Examiner: Dr D A S Rees		Date May 2018	
Unit Title: Modelling Techniques 2		Unit Code: ME20021	
Year: 2017/18	Question Number: 3.	Page 1 of 1	

Part		Mark
(a)	Given that $\theta = 0$ at $x = 0$ and $\theta_x = 0$ at $x = 1$, we use a quarter-range series. Hence let $\theta = T(t)\sin(n\pi x/2)$ for $n = 1, 3, 5, \cdots$.	5
	Hence $T' = -\alpha(n^2\pi^2/4)T$, and the solution is $$T = Be^{-\alpha n^2\pi^2 t/4}.$$	5
	Superpose all of these to get, $$\theta = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} B_n e^{-\alpha n^2\pi^2 t/4} \sin(n\pi x/2).$$ 3 marks off if neither $n = 1, 3, 5...$ nor 'n =odd' appears here.	3
(b)	At $t = 0$ we have $\theta = 2x - x^2$, hence $2x - x^2 = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} B_n \sin(n\pi x/2).$	5
	Using the given formula, $$\begin{aligned} B_n &= 2\int_0^1 (2x - x^2)\sin(n\pi x/2)\,dx, \\ &= 2\Big[\Big(2x - x^2\Big)\Big(-\frac{2}{n\pi}\cos(n\pi x/2)\Big) - \Big(2 - 2x\Big)\Big(-\frac{4}{n^2\pi^2}\sin(n\pi x/2)\Big) \\ &\qquad + \Big(-2\Big)\Big(\frac{8}{n^3\pi^3}\cos(n\pi x/2)\Big)\Big]_0^1 \\ &= \frac{32}{n^3\pi^3}. \end{aligned}$$	10
	Hence the full solution is $$\theta = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{32}{n^3\pi^3}\sin(n\pi x/2)e^{-\alpha n^2\pi^2 t/4}.$$	5
	Total	33

UNIVERSITY OF BATH
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Outline Solution to Examination Question

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Part		Mark
(a)	This is bookwork. $\mathcal{F}[f''] = \int_{-\infty}^{\infty} f'' e^{-j\omega x}\,dx =$ $= \Big[f'\Big]\Big[e^{-j\omega x}\Big]_{-\infty}^{\infty} - \Big[f\Big]\Big[-j\omega e^{-j\omega x}\Big]_{-\infty}^{\infty} + \int_{-\infty}^{\infty}\Big[f\Big]\Big[-\omega^2 e^{-j\omega x}\Big]\,dx$ $= -\omega^2 F(\omega)$ (since $f, f' \longrightarrow 0$ as $x \to \pm\infty$).	5
(b)	$\mathcal{F}[e^{-a\|x\|}] = \int_{-\infty}^{\infty} e^{-a\|x\|} e^{-j\omega x}\,dx = 2\int_0^{\infty} e^{-ax}\cos\omega x\,dx = \frac{2a}{a^2+\omega^2},$ using symmetry, and then either integration by parts or by taking the imaginary part of the integral of $e^{-(a+\omega j)x}$. The symmetry theorem now gives, $\mathcal{F}\Big[\frac{2a}{a^2+x^2}\Big] = 2\pi e^{-a\|\omega\|}.$ Dividing by 2π gives the required result.	8
(c)	The application of the Fourier Transform, using the result of part (a), gives the result immediately: $\frac{\partial^2\Theta}{\partial y^2} - \omega^2\Theta = 0.$	5
(d)	The general solution is, $\Theta = A(\omega)e^{\omega y} + B(\omega)e^{-\omega y}$, which may be compressed into the form, $\Theta = C(\omega)e^{-y\|\omega\|}$, given that decaying solutions are required as $y \to \infty$ (Bookwork).	5
	At $y = 0$ we have $\theta = \delta(x)$, and hence its transform is $\Theta = 1$. Therefore $C = 1$ and hence the solution is $\Theta = e^{-y\|\omega\|}$. Using the result of part (b) with a set equal to y we get $\theta = \frac{1}{\pi}\Big(\frac{y}{x^2+y^2}\Big).$	5
	The $\theta = 1/2\pi$ contour gives us $\frac{1}{2\pi} = \frac{1}{\pi}\Big(\frac{y}{x^2+y^2}\Big) \quad\Rightarrow\quad x^2 + (y-1)^2 = 1,$ which is a circle of radius, 1, centred at $(x, y) = (0, 1)$.	5
	Total	33