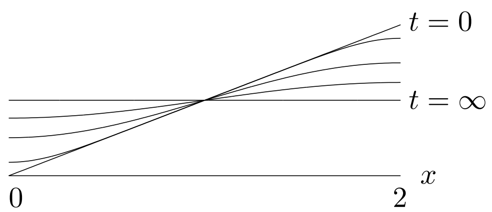
UNIVERSITY OF BATH
DEPARTMENT OF MECHANICAL ENGINEERING

Outline Solution to Examination Question

Examiner: Dr D A S Rees		Date May 2017
Unit Title: Modelling Techniques 2		Unit Code: ME20021
Year: 2016/17	Question Number: 3.	Page 1 of 1

Part		Mark
(a)	Separation of variables: let $\theta = T(t)\cos(n\pi x/2)$ for $n = 0, 1, 2, \cdots$. Hence $T' = -\alpha(n^2\pi^2/4)T$. When $n = 0$ we have $T = A$. When $n \neq 0$ we have $T = Ae^{-\alpha n^2\pi^2 t/4}$. Reconstructing θ and superposing yields (where only 7 marks is allocated if A_0 is missing), $$\theta = \tfrac{1}{2}A_0 + \sum_{n=1}^{\infty} A_n e^{-\alpha n^2\pi^2 t/4}\cos(n\pi x/2).$$	10
(b)	At $t = 0$ we have $\theta = x$, hence $x = \tfrac{1}{2}A_0 + \sum_{n=1}^{\infty} A_n \cos(n\pi x/2)$, where $$A_n = \int_0^2 x\cos(n\pi x/2)\,dx.$$ After integration we obtain $A_n = 0$ when n is even, and $A_n = -8/n^2\pi^2$ when n is odd. When $n = 0$ we have $$A_0 = \int_0^2 x\,dx = 2.$$ Hence the full solution is $$\theta = 1 - \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} 8\frac{\cos(n\pi x/2)}{n^2\pi^2}e^{-\alpha n^2\pi^2 t/4}.$$	15
(c)	The sketch is as below.  All curves must have zero slope at $x = 0$ and $x = 1$ for full marks (apart from when $t = 0$).	5
(d)	The value $\tfrac{1}{2}A_0$ is the mean initial temperature *and* the long-term temperature which is achieved.	3
	Total	33

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Part		Mark
(a)	This is bookwork. $\mathcal{F}_s[f''] = \int_0^\infty f'' \sin\omega x \, dx =$ $= \Big[f'\Big]\Big[\sin\omega x\Big]_0^\infty - \Big[f\Big]\Big[\omega\cos\omega x\Big]_0^\infty + \int_0^\infty \Big[f\Big]\Big[-\omega^2\sin\omega x\Big]\, dx$ $= \omega f(0) - \omega^2 F_s(\omega).$	5
(b)	$\mathcal{F}[e^{-x}] = \int_0^\infty e^{-x}\sin\omega x \, dx = \frac{\omega}{1+\omega^2}$ using either integration by parts or by taking the imaginary part of the integral of $e^{-(1-j)x}$.	8
(c)	The application of the Fourier Sine Transform to Fourier's equation together with the zero BC at $x = 0$ gives, $\frac{\partial\Phi_s}{\partial t} = -\alpha\omega^2\Phi_s.$ The general solution is, $\Phi_s = B(\omega)e^{-\alpha\omega^2 t}$. At $t = 0$ we have $\phi = e^{-x}$, and hence $\Phi_s = \omega/(1+\omega^2)$. Therefore $B = \omega/(1+\omega^2)$. Hence the solution is $\Phi_s = \frac{\omega e^{-\alpha\omega^2 t}}{1+\omega^2}$. Applying the inverse Fourier Sine Transform gives $\phi = \frac{2}{\pi}\int_0^\infty \frac{\omega e^{-\alpha\omega^2 t}\sin\omega x}{1+\omega^2}\, d\omega$.	20
	Total	33