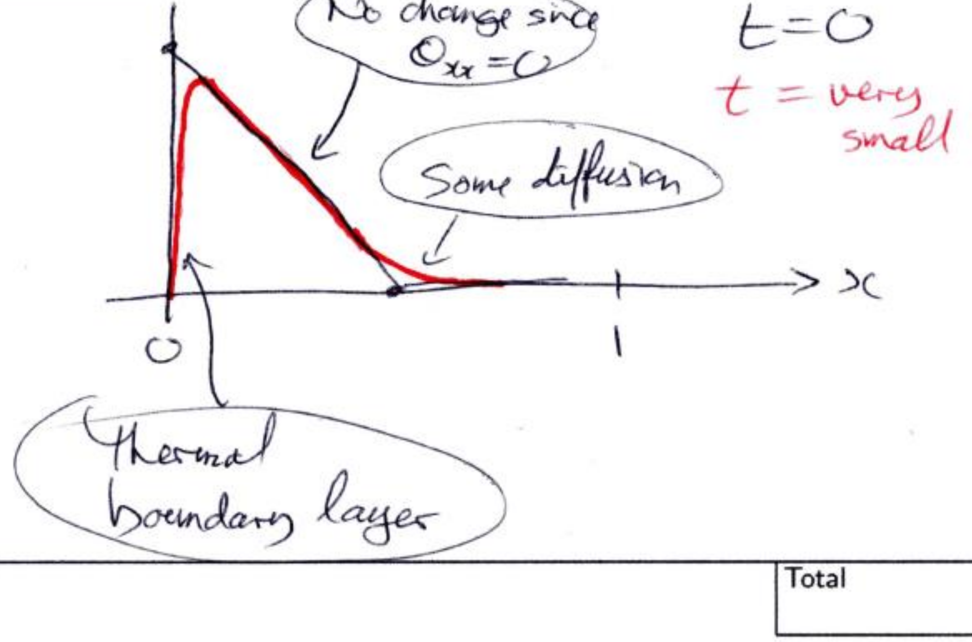
UNIVERSITY OF BATH
DEPARTMENT OF MECHANICAL ENGINEERING

Outline Solution to Examination Question

Examiner: Dr D A S Rees		Date May 2022
Unit Title: Modelling Techniques 2		Unit Code: ME20021
Year: 2021/22	Question Number: 3	Page 1 of 1

Part		Mark
(a)	Need $\Theta = T(t)\sin n\pi x \quad (n=1,2,3,\cdots)$. PDE $\Rightarrow T' = -\alpha n^2\pi^2 T \Rightarrow T = B_1 e^{-\alpha n^2\pi^2 t}$ ~~Super~~ $\Rightarrow \Theta = B e^{-\alpha n^2\pi^2 t}\sin n\pi x$. Superpose $\Rightarrow \Theta = \sum_{n=1}^{\infty} B_n e^{-\alpha n^2\pi^2 t}\sin n\pi x$	10
(b)	$t=0 \Rightarrow \Theta = \sum_{n=1}^{\infty} B_n \sin n\pi x = \text{given fct.}$ So $B_n = 2\int(\text{given fct})\sin n\pi x\,dx$ $= 2\int_0^{1/2}(1-2x)\sin n\pi x\,dx$ $= 2\Big[(1-2x)\Big(-\frac{\cos n\pi x}{n\pi}\Big) - (-2)\Big(-\frac{\sin n\pi x}{n^2\pi^2}\Big)\Big]_0^{1/2}$ $= \frac{2}{n\pi} - \frac{4\sin\frac{n\pi}{2}}{n^2\pi^2}$ Solution is $\Theta = \sum_{n=1}^{\infty}\Big[\frac{2}{n\pi} - \frac{4\sin\frac{n\pi}{2}}{n^2\pi^2}\Big]e^{-\alpha n^2\pi^2 t}\sin n\pi x$	15
(c)		8

Total	33

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Part		Mark
(a)	This is bookwork. $\mathcal{F}_s[f''] = \int_0^\infty f'' \sin \omega x \, dx =$ $= \Big[f'\Big]\Big[\sin \omega x\Big]_0^\infty - \Big[f\Big]\Big[\omega \cos \omega x\Big]_0^\infty + \int_0^\infty \Big[f\Big]\Big[-\omega^2 \sin \omega x\Big] \, dx$ $= \omega f(0) - \omega^2 F_s(\omega).$	8
(b)	The application of the Fourier Sine Transform to the wave equation (together with the nonzero boundary condition at $x = 0$) gives $\dfrac{\partial^2 Y_s}{\partial t^2} + c^2\omega^2 Y_s = c^2\omega.$ The general solution is, $Y_s = \dfrac{1}{\omega} + A(\omega)\cos c\omega t + B(\omega)\sin c\omega t.$	6
	At $t = 0$ we have zero displacement, i.e. $y = 0$, and hence $Y_s = 0$. Therefore $A = -1/\omega$.	2
	At $t = 0$ we also have a zero velocity, i.e. $\partial y/\partial t = 0$, hence $\partial Y_s/\partial t = 0$. Therefore $B = 0$.	3
	The solution is $Y_s = \dfrac{(1 - \cos c\omega t)}{\omega}$.	2
	Applying the inverse Fourier Sine Transform gives $y = \dfrac{2}{\pi}\int_0^\infty \dfrac{(1 - \cos c\omega t)}{\omega} \sin \omega x \, d\omega.$	2
(c)	The given analytical solution, $y = H(ct - x)$ is equal to 1 when $x < ct$ (i.e. when $ct - x > 0$), and equal to zero when $x > ct$. Therefore the Fourier Sine Transform of this function is $\mathcal{F}_s[H(ct - x)] = \int_0^\infty H(ct - x)\sin \omega x \, dx = \int_0^{ct} 1 \sin \omega x \, dx = \dfrac{(1 - \cos c\omega t)}{\omega}.$ This final answer is the same as Y_s above.	5
	This solution corresponds to a shock wave moving with velocity c in the positive x-direction.	5
	Total	33