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| Examiner: Dr D A S Rees            | Date May 2021                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| Unit Title: Modelling Techniques 2 | Unit Code: ME20021                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| Year: 2020/21                      | Question Number: <del>1</del> 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| Part                               | Mark                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| (a)                                | <p>Need to use <math>P(x, t) = T(t) \sin \frac{n\pi x}{2}</math> (n odd) to satisfy BCs at <math>x=0</math> &amp; <math>x=1</math>.</p> <p>PDE <math>\Rightarrow T'' = -\frac{c^2 n^2 \pi^2}{4} T</math></p> <p><math>\Rightarrow T = A \cos \frac{nc\pi t}{2} + B \sin \frac{nc\pi t}{2}</math>.</p> <p>Reconstruct P &amp; superpose:</p> $P = \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} (A_n \cos \frac{nc\pi t}{2} + B_n \sin \frac{nc\pi t}{2}) \sin \frac{n\pi x}{2}.$ <p>(b) <math>t=0 \Rightarrow \frac{\partial P}{\partial t} = 0 \Rightarrow B_n = 0</math>.</p> <p><math>t=0 \Rightarrow P=x \Rightarrow x = \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} A_n \sin \frac{n\pi x}{2}</math></p> <p>where <math>A_n = 2 \int_0^1 x \sin \frac{n\pi x}{2} dx</math></p> $= 2 \left[ \left[ x \right] \left[ -\frac{2}{n\pi} \cos \frac{n\pi x}{2} \right]_0^1 - \left[ 1 \right] \left[ -\frac{4}{n^2 \pi^2} \sin \frac{n\pi x}{2} \right]_0^1 \right]$ $= 2 \left[ 0 + \frac{4}{n^2 \pi^2} \sin \frac{n\pi}{2} \right] = \frac{8}{n^2 \pi^2} \sin \frac{n\pi}{2}$ <p>So <math>P = \sum_{\substack{n=1 \\ (n \text{ odd})}}^{\infty} \left( \frac{8}{n^2 \pi^2} \sin \frac{n\pi}{2} \right) \cos \frac{nc\pi t}{2} \sin \frac{n\pi x}{2}</math></p> <p>Nb. <math>\sin \frac{n\pi}{2} = (-1)^{(n-1)/2} = (-1)^{(n+3)/2}</math> etc.</p> <p>Alternative solution:</p> $P = \sum_{m=0}^{\infty} \frac{8(-1)^m}{(2m+1)^2 \pi^2} \cos \frac{(2m+1)c\pi t}{2} \sin \frac{(2m+1)\pi x}{2}.$ |
| Total                              | 25                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |

UNIVERSITY OF BATH  
DEPARTMENT OF MECHANICAL ENGINEERING

Outline Solution to Examination Question

| Examiner: Dr D A S Rees                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | Date May 2021      |
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| Unit Title: Modelling Techniques 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Unit Code: ME20021 |
| Year: 2020/21                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Question Number: 3 |
| Page 1 of 1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                    |
| Part                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | Mark               |
| <p>(a) <math>\mathcal{Y}_c[f''] = \int_0^\infty f'' \cos \omega x \, dx = [f']_0^\infty - [f]_0^\infty - \omega \int_0^\infty [f] \sin \omega x \, dx</math></p> <p style="text-align: center;"><math>= -f'(0) - \omega^2 \mathcal{Y}_c(f)</math></p> <p style="text-align: center;">where <math>f, f' \rightarrow 0</math> as <math>x \rightarrow \infty</math>.</p>                                                                                                                                                                                                                                           | 5                  |
| <p>(b) <math>\mathcal{Y}_c[g(x)] = \int_0^\infty (1-x) \cos \omega x \, dx</math></p> <p style="text-align: center;"><math>= [(1-x) \frac{\sin \omega x}{\omega}]_0^\infty - [-1] [-\frac{\cos \omega x}{\omega^2}]_0^\infty</math></p> <p style="text-align: center;"><math>= \frac{1}{\omega^2} (1 - \cos \omega)</math></p>                                                                                                                                                                                                                                                                                  | 5                  |
| <p>(c) PDE becomes <math>\frac{\partial \Theta}{\partial t} = \alpha [-\omega^2 \Theta] \Rightarrow \Theta = A(\omega) e^{-\alpha \omega^2 t}</math></p> <p>IC in that <math>\Theta = g(x) \Rightarrow \Theta = \frac{1 - \cos \omega}{\omega^2}</math></p> <p>Hence <math>A = \frac{1 - \cos \omega}{\omega^2}</math></p> <p><math>\Rightarrow \Theta = \left( \frac{1 - \cos \omega}{\omega^2} \right) e^{-\alpha \omega^2 t}</math></p> <p><math>\Rightarrow \Theta = \frac{2}{\pi} \int_0^\infty \left( \frac{1 - \cos \omega}{\omega^2} \right) e^{-\alpha \omega^2 t} \cos \omega x \, d\omega</math></p> | 10                 |
| <p>(d) </p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 5                  |
| Total                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 25                 |