UNIVERSITY OF BATH
DEPARTMENT OF MECHANICAL ENGINEERING

Outline Solution to Examination Question

Examiner: Dr D A S Rees		Date May 2020
Unit Title: Modelling Techniques 2		Unit Code: ME20021
Year: 2019/20	Question Number: 3	Page 1 of 1

Part		Mark
(a)	BCs $\Rightarrow$ need to use $\sin n\pi x$ $(n=1,2,3,\ldots)$. Hence let $T = Y(y)\sin n\pi x$	
	Subst. into PDE $\Rightarrow$ $Y'' - n^2\pi^2 Y = 0$.	
	$\Rightarrow Y = Ae^{n\pi y} + Be^{-n\pi y}$ $\Rightarrow T = (Ae^{n\pi y} + Be^{-n\pi y})\sin n\pi x$ $(n=1,2,\ldots)$	
	Superpose: $\boxed{T = \sum_{n=1}^{\infty}(A_n e^{n\pi y} + B_n e^{-n\pi y})\sin n\pi x}$ — (1)	8
(b)	Expect $T \to 0$ as $y \to \infty$ $\Rightarrow A_n = 0$.	
	At $y=0$, $T = x - x^3$ $\Rightarrow x - x^3 = \sum_{n=1}^{\infty} B_n \sin n\pi x$	
	where $B_n = 2\int_0^1 (x-x^3)\sin n\pi x = \ldots = \frac{12(-1)^{n+1}}{n^3\pi^3}$	
	Hence $\boxed{T = \sum_{n=1}^{\infty} \frac{12(-1)^{n+1}}{n^3\pi^3} e^{-n\pi y} \sin n\pi x}$	12
(c)	$\boxed{T = \sum_{n=1}^{\infty} \frac{12(-1)^{n+1}}{n^3\pi^3} e^{-n\pi x} \sin n\pi y}$ (Interchange x + y)	5
(d)	In (1) above need $T=0$ at $y=1$ $\Rightarrow A_n e^{n\pi} + B_n e^{-n\pi} = 0$	
	$\Rightarrow A_n = -B_n e^{-2n\pi}$	
	$\Rightarrow T = \sum_{n=1}^{\infty} B_n [e^{-n\pi y} - e^{n\pi y - 2n\pi}] \sin n\pi x$	
	Same analysis as part (b) $\Rightarrow B_n[1 - e^{-2n\pi}] = \frac{12(-1)^{n+1}}{n^3\pi^3}$	
	$\Rightarrow T = \sum_{n=1}^{\infty} \frac{12(-1)^{n+1}}{n^3\pi^3} \frac{[e^{-n\pi y} - e^{n\pi y - 2n\pi}]}{[1 - e^{-2n\pi}]} \sin n\pi x$	
	$= \sum_{n=1}^{\infty} \frac{12(-1)^{n+1}}{n^3\pi^3}\left[\frac{e^{-n\pi(y-1)} - e^{n\pi(y-1)}}{e^{n\pi} - e^{-n\pi}}\right] \sin n\pi x$	
	$= \boxed{\sum_{n=1}^{\infty} \frac{12(-1)^{n+1}}{n^3\pi^3} \frac{\sinh n\pi(1-y)}{\sinh n\pi} \sin n\pi x}$.	8

	Total	33

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Part		Mark

(a) $\mathcal{F}[f''] = \int_{-\infty}^{\infty} f'' e^{-j\omega x}\,dx = \cancel{[f'][e^{-j\omega x}]} - \cancel{[f][-j\omega e^{-j\omega x}]} + \int_{-\infty}^{\infty}[f][j\omega^2 e^{-j\omega x}]\,dx$

$=0$ if $f' \to 0$ $\quad =0$ if $f \to 0$

$= -\omega^2 \int f e^{-j\omega x} = \boxed{-\omega^2 F(\omega)}$ — 8

(b) $\int_{-\infty}^{\infty} f(x+a) e^{-j\omega x}\,dx = \int_{-\infty}^{\infty} f(\xi) e^{-j\omega(\xi - a)}\,d\xi \quad [\xi = x+a \ldots]$

$= \boxed{e^{j\omega a} F(\omega)}.$ — 5

(c) FT of $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is $\boxed{\frac{\partial^2 U}{\partial t^2} = -\omega^2 c^2 U}$ — 5

(d) Solution is $U = A\cos\omega c t + B\sin\omega c t$ [or $Ae^{j\omega ct} + Be^{-j\omega ct}$]

IC: $\frac{\partial u}{\partial t} = 0$ at $t=0 \Rightarrow \frac{\partial U}{\partial t} = 0 \Rightarrow B = 0 \Rightarrow U = A\cos\omega c t$

[or $A - B = 0$]

I.C. $u = f(x)$ at $t=0 \Rightarrow U = F. \Rightarrow A = F \Rightarrow U = F\cos\omega c t$

[or $A+B=F \Rightarrow A = B = F/2 \Rightarrow U = \frac{1}{2}F(e^{j\omega ct} + e^{-j\omega ct})$]

As $\cos\omega c t = \frac{1}{2}[e^{j\omega ct} + e^{-j\omega ct}]$ then $U = \frac{1}{2}F(e^{j\omega ct} + e^{-j\omega ct})$

Using the shift theorem in part (b) we get

$$\boxed{u = \tfrac{1}{2}[f(x+ct) + f(x-ct)]}$$ — 10

These are waves travelling at velocities $-c$ and $+c$, respectively, with the same shape as the initial disturbance and half the amplitude. — 5

	Total	33