse. The sketch should have featured the evolution in time of the initial triangular pulse.

- Final sketch: Some good attempts at a 3D sketch.
- Notice the difference between the four integrals:

$$\int_0^\infty f \times -\omega^2 \cos \omega x \, dx \qquad \int_0^\infty f \cdot -\omega^2 \cos \omega x \, dx \qquad \int_0^\infty f(-\omega^2) \cos \omega x \, dx \qquad \int_0^\infty -\omega^2 f \cos \omega x \, dx$$

I dislike the first two intensely and I get double-operator rage. The same goes for what appears to be a double minus: 4--3, which is the subtraction of a negative number, and should be written as 4-(-3). The symbol, -, plays two different roles and they need to be well-separated. The second integral is sometimes misread by the person who wrote it but in such a way that the dot goes missing on the next line — this happened a few times in the present exam. Generally it is better to place the $-\omega^2$ within its own bracket: $(-\omega^2)$. However, in this context the third integral isn't a good way of doing it because that looks as though f is a function of $(-\omega^2)$ whereas it is actually a function of x in this question. The fourth is good.

My advice: when using the centred dot to represent multiplication, then make it big! Don't use a dot between numbers because it may look like a decimal: is $2 \cdot 3 = 2 \cdot 3 = 2 \cdot 3$? Things like: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ are unambiguous and are fine.

• The following evaluation appeared after some integation and the correct solution is given on the right hand side:

$$-\left[-1\right]\left[-\frac{\cos\omega x}{\omega^2}\right]_0^1 = \frac{1-\cos\omega}{\omega^2}.$$
(1)

A quadruple negative arises as part of the evaluation and this often messes things up. Here are some alternative solutions which appeared on the scripts — try to work out which ones are correct before I make my comments on the next page:

$$(a) - \frac{\cos \omega - 1}{\omega^2} \qquad (b) - \frac{\cos \omega - 1}{\omega^2} \qquad (c) - \frac{\cos \omega - 1}{\omega^2}$$
$$(d) - \left(\frac{\cos \omega - 1}{\omega^2}\right) \qquad (e) - \frac{\cos \omega + 1}{\omega^2} \qquad (f) - \frac{\cos \omega + 1}{\omega^2}$$

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Options (a), (d) and (f) are correct, and of the three I prefer (d) because it is the clearest . That said, Equation (1) above is the very best of the lot. The problem with (b) is that I don't know how to interpret the leading minus sign; it could equally well be as either (a) or as (c). And likewise, is (e) the same as (f)? I shouldn't need to guess your intentions while I am marking! Please please please avoid ambiguity.

- So many wrote the inverse Fourier Cosine Transform integral as being with respect to x, rather than to ω .
- Note that all three of $1 \cos \omega e^{-\alpha \omega^2 t}$, $1 \cos \omega e^{-\alpha \omega^2 t}$ and $1 \cos \omega \cdot e^{-\alpha \omega^2 t}$ are different from $(1 \cos \omega) e^{-\alpha \omega^2 t}$. There were many of these.

Also, if one wrote $\cos \omega e^{-\alpha \omega^2 t}$ was the intention either $(\cos \omega) e^{-\alpha \omega^2 t}$ or $\cos(\omega e^{-\alpha \omega^2 t})$?